How does the near-horizon geometry of black rings encode its charges ?

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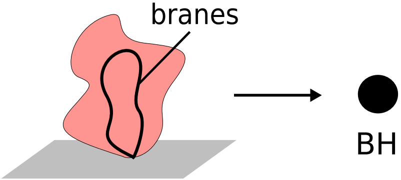
Contents

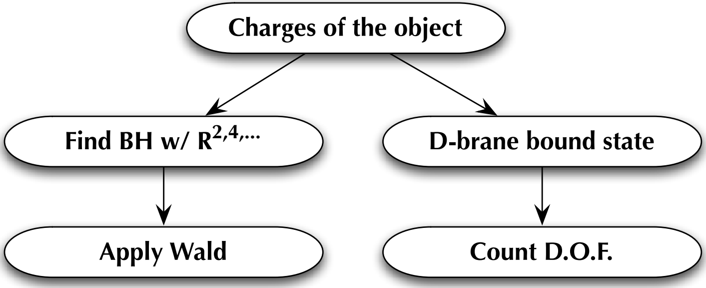
Some success of string theory

- Open string d.o.f. accounts for Bekenstein-Hawking entropy for certain extremal Black Holes in 4D



- Even subleading corrections from Wald's formula reproduced

Matching of the entropy



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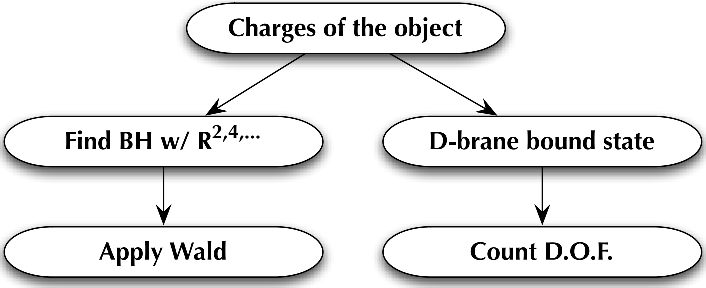
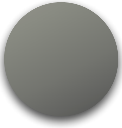
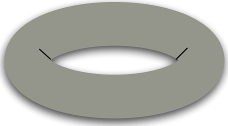


The two methods give the same entropy !

Black things in 5D

- Uniqueness theorem in 4D
- In 5D, the horizon can be S^3 or $S^2 \times S^1$, Black Rings.





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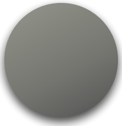

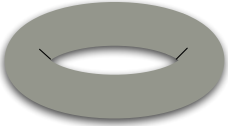

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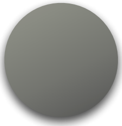
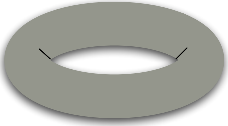
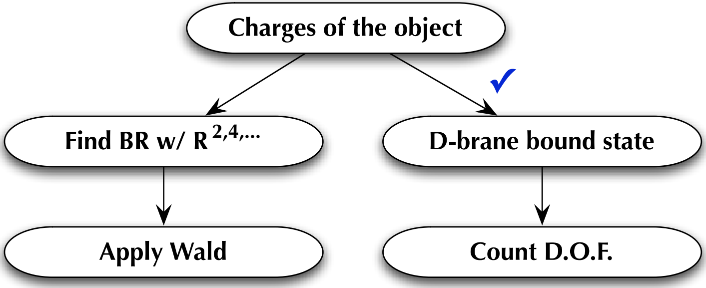




- More fun !

- Entropy matching should be even more interesting.
- However ...

Matching of the entropy in 5D



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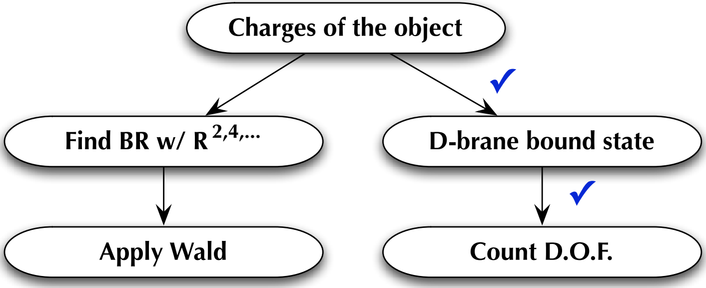

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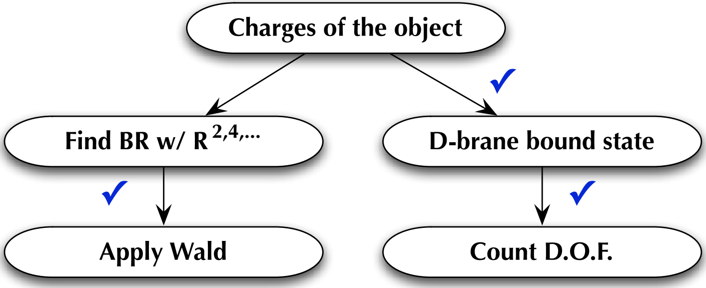

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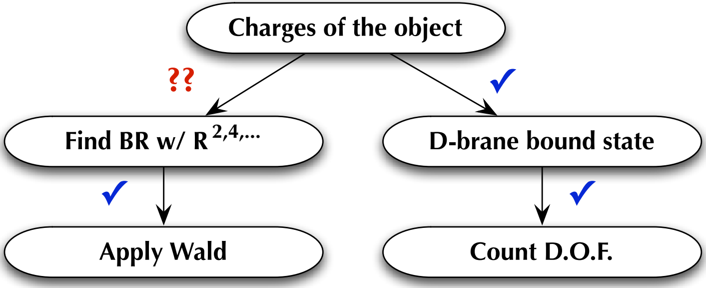

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- Assume BR is Extremal $\longrightarrow$ Near-horizon is $AdS_3 \times S^2$
- Charges ??

Dreaded Chern-Simons

5d minimal supergravity

$$S = \frac{1}{8\pi G}\int\left(\frac{1}{2}\star R - F\wedge\star F - \frac{4}{3\sqrt{3}}A\wedge F\wedge F\right)$$

- $d\star F = F\wedge F \longrightarrow \int_\Sigma \star F$ dependent on Σ
- Electric charge distributes throughout the spacetime outside the near-horizon region
- Also true for angular momenta

Page charge comes to the rescue ?

- $d\,(\star F - A \wedge F) = 0$
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Page charge comes to the rescue ?

- $d\,(\star F - A \wedge F) = 0$
  $\longrightarrow \displaystyle\int_\Sigma (\star F - A \wedge F)$ independent on Σ
- Called the Page charge
- Not well-defined near horizon because
  - $\displaystyle\int_{S^2} F$ is non-zero
  - and thus A is not a globally well-defined one-form
- More complicated for angular momenta

Yes !

- We solve the problem by using two coordinate patches

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- BH, BR in $\mathbb{R}^{3,1} \times S^1 \Longleftrightarrow$ BH in $\mathbb{R}^{3,1}$
- Large gauge transformation $\Longleftrightarrow$ Witten effect on dyons

Contents

the Action

Minimal Supergravity in 5D

$$S = \frac{1}{8\pi G}\int \left(\frac{1}{2}\star R - F\wedge\star F - \frac{4}{3\sqrt{3}}A\wedge F\wedge F\right)$$

EOM

$$R_{\mu\nu} = -\frac{1}{3}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma} + 2F_{\mu\rho}F_{\nu}{}^{\rho},$$
$$d\star F = -\frac{2}{\sqrt{3}}F\wedge F.$$

Electric Charge

$$F_{\mu\nu} = \begin{pmatrix} B_{ij} & E_i \\ -E_j & 0 \end{pmatrix} \quad \longrightarrow \quad Q = \int_\infty d\vec{S} \cdot \vec{E} = \int_\infty \star F$$

defined at asymptotic infinity

gauge-invariant and conserved

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$$\int_\infty \star F - \int_\Sigma \star F = \int_B d \star F = \int_B F \wedge F$$

$\longrightarrow$ $\rho^\mu = \epsilon^{\mu\nu\rho\sigma\tau} F_{\nu\rho} F_{\sigma\tau}$ is the electric current

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charge diffusely distributed throughout the space

Page charge

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Page charge

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- introduced by D. Page
- $A \wedge F$ taken to vanish sufficiently fast near ∞ $\longrightarrow$

$$\int_\infty \star F = \int_\infty (\star F - A \wedge F)$$

- invariant under small gauge transformation $A \to A + d\chi$ because

$$\int_\Sigma (A + d\chi) \wedge F = \int_\Sigma A \wedge F + d(\chi \wedge F) = \int_\Sigma A \wedge F.$$

Gauge Non-invariance of Page charge

- Not invariant under Large gauge transformation $A \to A + g^{-1}dg$ where $g = \exp(i\phi)$ winds nontrivially along some path C

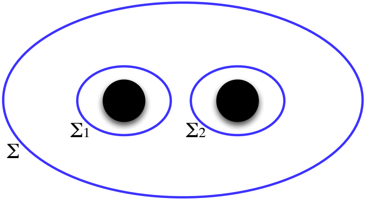
$$\int_{C\times S^2} (g^{-1}dg) \wedge F = (\phi(2\pi) - \phi(0)) \int_{S^2} F$$
$$= (2\pi)^2 \times \text{integer}$$

- can be fixed at asymptotic infinity: $g^{-1}dg$ only decay $\sim 1/r$

Gauge Non-invariance of Page charge

- cannot really be fixed if there are multiple black objects :

$$\underbrace{\int_{\Sigma} (\star F - A \wedge F)}_{Q} = \underbrace{\int_{\Sigma_1} (\star F - A \wedge F)}_{Q_1} + \underbrace{\int_{\Sigma_2} (\star F - A \wedge F)}_{Q_2}$$



- Q can be fixed, but Q_1 and Q_2 might change.

Even worse.

- A not a globally well-defined one-form if $\displaystyle\int_{S^2} \neq 0$

  $\longrightarrow$ $\displaystyle\int A \wedge F$ meaningless.

- Way out :

1. Introduce the Dirac surface.

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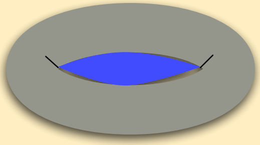

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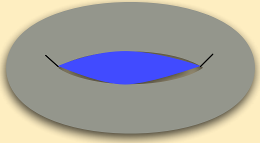
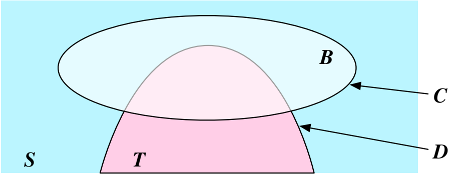


: extends outside near-horizon region.

2. Introduce two coordinate patches.

Solution



Let $A_S = A_T + \beta$ where $\beta = g^{-1}dg$

$$
\begin{aligned}
\int_B F \wedge F &= \int_{B\cap S} F \wedge F + \int_{B\cap T} F \wedge F \\
&= \int_{C\cap S + D\cap B} A_S \wedge F + \int_{C\cap T - D\cap B} A_T \wedge F \\
&= \left(\int_{C\cap S} A_S \wedge F + \int_{C\cap T} A_T \wedge F\right) + \int_{D\cap B} (A_S \wedge F - A_T \wedge F) \\
&= \left(\int_{C\cap S} A_S \wedge F + \int_{C\cap T} A_T \wedge F\right) + \int_{C\cap D} A_S \wedge \beta.
\end{aligned}
$$

Solution

- Define

$$“\int_M A \wedge F” \equiv \int_{M\cap S} A_S \wedge F + \int_{M\cap T} A_T \wedge F + \int_{D\cap M} A_S \wedge \beta$$

- Then

$$\int_\infty (\star F - A \wedge F) = “\int_\Sigma (\star F - A \wedge F)”$$

where you can take Σ to be the horizon.

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- Still non-invariant under large gauge transformation.

Angular momenta

- Assume spacetime to be stationary,
  let ξ be Killing vector generating rotation
- Komar's formula
$$J_\xi = -\frac{1}{16\pi G}\int_\infty \star\nabla\xi$$
  where $\nabla\xi = \nabla_\mu\xi_\nu dx^\mu \wedge dx^\nu = d\xi$.
- $d \star d\xi = 2 \star R_{\mu\nu}\xi^\mu dx^\nu$ $\longrightarrow$
  Komar's integral independent of the surface for vacuum gravity
  not independent otherwise.

Wald's Noether charge

- Wald's analysis guarantees one can always find

$$\begin{aligned} d \star \nabla \xi &= 2 \star R_{\mu\nu} \xi^\mu dx^\nu \\ &= 4 \star F_{\mu\rho} F^\rho{}_\nu \xi^\mu dx^\nu = d(\text{something}) \end{aligned}$$

so that

$$\int_\Sigma (\star \nabla \xi + \text{something})$$

is independent of Σ.

- that "something" is

$$\star(\xi \cdot A) F + \frac{4}{3\sqrt{3}} (\xi \cdot A) A \wedge F.$$

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- Not well-defined because A is not globally well defined at the horizon.

Two patches

- can be made well-defined using two patches as before.
- rather than being concrete, let us give the general prescription:

1. $d\omega = \rho$ where $\rho = F \wedge F$ and/or $2 \star R_{\mu\nu}\xi^\mu dx^\nu$.

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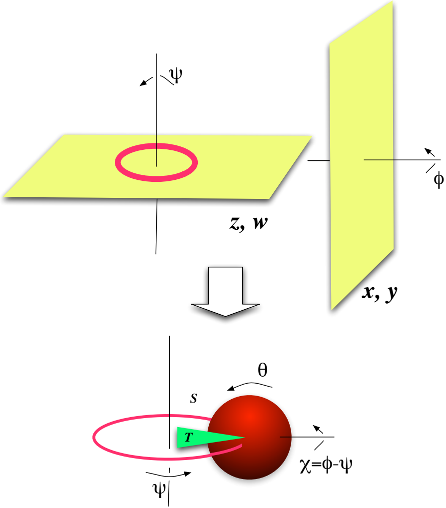
6. which is the Descent equation !

Checks

Supersymmetric Single Black Ring:



$$A_S = -\left(q + \frac{Q}{q}\right) d\psi - q\cos\theta d\chi,$$
$$A_T = A_S + 2q d\phi$$

electric charge

$$= \int_{S\cap\Sigma} A_S \wedge F + \int_{D\cap\Sigma} A_T \wedge A_S$$
$$= \frac{Q+q^2}{2} + \frac{Q-q^2}{2} = Q$$

Checks

- Supersymmetric Multiple Black Rings

$$\mathrm{Q}_{\text{total}} = \frac{\sqrt{3}\pi}{2G}\left[\sum_{i=1}^{N}\left(Q_i - q_i^2\right) + \left(\sum_{i=1}^{N} q_i\right)^2\right],$$

  where Q_i and q_i are the electric and magnetic charges of i-th ring.
- "Interaction" comes from the compensating term

Contents

Black Ring in Taub-NUT

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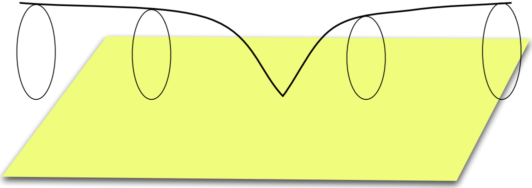
- Let us instead consider BR in $\mathbb{R}_t \times$ Euclidean Taub-NUT



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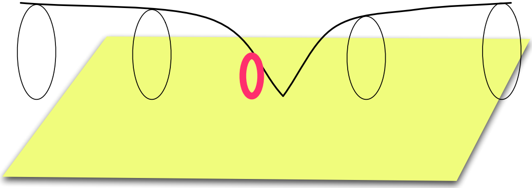

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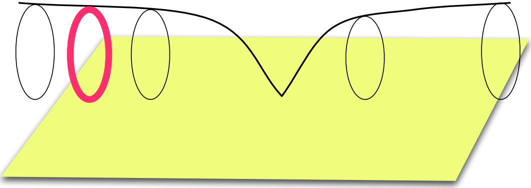

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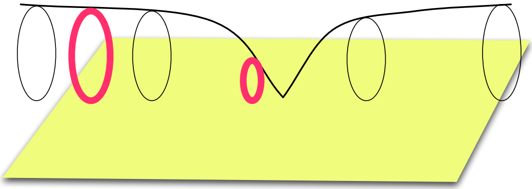

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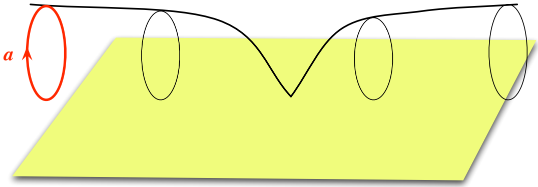
- Near infinity, it's just a Black String wound in KK circle
- Near-horizon geom. the same in proper coordinates

Gauge dependence



- $a = \int A_\psi d\psi$ is a scalar field in 4d
- "$a(x) \to 0$ when $|x| \to \infty$" is not the right thing
- large gauge transformation $A \to A + g^{-1}dg$

  $\longrightarrow$ $a \to a + 2\pi n$
- charge q, electric charge Q, angular momentum J changes

$$(q, Q, J) \to (q, Q + q, J + q^2 + qQ)$$

because $Q \supset \int AF, \qquad J \supset \int AAF$

KK reduction

$$S_{5D} = \frac{1}{2} \star R - F^I \wedge \star F^J - \frac{4}{3\sqrt{3}} A \wedge F \wedge F^K$$

$+$

$$ds^2_{5d} = e^{2\rho}(d\psi - A^0)^2 + e^{-\rho} ds^2_{4d}$$
$$A_{5d} = a(d\psi - A^0) + A^1_{4d}$$

$\downarrow$

$$\begin{aligned} S_{\text{gauge fields}} = &\left[\frac{1}{2}e^{3\rho} + 2e^\rho a^2\right] F^0 \wedge \star F^0 - \left(\frac{2a}{\sqrt{3}}\right)^3 F^0 \wedge F^0 \\ &+ 4e^\rho a F^0 \wedge \star F^1 + \frac{4}{\sqrt{3}} a^2 F^0 \wedge F^1 \\ &- 2e^\rho F^1 \wedge \star F^1 - \frac{4}{\sqrt{3}} a F^1 \wedge F^1 \end{aligned}$$

Scalar-dependent θ term

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- $\epsilon^{\mu\nu\rho\sigma}\theta F_{\mu\nu}F_{\rho\sigma}$ with scalar dependent θ

$$-\left(\frac{2a}{\sqrt{3}}\right)^3 F^0 \wedge F^0 + \frac{4}{\sqrt{3}}a^2 F^0 \wedge F^1 - \frac{4}{\sqrt{3}}aF^1 \wedge F^1$$

Witten Effect

- Consider $\mathcal{L} = F_{\mu\nu}F^{\mu\nu} + \theta\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$
- θ doesn't change the EOM, but changes

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- Adiabatic change $\theta \to \theta + 2\pi$ brings $(Q, q) \longrightarrow (Q + 2\pi q, q)$

5d vs 4d

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- Q^0 : KK electric charge = J : angular momentum along ψ
- $a = \int A_\psi d\psi$

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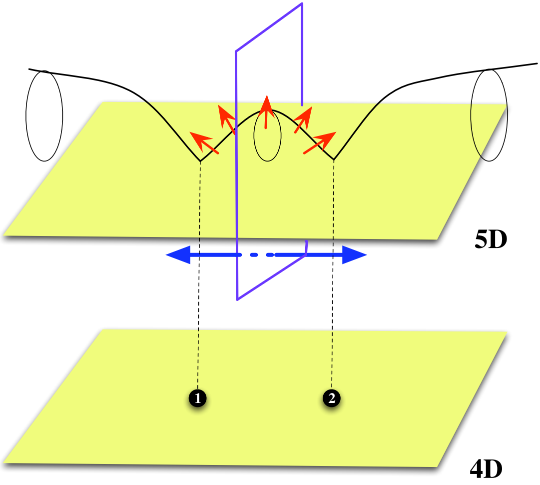
- Equivalent under large gauge transformation / Witten effect.

Final example



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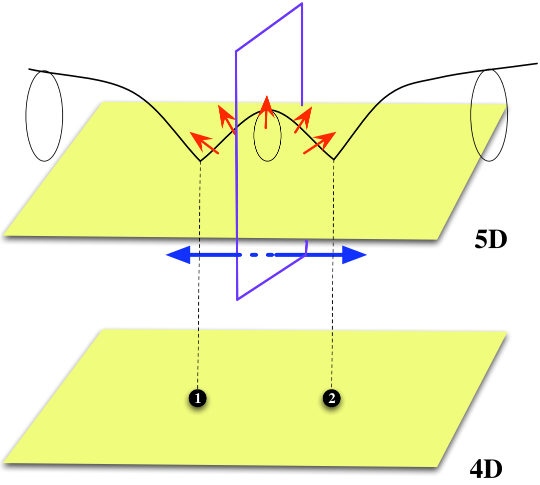

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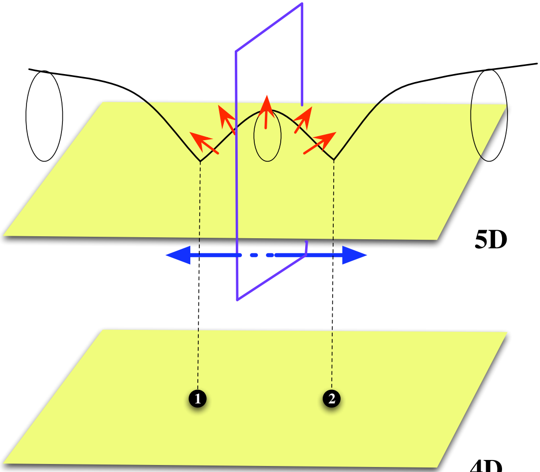

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x can be chosen at will;

BH_1 BH_2 different !

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- Higher-derivative interaction ?? [Castro-Davis-Kraus-Larsen]
- NUT gets electric charges from $\boldsymbol{A} \wedge \boldsymbol{R} \wedge \boldsymbol{R}$??