Curvature-Squared Terms in 5d Supergravity and AdS/CFT correspondence

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5d Supergravity

- Minimal susy, dubbed $\mathcal{N} = \mathbf{2}$, with eight supercharges [Günaydin-Sierra-Townsend]
- M-theory on Calabi-Yau
- IIB on Sasaki-Einstein

5d Supergravity

- Minimal susy, dubbed $\mathcal{N} = 2$, with eight supercharges [Günaydin-Sierra-Townsend]
- M-theory on Calabi-Yau
- IIB on Sasaki-Einstein

- Rich black objects with horizon topology
  - $\sim S^3$ and
  - $\sim S^2 \times S^1$
- Contains a lot of info about AdS_5/CFT_4 correspondence
  - isometry of AdS = 4d conformal group = $SO(4, 2)$
  - 4d $\mathcal{N} = 1$ SCFT ⟶ eight supercharges
  - a-maximization

a-maximization and 5d supergravity

a-maximization [Intriligator-Wecht]

- method to determine R-symmetry
- superconformal algebra
  $\ni$ one combination $R = r^I G_I$ from $U(1)$ symmetries G_I
- R controls the dynamics → important to know r^I.

Procedure

$$\begin{aligned} c_{IJK} &= \mathrm{tr}\, G_I G_J G_K &: \quad & U(1)_I\text{-}U(1)_J\text{-}U(1)_K \text{ anomaly} \\ c_I &= \mathrm{tr}\, G_I &: \quad & U(1)_I\text{-grav.-grav. anomaly} \end{aligned}$$

↓

Maximize $a(r) = 3c_{IJK} r^I r^J r^K + c_I r^I$!

a-maximization and 5d supergravity

- mapped to the SUSY condition in the AdS side
  [YT, Barnes-Gorbatov-Intriligator-Wright]
- $c_{IJK} = \mathrm{tr}\, G_I G_J G_K \longrightarrow c_{IJK} A^I \wedge F^J \wedge F^K$
- SUSY determines most of the terms in the Lagrangian
- r^I in $R = r^I G_I$ mapped to scalars in the vector multiplet
  $\longrightarrow$ susy vac. condition = a-maximization

a-maximization and 5d supergravity

- mapped to the SUSY condition in the AdS side

  [YT, Barnes-Gorbatov-Intriligator-Wright]
- $c_{IJK} = \mathrm{tr}\, G_I G_J G_K \longrightarrow c_{IJK} A^I \wedge F^J \wedge F^K$
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- r^I in $R = r^I G_I$ mapped to scalars in the vector multiplet

  $\longrightarrow$ susy vac. condition = a-maximization
- $c_I = \mathrm{tr}\, G_I \longrightarrow c_I A^I \wedge \mathrm{tr}\, R \wedge R$.
- Supersymmetric completion of $A \wedge R \wedge R$ terms essential to complete the story.

4d $\mathcal{N} = 2$ supergravity

- [de Wit et al.] : an infinite series of higher derivative 'F-terms',
- studied black hole entropy using Wald's formula
- [Ooguri-Strominger-Vafa] : the relation with topological strings,
- detailed matching of macro- and microscopic blackhole entropy.
- What'll happen in 5d ?

  → We need supersymmetric higher derivative terms !

Today's Objective

- to supersymmetrize $A \wedge \mathrm{tr}\, R \wedge R$ term
- study its effect on a-maximization

- Having an off-shell formulation is helpful.
  → Superconformal Tensor Calculus.

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- to supersymmetrize $A \wedge \mathrm{tr}\, R \wedge R$ term
- study its effect on a-maximization

- Having an off-shell formulation is helpful.
  → Superconformal Tensor Calculus.
- It has a quite long history
- For 5d, it was done by [Kugo-Fujita-Ohashi]
  and by [Bergshoeff-Cucu-Derix-de Wit-Halbersma-Van Proeyen]
  in 2000, 2001

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Approaches to Supergravity

On-shell

Off-shell

- Superspace
- Superconformal tensor calculus

Approaches to Supergravity

To construct higher derivative terms,

On-shell

Need to simultaneously modify the action and the susy tr.

Off-shell

Susy transformation independent of the action

- Superspace
- Superconformal tensor calculus

Superconformal Tensor Calculus

Superspace

- construct superspace
- consider superfields on it
- put constraints on them; expand to components

Superconformal Tensor Calculus

- directly constructs components of multiplets
- rules to combine multiplets to another multiplet
- rules to construct Lagrangian density to be integrated
- It is superconformal.

Steps

1st

Gauge the 5d superconformal group.

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2nd

construct an action invariant under local superconformal symmetry.

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3rd

Compensator gets a vev $\longrightarrow$
symmetry Higgsed to the ordinary Poincaré supergravity.

- Let us study toy versions first.

Gauging the Poincaré group

- Symmetries P_a and M_{ab} ; Gauge fields e^a_μ and ω^{ab}_μ

  $\longrightarrow$ $\nabla_\mu = \partial_\mu - e^a_\mu P_a - \frac{1}{2}\omega^{ab}_\mu M_{ab}$

- Let $[\nabla_\mu, \nabla_\nu] = -\hat{R}^a_{\mu\nu}(P) P_a - \frac{1}{2}\hat{R}^{ab}_{\mu\nu}(M) M_{ab}$

  where $\begin{cases} \hat{R}^a_{\mu\nu}(P) &= \partial_{[\mu} e^a_{\nu]} - \omega^a{}_{b[\mu} e^b_{\nu]}, \\ \hat{R}^{ab}_{\mu\nu}(M) &= \partial_{[\mu}\omega^{ab}{}_{\nu]} - \omega^a{}_{c[\mu}\omega^{cb}_{\nu]}. \end{cases}$

- Impose $\hat{R}^a_{\mu\nu}(P) = 0$ $\longrightarrow$ ω expressed by e^a_μ

- Write $\nabla_\mu \phi = \hat{\mathcal{D}}_\mu \phi - e^a_\mu P_a \phi$; demand $\nabla_\mu \phi = 0$

  $\longrightarrow$ e^a_μ = vielbein, ω^{ab}_μ = spin connection,

  P_a = covariant derivative $\hat{\mathcal{D}}_a$.

Gauging the Conformal group

Generators

P_a	e_μ^a	Translation
M_{ab}	ω_μ^{ab}	Rotation
D	b_μ	Dilation; Weyl tr.
K_a	f_μ^a	Special conformal

Constraints

$$\hat{R}^a_{\mu\nu}(P) = 0, \qquad e^\mu_a \hat{R}^{ab}_{\mu\nu}(M) = 0.$$

- ${\omega^{ab}}_\mu = {\omega_0^{ab}}_\mu - 2e_\mu^{[a}b^{b]}, \qquad f_\mu^a = \alpha R_\mu^a + \beta e_\mu^a R$
- Consider a scalar field ϕ with Weyl weight 1
- form invariant Lagrangian

$$\mathcal{L} = e\phi^{d-3}\hat{\mathcal{D}}^a\hat{\mathcal{D}}_a\phi = e\phi^{d-3}\hat{\mathcal{D}}^a(\mathcal{D}_a\phi - b_a\phi) \sim e\phi^{d-3}\mathcal{D}^a\mathcal{D}_a\phi + e\phi^{d-3}f_a^a\phi$$

- set $\phi = 1 \longrightarrow \mathcal{L} = \sqrt{-g}R.$

Gauging 5d Superconformal Group

Generators

P_a	e_μ^a	Translation
M_{ab}	ω_μ^{ab}	Rotation
D	b_μ	Dilation; Weyl tr.
K_a	f_μ^a	Special conformal
Q_i	ψ_μ^i	Susy
S_i	ϕ_μ^i	Conformal susy
U_{ij}	V_μ^{ij}	$SU(2)$ R-symmetry

Constraints

$$\hat{R}^a_{\mu\nu}(P) = 0,$$
$$e^\mu_a \hat{R}^{ab}_{\mu\nu}(M) = 0,$$
$$\gamma^\mu \hat{R}^i_{\mu\nu}(Q) = 0.$$

Independent gauge fields

e_μ^a, b_μ, ψ_μ^i and V_μ^{ij}.

$\mathbb{W}$: Weyl Multiplet

Components of $\mathbb{W}$

e^a_μ, b_μ, ψ^i_μ, V^{ij}_μ, and v_{ab}, χ^i, D

- $\delta \equiv \delta_Q(\epsilon) + \delta_S(\eta) + \delta_K(\xi_K) = \epsilon^i Q_i + \eta^i S_i + \xi^a_K K_a$ →

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] = \delta_U(-4i\bar{\epsilon}^i_1 \gamma \cdot v \epsilon^j_2) + \cdots$$
$$[\delta_S(\eta), \delta_Q(\epsilon)] = \delta_U(-6i\bar{\epsilon}^{(i}\eta^{j)}) + \cdots$$

- Spinors are $SU(2)$-Majorana, i.e. $\chi^i = \epsilon^{ij} C(\chi^j)^*$

$\mathbb{V}$: Vector Multiplet

Components of $\mathbb{V}$

$$W^I_\mu, \quad M^I, \quad \Omega^I_i, \quad Y^I_{ij}$$

- Contain a gauge field W^I_a for $G = U(1)^n$, $I = 1, \ldots, n$
- $\delta W_\mu = -2i\bar{\epsilon}^i\gamma_\mu\Omega_i - 2i\bar{\epsilon}^i\psi_{i\mu}M$

  $\longrightarrow \; f_{GQ}{}^Q \sim f_{QQ}{}^G \sim M \; \longrightarrow$

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] = \delta_U(-4i\bar{\epsilon}^i_1\gamma\cdot v\epsilon^j_2) + \cdots + \delta_G(-2i\bar{\epsilon}_1\epsilon_2 M)$$

$\mathbb{L}$: Linear Multiplet

Components of $\mathbb{L}$

$$L^{ij}, \quad \varphi^i, \quad E_a, \quad N$$

- Contain a conserved current $\hat{\mathcal{D}}_a E^a = 0$.
- 'dual' to vector multiplet, compare the components.
- $\delta L^{ij} = 2i\bar{\epsilon}^{(i}\varphi^{j)}$
- $SU(2)$-triplet, S-invariant scalar L^{ij} generates a linear multiplet.

Two Formulae

$\mathbb{V} \cdot \mathbb{L}$ action formula

- $\mathbb{V}$: vector multiplet, $\mathbb{L}$: linear multiplet
  → invariant action $\mathcal{L}(\mathbb{V} \cdot \mathbb{L})$
- Supersymmetrization of $W_a E^a$.

$\mathbb{V} \times \mathbb{V}' \longrightarrow \mathbb{L}$ Embedding formula

- $\mathbb{V}$, $\mathbb{V}'$: vector multiplets → $\mathbb{L}[\mathbb{V}, \mathbb{V}']$: linear multiplet
- Supersymmetrization of $W_a, W'_a \to E_a = \epsilon_{abcde} F^{bc} F'^{de}$.

Action for the Vector Multiplet

- Combine $\mathbb{V}^J \times \mathbb{V}^K \to \mathbb{L}^{JK}$ and the $\mathbb{V}^I \cdot \mathbb{L}^{JK}$ action formula.
- $E^a[\mathbb{L}^{JK}] = \epsilon^{abcde} F^J_{bc} F^K_{de}$
  → Supersymmetrization of $\epsilon^{abcde} W^I_a F^J_{bc} F^K_{de}$!
- symmetric in I, J, K
  → defined by a purely cubic $\mathcal{N} = c_{IJK} M^I M^J M^K / 6$

Comparison to 4d

- Chiral multiplet $\mathbb{C}_w$ with Weyl weight w, which is annihilated by chiral half of Q.
- $\mathbb{C}_w \times \mathbb{C}_{w'} \longrightarrow \mathbb{C}_{w+w'}$
- F-term Action formula $\mathcal{L}_F(\mathbb{C}_2)$
- Embedding $\mathbb{V} \longrightarrow \mathbb{C}_1[\mathbb{V}]$ and $\mathbb{C}_1 \longrightarrow \mathbb{L}[\mathbb{C}_1]$
  cf. $V \longrightarrow W_\alpha$ in 4d susy
- $\mathbb{V}^I$: vector multiplets, let $\mathbb{X}^I = \mathbb{C}_1[\mathbb{V}^I]$
- Take $F(\mathbb{X})_2$: arbitrary degree 2
  $\longrightarrow$ invariant action $\mathcal{L}_F(F(\mathbb{X}))$

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- $\mathbb{V}^I$: vector multiplets, let $\mathbb{X}^I = \mathbb{C}_1[\mathbb{V}^I]$
- Take $F(\mathbb{X})_2$: arbitrary degree 2
  ⟶ invariant action $\mathcal{L}_F(F(\mathbb{X}))$ which satisfies

$$\mathcal{L}_F(F(\mathbb{X})) = \mathcal{L}_F(\mathbb{X}^I \cdot F_I(\mathbb{X})) = \mathcal{L}(\mathbb{V}^I \cdot \mathbb{L}[F_I(\mathbb{X})])$$

- only the last form available in 5d and in 6d

$\mathbb{H}$: Hypermultiplet

Components of $\mathbb{H}$

$$\mathcal{A}^i_\alpha, \quad \zeta_\alpha, \quad \mathcal{F}^i_\alpha$$

- $\alpha = 1, 2, \ldots, 2r$, acted by $USp(2r)$
- $\mathcal{A}^i_\alpha = \epsilon^{ij} J_{\alpha\beta} (\mathcal{A}^j_\beta)^*$
- $\delta\zeta_\alpha = \cdots - P_I^{\alpha\beta} M^I \mathcal{A}^j_\beta \epsilon_j$
  where $P_I^{\alpha\beta}$ the charge matrix of I-th $U(1)$
- We use with $r = 1$ as the compensator.
- Used to fix extra symmetries.

Action

Bosonic part of the Action

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\mathbb{V}} &= \mathcal{N}\left(-\tfrac{1}{2}D + \tfrac{1}{4}R - 3v^2\right) + \mathcal{N}_I\left(-2v^{ab}F^I_{ab}\right) \\
&\quad + \mathcal{N}_{IJ}\left(-\tfrac{1}{4}F^I_{ab}F^{abJ} + \tfrac{1}{2}\mathcal{D}^a M^I \mathcal{D}_a M^J + Y^I_{ij}Y^{Jij}\right) \\
&\quad - e^{-1}\tfrac{1}{24}\epsilon^{\lambda\mu\nu\rho\sigma}\mathcal{N}_{IJK}W^I_\lambda F^J_{\mu\nu}F^K_{\rho\sigma}. \\
e^{-1}\mathcal{L}_{\mathbb{H}} &= \mathcal{D}^a\mathcal{A}^{\bar\alpha}_i\mathcal{D}_a\mathcal{A}^i_\alpha + \mathcal{A}^{\bar\alpha}_i(PM)^2\mathcal{A}^i_\alpha \\
&\quad + \mathcal{A}^2\left(\tfrac{1}{8}D + \tfrac{3}{16}R - \tfrac{1}{4}v^2\right) + 2P_{I\alpha\beta}Y^{Iij}\mathcal{A}^{\bar\alpha}_i\mathcal{A}^\beta_j
\end{aligned}
$$

- Gauge fix & Eliminate Auxiliaries
  → Reproduces [Günaydin-Sierra-Townsend]

Symmetry breaking pattern

Action

$$e^{-1}\mathcal{L}_{\mathbb{H}} = \mathcal{D}^a \mathcal{A}_i^{\bar{\alpha}} \mathcal{D}_a \mathcal{A}_\alpha^i + \mathcal{A}_i^{\bar{\alpha}} (PM)^2 \mathcal{A}_\alpha^i + \mathcal{A}^2 \left(\frac{1}{8}D + \frac{3}{16}R - \frac{1}{4}v^2\right) + 2P_{I\alpha\beta} Y^{Iij} \mathcal{A}_i^{\bar{\alpha}} \mathcal{A}_j^\beta$$

- Suppose $P_{I\alpha\beta} = P_I i\sigma^3_{\alpha\beta}$.
- $\mathcal{D}_\mu = \partial_\mu + V_\mu^{ij} - P_{I\alpha\beta} W_\mu^I$.
- fix $\mathcal{A}_\alpha^i \propto \delta_\alpha^i$
  → $SU(2)_R \times U(1)^n$ broken to $U(1)^n$.
- effectively sets $V_\mu^{ij} = i\sigma^{3ij} P_I W_\mu^I$

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Strategy

Supersymmetrize $W^I \wedge \mathrm{tr}\, R \wedge R$

- Weyl multiplet $\mathbb{W}$
  → Linear multiplet $\mathbb{L}[\mathbb{W}^2]$ with $E^a \sim \epsilon^{abcde} R^f{}_{gbc} R^g{}_{fde}$
- Use $\mathbb{V} \cdot \mathbb{L}$ action formula
  → $c_I W^I_a E^a \sim c_I W^I \,\mathrm{tr}\, R \wedge R$

- $R(Q) \xrightarrow{Q} R(M)$ so that

$$L_{ij} \xrightarrow{Q} \phi \xrightarrow{Q} \begin{array}{c} E_a \\ \Cup \\ R(M)R(M) \end{array}$$

- L_{ij} should be triplet, Weyl weight 3, covariant :

$$L^{ij}[\mathbb{W}^2] = i\bar{\hat{R}}_{ab}{}^{(i}(Q)\hat{R}^{abj)}(Q) + A_1 i\bar{\chi}^{(i}\chi^{j)} + A_2 v^{ab}\hat{R}_{ab}{}^{ij}(U)$$

- $\delta_S L_{ij} = 0$ fixes $A_1 = \frac{1}{12}$ and $A_2 = -\frac{4}{3}$.
- Get other components by SUSY variation !

- $R(Q) \xrightarrow{Q} R(M)$ so that

$$
\begin{array}{ccccc}
L_{ij} & \xrightarrow{Q} & \phi & \xrightarrow{Q} & E_a \\
 & & \Cup & & \Cup \\
 & & R(Q)R(M) & \xrightarrow{Q} & R(M)R(M)
\end{array}
$$

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- Get other components by SUSY variation !

Result

$\mathbb{W} \times \mathbb{W} \to \mathbb{L}$ formula

$$
\begin{aligned}
L^{ij} &= i\bar{\hat{R}}_{ab}{}^{(i}(Q)\hat{R}^{abj)}(Q) + \tfrac{1}{12}i\bar{\chi}^{(i}\chi^{j)} - \tfrac{4}{3}v^{ab}\hat{R}_{ab}{}^{ij}(U),\\
\varphi^i &= \cdots,\\
E_a &= -\tfrac{1}{8}\epsilon_{abcde}\hat{R}^{bcfg}(M)\hat{R}^{de}{}_{fg}(M) + \tfrac{1}{6}\epsilon_{abcde}\hat{R}^{bcij}(U)\hat{R}^{de}{}_{ij}(U)\\
&\quad + \hat{\mathcal{D}}^b\Big(-\tfrac{2}{3}v_{ab}D + 2\hat{R}_{abcd}(M)v^{cd} - \tfrac{8}{3}\epsilon_{abcde}v^{cf}\hat{\mathcal{D}}_f v^{de}\\
&\quad - 4\epsilon_{abcde}v^c{}_f\hat{\mathcal{D}}^d v^{ef} + \tfrac{16}{3}v_{ac}v^{cd}v_{db} + \tfrac{4}{3}v_{ab}v^2\Big),\\
N &= \tfrac{1}{6}D^2 + \tfrac{1}{4}\hat{R}^{abcd}(M)\hat{R}_{abcd}(M) - \tfrac{2}{3}\hat{R}_{abij}(U)\hat{R}^{abij}(U)\\
&\quad - \tfrac{2}{3}\hat{R}_{abcd}(M)v^{ab}v^{cd} + \tfrac{16}{3}v_{ab}\hat{\mathcal{D}}^b\hat{\mathcal{D}}_c v^{ac} + \tfrac{8}{3}\hat{\mathcal{D}}^a v^{bc}\hat{\mathcal{D}}_a v_{bc}\\
&\quad + \tfrac{8}{3}\hat{\mathcal{D}}^a v^{bc}\hat{\mathcal{D}}_b v_{ca} - \tfrac{4}{3}\epsilon_{abcde}v^{ab}v^{cd}\hat{\mathcal{D}}_f v^{ef}\\
&\quad + 8v_{ab}v^{bc}v_{cd}v^{da} - 2(v_{ab}v^{ab})^2.
\end{aligned}
$$

Comments

E_a term

$$E_a[\mathbb{W}^2] = \epsilon_{abcde}\left(-\tfrac{1}{8}\hat{R}^{bcfg}(M)\hat{R}^{de}{}_{fg}(M) + \tfrac{1}{6}\hat{R}^{bcij}(U)\hat{R}^{de}{}_{ij}(U)\right)$$
$$+ \hat{\mathcal{D}}^b\left(-\tfrac{2}{3}v_{ab}D + 2\hat{R}_{abcd}(M)v^{cd} - \tfrac{8}{3}\epsilon_{abcde}v^{cf}\hat{\mathcal{D}}_f v^{de}\right.$$
$$\left. - 4\epsilon_{abcde}v^c{}_f\hat{\mathcal{D}}^d v^{ef} + \tfrac{16}{3}v_{ac}v^{cd}v_{db} + \tfrac{4}{3}v_{ab}v^2\right)$$

- $\hat{\mathcal{D}}^a E_a = 0$: check of consistency
- $-\frac{1}{8}\,\mathrm{tr}\, R(M)\wedge R(M)$ accompanied by $\frac{1}{6}\,\mathrm{tr}\, R(U)\wedge R(U)$
- $R(U)^{ij}_{ab}$: curvature of $SU(2)_R$ gauge field
- $\hat{R}(M)^{ab}{}_{bc} = 0$

  → bosonic components of $\hat{R}(M)_{abcd}$ = Weyl tensor.

Correction to the Chern-Simons

CS terms

$$c_I W_a^I E^a[\mathbb{W}^2]$$
$$= \epsilon_{abcde} c_I W^{aI} \left(\frac{1}{16} \hat{R}^{bcfg}(M) \hat{R}^{de}{}_{fg}(M) - \frac{1}{12} \hat{R}^{bc}{}_{jk}(U) \hat{R}^{dejk}(U) \right)$$
$$= \epsilon_{abcde} c_I W^{aI} \left(\frac{1}{16} \hat{R}^{bcfg}(M) \hat{R}^{de}{}_{fg}(M) - \frac{1}{6} P_J P_K F^{Jbc} F^{Kde} \right)$$

- Recall $V_{ij\mu} = P_I(i\sigma^3_{ij}) W^I_\mu$
  $\longrightarrow \hat{R}(U)^{ij}_{ab} = P_I(i\sigma^3_{ij}) F^I_{ab}$.
- P_I: charge of the compensator under I-th $U(1)$.
- $c_{IJK} \to c_{IJK} + c_{(I} P_J P_{K)}$.

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4d superconformal algebra

Susy algebra

$$\{Q_\alpha, Q^\dagger_\beta\} \sim P_\mu \gamma^\mu_{\alpha\beta}$$

Conformal algebra

$$[K_\mu, P_\nu] \sim \delta_{\mu\nu} D$$

where $D : x^\mu \to (1+\epsilon)x^\mu$

4d superconformal algebra

Susy algebra

$\{Q_\alpha, Q^\dagger_\beta\} \sim P_\mu \gamma^\mu_{\alpha\beta}$

Conformal algebra

$[K_\mu, P_\nu] \sim \delta_{\mu\nu} D$
where $D : x^\mu \to (1+\epsilon)x^\mu$

Superconformal algebra in 4d

- $[K_\mu, Q_\alpha] \sim \gamma^\mu_{\alpha\beta} S^\beta$
- $\{Q_\alpha, S_\beta\} \sim \epsilon_{\alpha\beta} R$
- R controls the dynamics, central charge, ...
- $R = r^I G_I$, $\quad G_I$: I-th $U(1)$ charge
- How do we determine r^I ?

a-maximization

Input

- $\hat{c}_{IJK} = \mathrm{tr}\, G_I G_J G_K$: $U(1)_I$-$U(1)_J$-$U(1)_K$ anomaly
- $\hat{c}_I = \mathrm{tr}\, G_I$: $U(1)_I$-grav.-grav. anomaly
- $[G_I, Q_\alpha] = P_I Q_\alpha$

Output

- Form $a(r^I) = 3\hat{c}_{IJK} r^I r^J r^K - \hat{c}_I r^I$
- Maximize it under the condition $P_I r^I = 1$
- Bonus: a at the maximum = central charge

Translation of Parameters

't Hooft anomaly → Chern-Simons

$$\hat{c}_{IJK} = \mathrm{tr}\, G_I G_J G_K \longrightarrow \hat{c}_{IJK} W^I \wedge F^J \wedge F^K$$
$$\hat{c}_I = \mathrm{tr}\, G_I \longrightarrow \hat{c}_I W^I \wedge \mathrm{tr}\, R \wedge R$$

- We'll see $[G_I, Q_\alpha] = P_I Q_\alpha \longrightarrow P_I$: charge of the compensator
- We'll also see $R = r^I G_I \longrightarrow r^I \propto M^I$
- a-maximization should arise as the susy condition ...

Supersymmetric AdS solutions

- Finally hard work really pays off.
- 4d superconformal algebra $\subset$ local 5d superconformal algebra !
- Suppose the metric is AdS with radius L

  ⟶ eight solutions to $\mathcal{D}_\mu \epsilon + \frac{i}{2L}\gamma_\mu \epsilon = 0$
- Its complex conjugate : $\mathcal{D}_\mu \epsilon^* - \frac{i}{2L}\gamma_\mu \epsilon^* = 0$
- $SU(2)$-Majorana spinor ϵ satisfies $(\epsilon^1)^* = \epsilon^2$

  ⟶ need to choose $\vec{q}$: constant triplet of $SU(2)_R$ to have

$$D_\mu \epsilon^i - \frac{1}{2L}\gamma_\mu i(\vec{q}\cdot\vec{\sigma})^i{}_j \epsilon^j = 0$$

Gravitino

Gravitino Variation

$$\delta\psi^i_\mu = \mathcal{D}_\mu\epsilon^i - \gamma_\mu\eta^i$$

- Remaining susy: $\delta'_Q(\epsilon) = \delta_Q(\epsilon) + \delta_S(\eta)$
- where $\eta^i = \frac{1}{2L}(i\vec{q}\cdot\vec{\sigma})^i{}_j\epsilon^j$
- $\delta\chi = 0 \longrightarrow D = 0$.

Hyperino Variation

$$\delta\zeta^\alpha = -P_I{}^\alpha_\beta M^I \mathcal{A}^\alpha_i \epsilon^i + 3\mathcal{A}^\alpha_i \eta^i$$

- $\mathcal{A}^\alpha_i \propto \delta^\alpha_i$
- $\eta^i = \frac{1}{2L}(i\vec{q}\cdot\vec{\sigma})^i{}_j \epsilon^j$

  → $P_{I\alpha\beta} = P_I(i\vec{q}\cdot\vec{\sigma})_{\alpha\beta}$ and $L = \frac{3}{2}(P_I M^I)^{-1}$

4d and 5d algebra

5d Commutators

$$\begin{aligned}
[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] &= \delta_G(-2i\bar{\epsilon}_1\epsilon_2 M) \\
[\delta_Q(\epsilon), \delta_S(\eta)] &= \delta_U(-6i\bar{\epsilon}^{(i}\eta^{j)}) \\
[\delta_U(\sigma^i_j), \delta_Q(\epsilon)] &= \delta_Q(\sigma^i_j\epsilon^j)
\end{aligned}$$

Unbroken Generators

$$\begin{aligned}
\delta'_Q(\epsilon) &= \delta_Q(\epsilon) + \delta_S(\eta), \\
\delta'_G(\theta) &= \delta_G(\theta) - \delta_U(\sigma)
\end{aligned}$$

where

$$\begin{aligned}
\eta^i &= (i\vec{q}\cdot\vec{\sigma})^i{}_j\epsilon^j/(2L) \\
\sigma &= P_I\theta^I(i\vec{q}\cdot\vec{\sigma})^{ij}
\end{aligned}$$

4d and 5d algebra

5d Commutators

$$\begin{aligned} [\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] &= \delta_G(-2i\bar\epsilon_1\epsilon_2 M) \\ [\delta_Q(\epsilon), \delta_S(\eta)] &= \delta_U(-6i\bar\epsilon^{(i}\eta^{j)}) \\ [\delta_U(\sigma^i_j), \delta_Q(\epsilon)] &= \delta_Q(\sigma^i_j\epsilon^j) \end{aligned}$$

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4d commutators

$$\begin{aligned} [\delta'_Q(\epsilon), \delta'_Q(\epsilon')] &= \delta'_G(-2iM^I\bar\epsilon\epsilon') \\ [\delta'_G(\theta), \delta'_Q(\epsilon')] &= \delta'_Q(P_I\theta^I(i\vec q\cdot\vec\sigma)^i{}_j\epsilon^j) \end{aligned}$$

- $\delta'_Q(\epsilon)$ gives Q_{4d} and S_{4d}

  → $[G_I, Q_{4d}] = P_I Q_{4d}$, $[Q_{4d}, S_{4d}] \sim M^I G_I$
- $r^I \sim M^I$!

Gaugino

Gaugino Variation

$$\delta\Omega^I_i = Y_{ij}\epsilon^j - M\eta_i$$

- $\eta^i = \frac{1}{2L}(i\vec{q}\cdot\vec{\sigma})^i{}_j\epsilon^j \longrightarrow Y^I_{ij} = (i\vec{q}\cdot\vec{\sigma})_{ij}M^I$
- Y^I_{ij} determined by EOM.
- This is the ONLY place where the detailed action comes in !

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- $\mathcal{L}_{\text{tot}} = \mathcal{L}(c_{IJK}\mathbb{V}^I\cdot\mathbb{L}[\mathbb{V}^J,\mathbb{V}^K]) + \mathcal{L}(c_I\mathbb{V}^I\cdot\mathbb{L}[\mathbb{W}^2]) + \mathcal{L}_{\mathbb{H}}$

  $\longrightarrow P_I = \frac{3}{2L}c_{IJK}M^JM^K$ independent of c_I

Susy condition

Chern-Simons terms

			$\mathcal{L}(\mathbb{V}\cdot\mathbb{L}[\mathbb{V},\mathbb{V}])$	$+\mathcal{L}(\mathbb{V}\cdot\mathbb{L}[\mathbb{W}^2])$
$W^I F^J F^K$	$\frac{1}{24\pi^2}\hat{c}_{IJK}$	$=$	$\frac{1}{2}c_{IJK}$	$+\frac{2}{3}c_{(I}P_J P_{K)}$
$W^I \,\mathrm{tr}\, RR$	$\frac{1}{192\pi^2}\hat{c}_I$	$=$		$-\frac{1}{4}c_I$

- EOM: $P_I = \frac{3}{2L}c_{IJK}M^J M^K$
  → $P_I \propto 3\hat{c}_{IJK}M^J M^K - \hat{c}_{(I}P_J P_{K)}M^J M^K$
- which is exactly what you get if you maximize

$$3\hat{c}_{IJK}M^I M^J M^K - \hat{c}_I M^I P_J M^J P_K M^K$$

under the condition $P_I M^I = 1$.

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under the condition $P_I M^I = 1$.
- $a(r^I) = \frac{3}{32}(3\hat{c}_{IJK}r^I r^J r^K - \hat{c}_I r^I)$!

Value of a at the maximum

Formula for a and c

$$e^{-1}\mathcal{L} = \frac{1}{2}\left(\frac{12}{L^2} - \frac{80\alpha+16\beta+8\gamma}{L^4} - R + \alpha R^2 + \beta R_{\mu\nu}^2 + \gamma R_{\mu\nu\rho\sigma}^2\right),$$

$$\longrightarrow \quad \begin{aligned} a &= \pi^2 L^3 (1 - 40\alpha - 8\beta - 4\gamma)\,, \\ c &= \pi^2 L^3 (1 - 40\alpha - 8\beta + 4\gamma)\,. \end{aligned}$$

- Derived from $\langle TT \rangle$ correlator.
- For us, $(\alpha, \beta, \gamma) = \frac{1}{4} c_I M^I (\frac{1}{6}, -\frac{4}{3}, 1)$ $\longrightarrow$

$$\begin{aligned} a &= \pi^2 L^3 = \frac{27}{8}\pi^2 (M_I P^I)^{-3} = \frac{27}{8}\pi^2 c_{IJK} r^I r^J r^K \\ &= \frac{3}{32}\left(3\hat{c}_{IJK} r^I r^J r^K - \hat{c}_I r^I\right) \end{aligned}$$

where $r^I \propto M^I$ is normalized so that $P_I r^I = 1$
and $c_{IJK} M^I M^J M^K = 1$ was used.

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- More application should be possible.
- Black rings [Castro-Davis-Kraus-Larsen, 0702072]