$6=4+2$

を考える。

立川裕二 (IPMU)

Based on a series of papers in collaboration with:

Alday, Benini, Benvenuti, Gaiotto, Gukov,
H. Kanno, S. Kanno, Keller, Matsuo, Mekareeya,
Moore, Neitzke, Nishioka, Shiba, Song,
Terashima, H. Verlinde, Wecht, Xie, Yamazaki

I also greatly benefitted from discussions with

Argyres, Chacaltana, Distler, Hosomichi,
Maruyoshi, Okuda, Shapere, Taki, Teschner, Yagi

What is “6=4+2” ?

How does it work ?

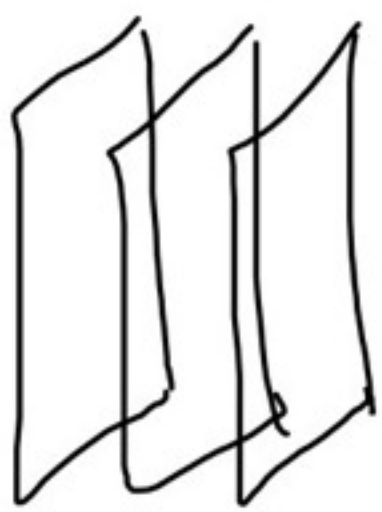
What does it mean ?

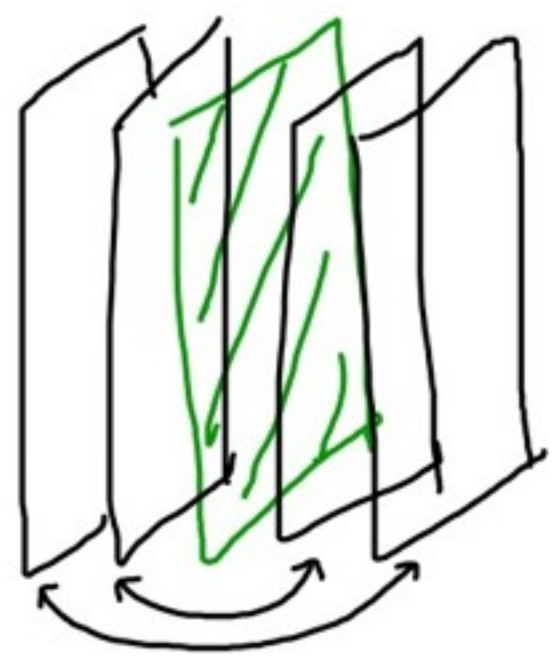
What is

6=4+2 ?

A 6d $\mathcal{N}$=(2,0) theory is one of :

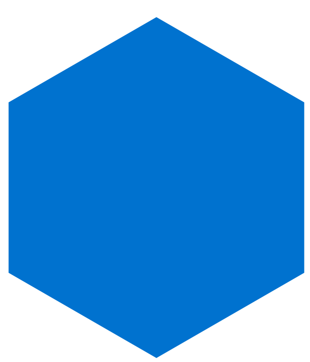
A_{K-1}	D_K	$E_{6,7,8}$
K M5s	2K M5s + M-orientifold	???

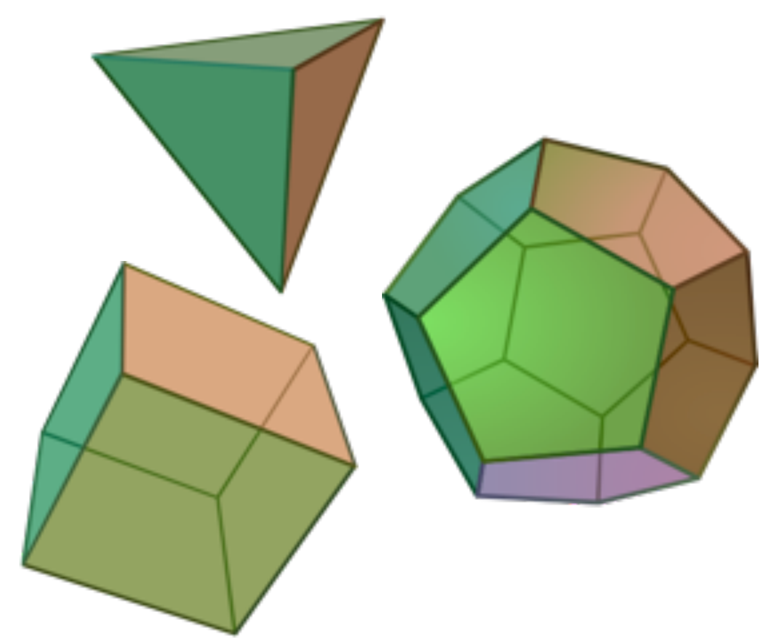




A 6d $\mathcal{N}$=(2,0) theory is one of :

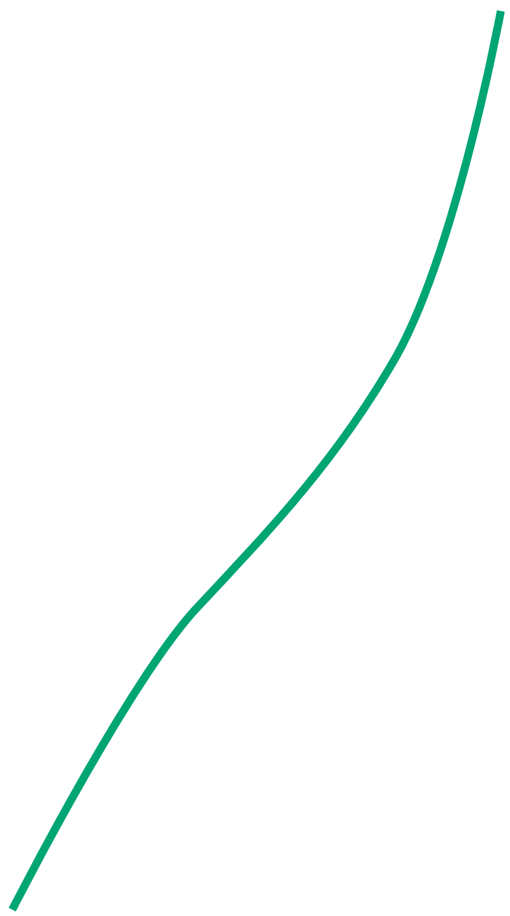
	A_{K-1}	D_K	$E_{6,7,8}$
Type IIB on	C^2/Z_K	C^2/dihed	C^2/tetra C^2/octa C^2/icosa





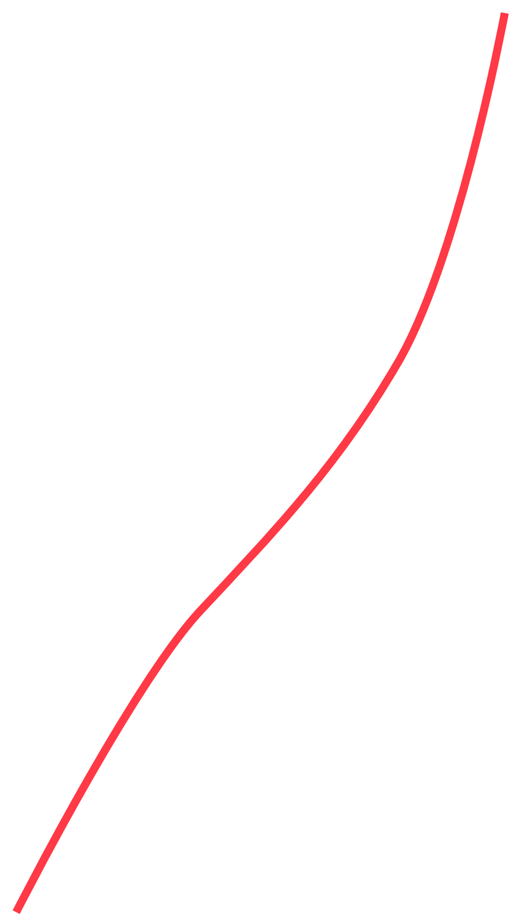
- 6d theory of type X on T^2
  = 4d $\mathcal{N}$=4 SYM with gauge group X
- Exchanging edges of T^2
  = S-duality of 4d $\mathcal{N}$=4 SYM with X
- There's no theory of type B, C, F, G.
  .... if so, the S-dual of $\mathcal{N}$=4 SYM of SO(odd) would be $\mathcal{N}$=4 SYM of SO(odd) again, contradicting Montonen-Olive.

The basic defects of 4d *N*=4 SYM of gauge group G are:



Wilson loop

R : rep. of G



’t Hooft loop

$R^{\vee}$: rep. of $G^{\vee}$

The basic defects of 6d $\mathcal{N}$=(2,0) theory of type X are:

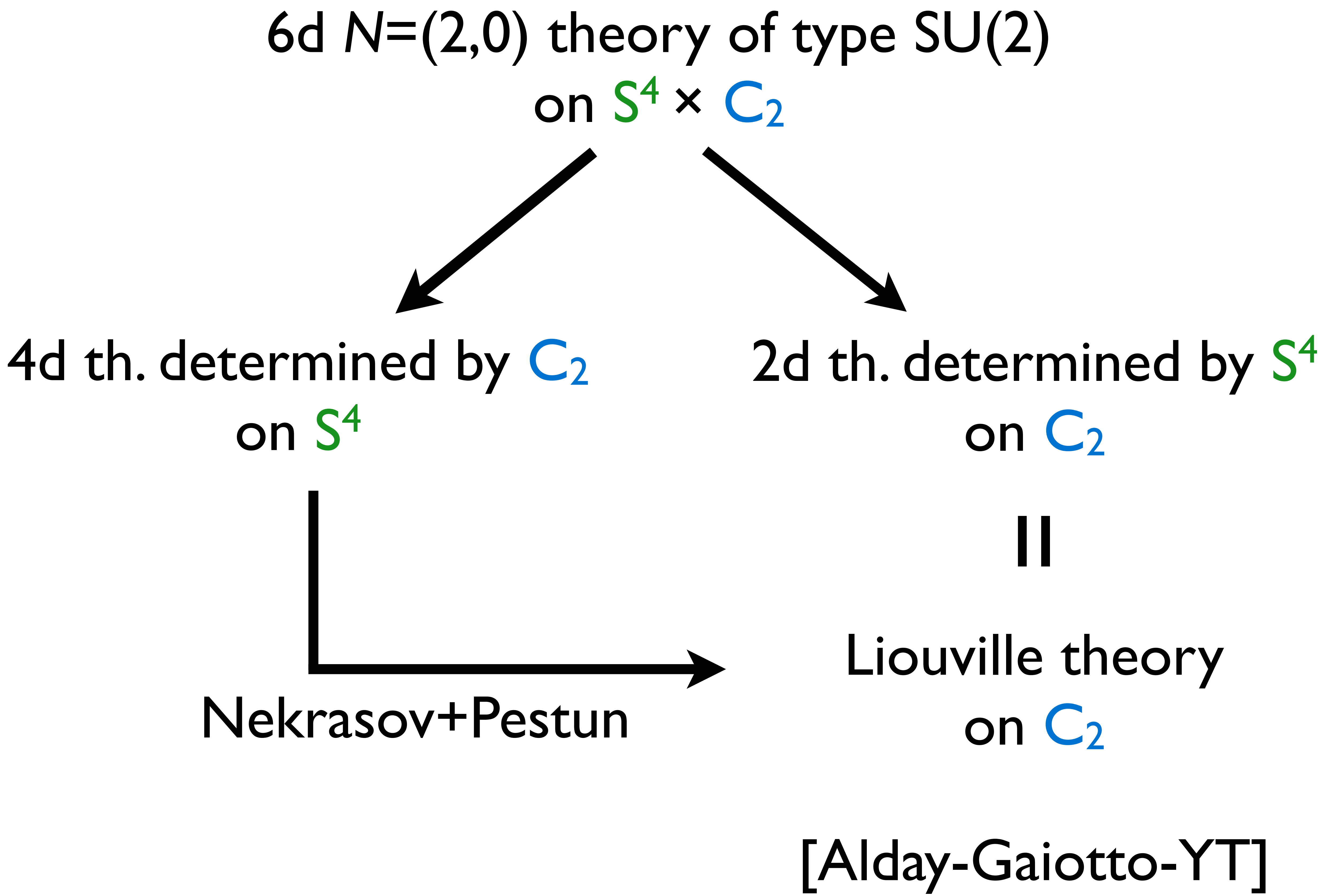
$$\mathbb{R}^2 \subset \mathbb{R}^6 \qquad\qquad \mathbb{R}^4 \subset \mathbb{R}^6$$

2d defect	4d defect
"Wilson surface"	"Fuzzy spheres"
R : rep. of X	ρ : SU(2) → X

6d $\mathcal{N}$=(2,0) theory of type SU(2)
on $S^4 \times C_2$

4d th. determined by C_2
on S^4

2d th. determined by S^4
on C_2

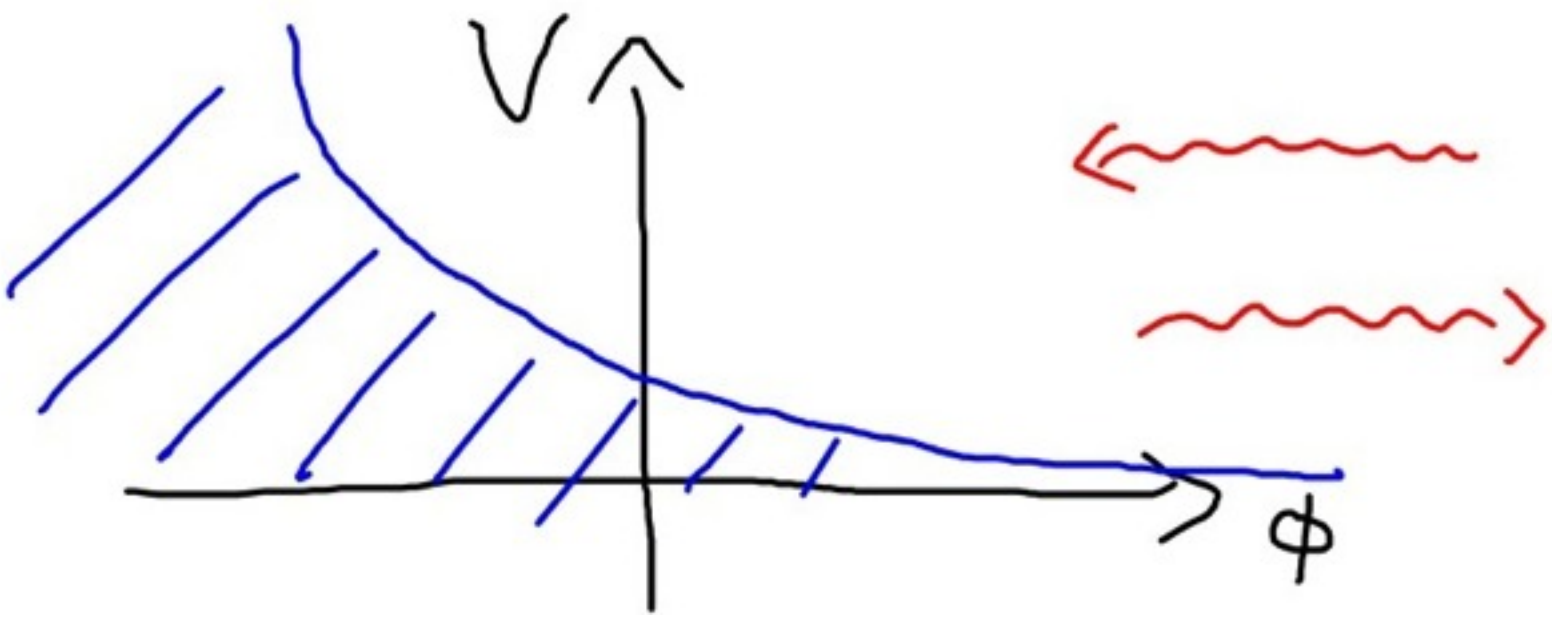


6d $\mathcal{N}$=(2,0) theory of type SU(2)
on $S^4 \times C_2$
4d th. determined by C_2
on S^4
2d th. determined by S^4
on C_2
=
Liouville theory
on C_2
Nekrasov+Pestun
[Alday-Gaiotto-YT]

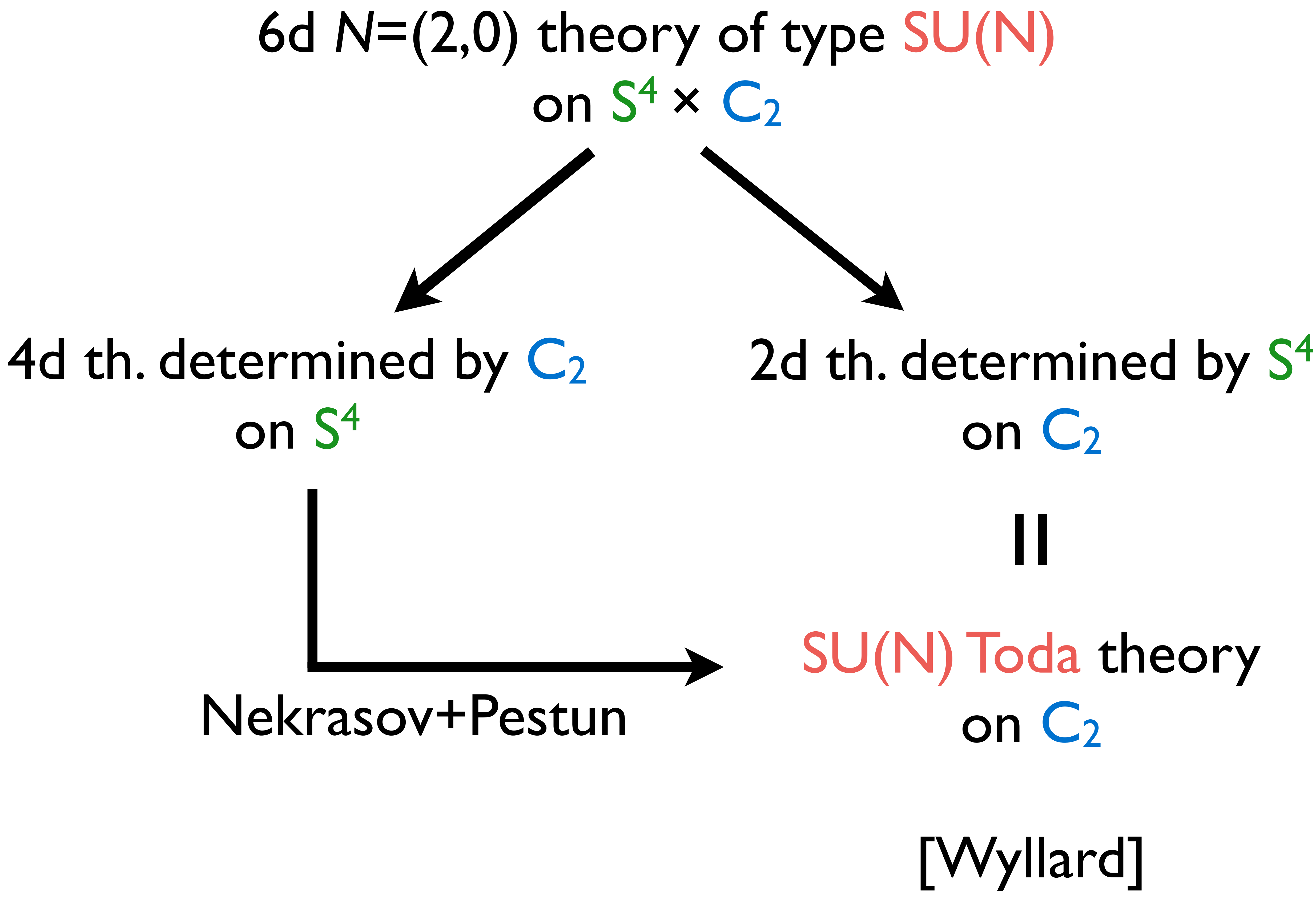
Liouville theory:

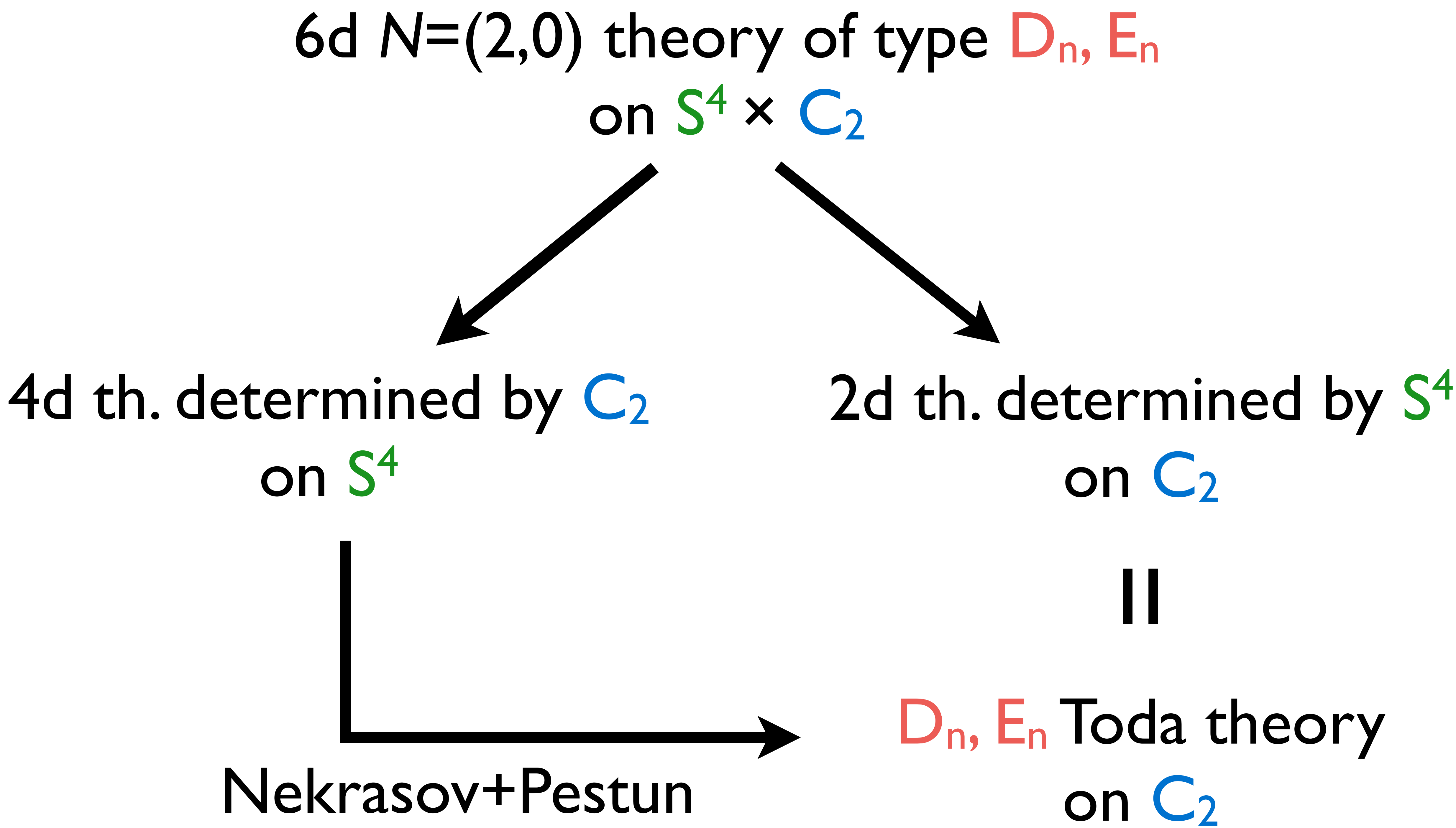
$$S = \int d^2x \sqrt{g} \left(\partial_\mu \phi \partial_\mu \phi + \sqrt{2}\, Q \phi R + \mu e^{-\sqrt{2} b \phi} \right)$$

It describes reflection against the exponential wall :

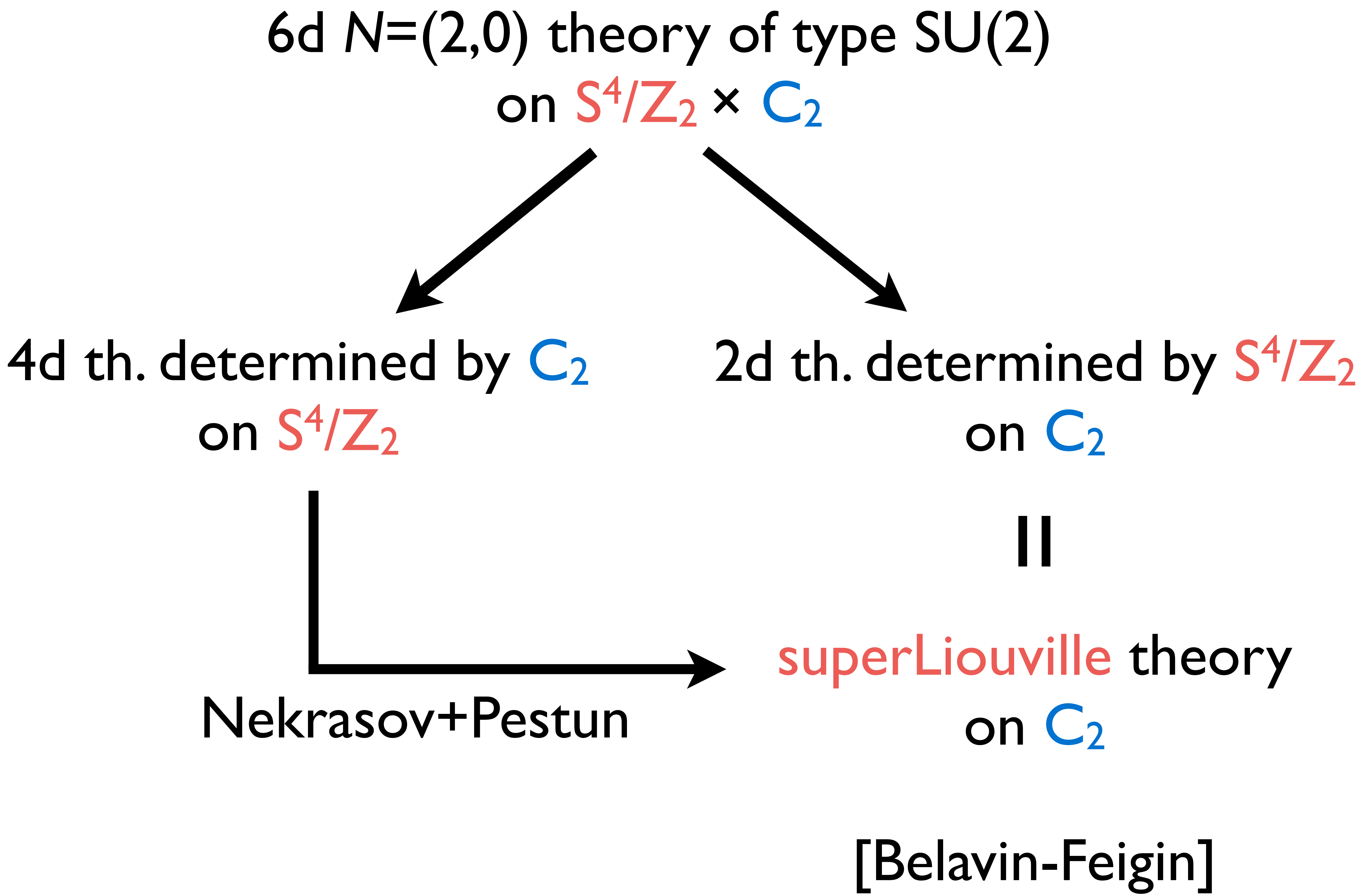


Well studied !


6d N=(2,0) theory of type SU(N)
on S4 × C2
4d th. determined by C2
on S4
2d th. determined by S4
on C2
=
SU(N) Toda theory
on C2
Nekrasov+Pestun
[Wyllard]



[Keller-Mekareeya-Song-YT]


6d $\mathcal{N}$=(2,0) theory of type SU(2)
on $S^4/Z_2 \times C_2$
4d th. determined by C_2
on S^4/Z_2
2d th. determined by S^4/Z_2
on C_2
=
superLiouville theory
on C_2
Nekrasov+Pestun
[Belavin-Feigin]

6d $\mathcal{N}$=(2,0) theory of type SU(N)
on $S^3 \times S^1 \times C_2$

4d th. determined by C_2
on $S^3 \times S^1$

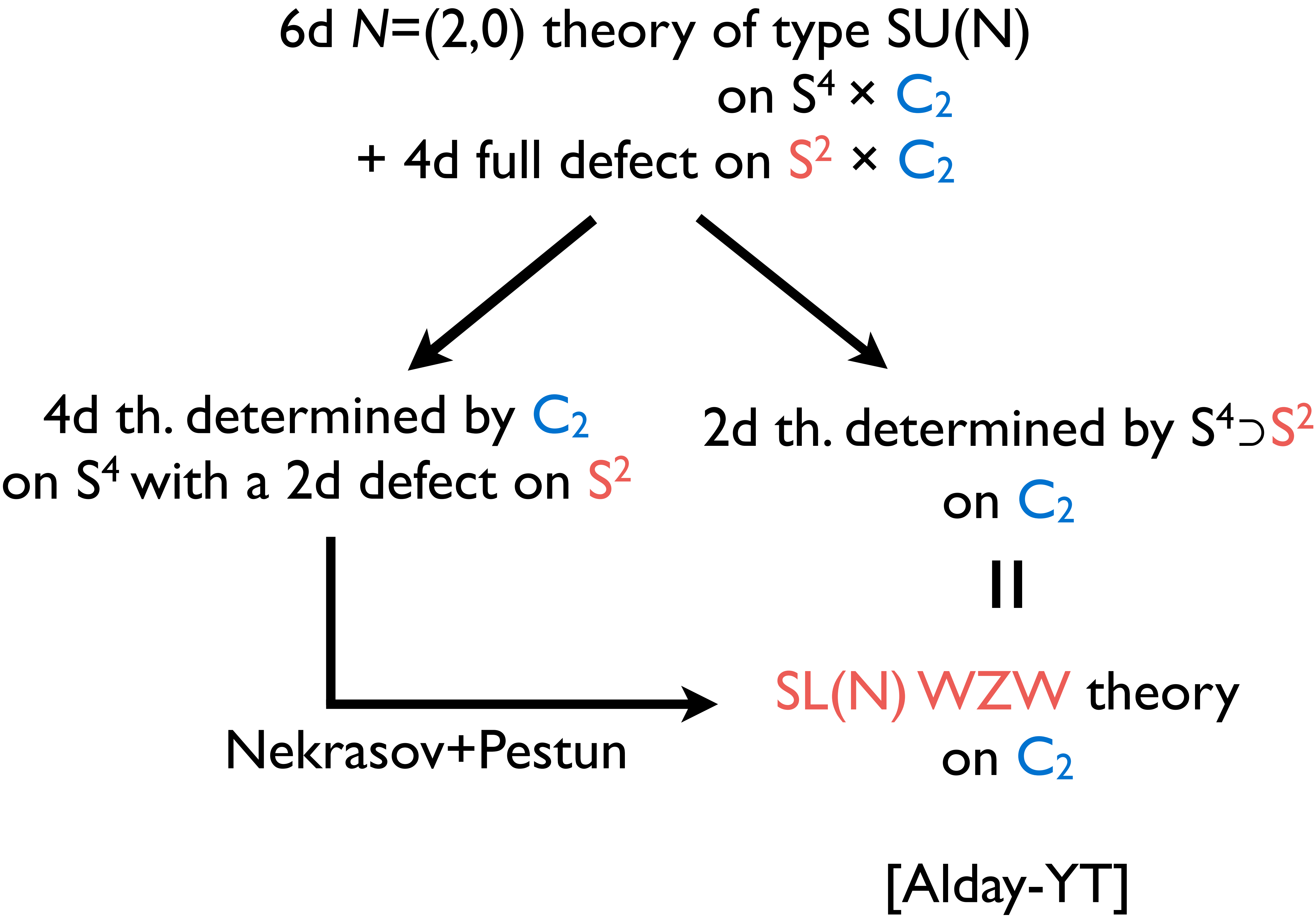
2d th. determined by $S^3 \times S^1$
on C_2

=

q-deformed Yang-Mills
on C_2

Kinney-Maldacena-Minwalla-Raju

[Gadde-Pomoni-Rastelli-Razamat-Yan]


6d $\mathcal{N}$=(2,0) theory of type SU(N)
on $S^4 \times C_2$
+ 4d full defect on $S^2 \times C_2$
4d th. determined by C_2
on S^4 with a 2d defect on S^2
2d th. determined by $S^4 \supset S^2$
on C_2
=
SL(N) WZW theory
on C_2
Nekrasov+Pestun
[Alday-YT]

6d $\mathcal{N}$=(2,0) theory of type SU(N)
on $S^4 \times C_2$
+ 4d defect of type ρ on $S^2 \times C_2$

4d th. determined by C_2
on S^4 with a 2d defect
of type ρ on S^2

2d th. determined by $S^4 \supset S^2$
and ρ on C_2

=

some theory with $W(\rho)$
symmetry on C_2

Nekrasov+Pestun

[Wyllard]

$$6=3+3$$

を考える。

6d $\mathcal{N}$=(2,0) theory of type SU(2)
on $S^3 \times M_3$

3d th. determined by M_3
on S^3

3d th. determined by S^3
on M_3

=

SL(2) CS-theory
on M_3

Kapustin-Yaakov-Willett

[Dimofte-Gaiotto-Gukov]

$$4=3+1$$

を考える。

4d $\mathcal{N}=4$ SYM with gauge group G
on S^3 × an interval with boundary conditions

3d th. on S^3 determined by
boundary conditions

1d th. determined by S^3
on the interval

=

a certain QM

Kapustin-Yaakov-Willett

[Nishioka-YT-Yamazaki]

$$5 = 3 + 2 \ ?$$

$$6 = 2 + 2 + 2 \ ?$$

How does it work ?

Cherednik algebras, W algebras and the equivariant cohomology of the moduli space of instantons on A^2

Olivier Schiffmann, Eric Vasserot

(Submitted on 13 Feb 2012)

We construct a representation of the affine W-algebra of gl_r on the equivariant homology space of the moduli space of U_r-instantons on A^2, and identify the corresponding module. As a corollary we prove the AGT conjecture (in the massless case). Our approach uses a suitable deformation of the universal enveloping algebra of the Witt algebra $W_{1+\infty}$, which is shown to act on the above homology spaces (for any r) and which specializes to all $W(gl_r)$. This deformation is in turn constructed from a limit, as n tends to infinity, of the spherical degenerate double affine Hecke algebra of GL_n, or equivalently as a degeneration of the (spherical) Hall algebra of an elliptic curve.

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Subjects:	**Quantum Algebra (math.QA)**; Mathematical Physics (math-ph); Algebraic Geometry (math.AG)
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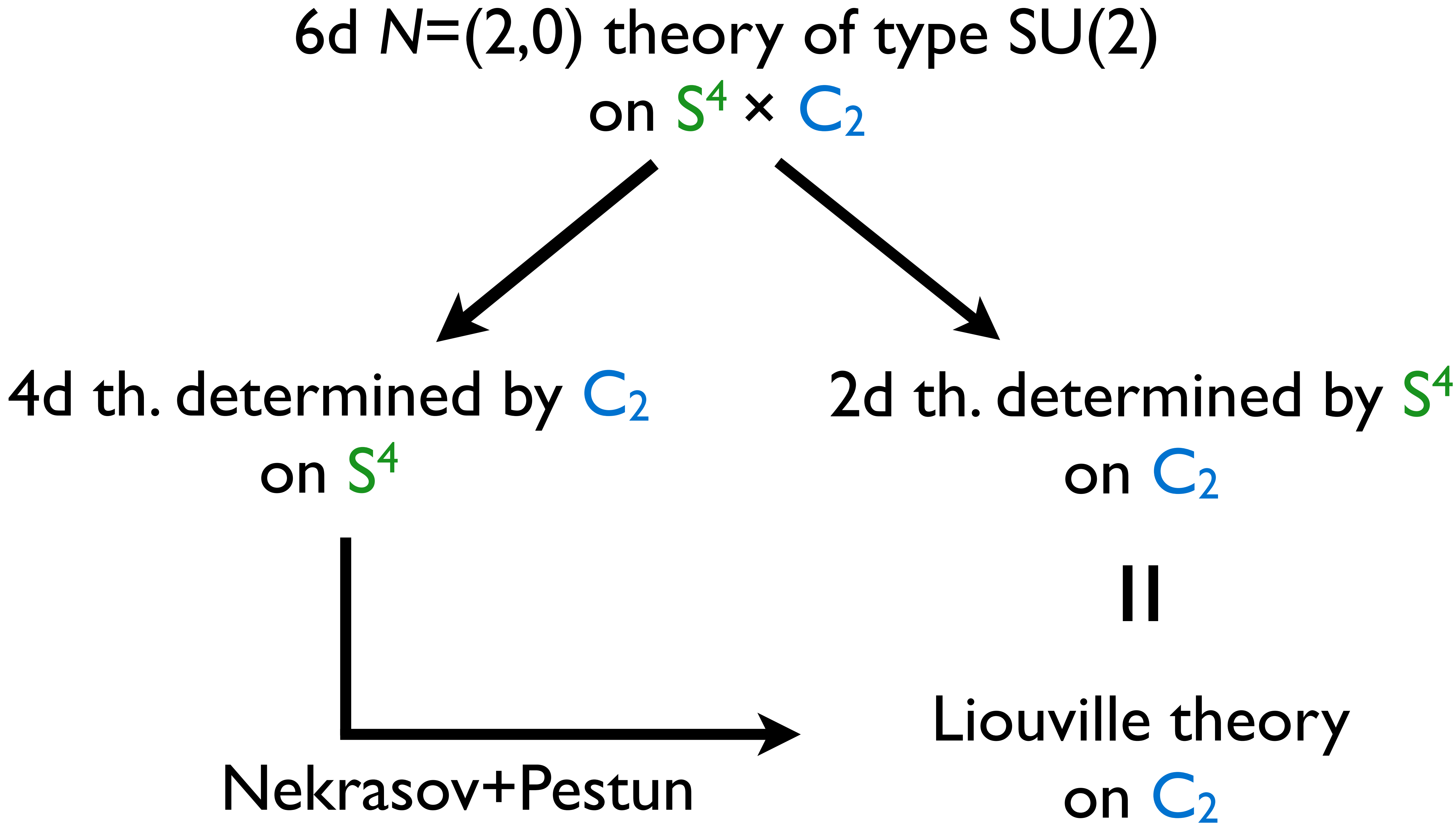
6d $\mathcal{N}$=(2,0) theory of type SU(2)
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2d th. determined by S^4
on C_2
=
Liouville theory
on C_2
Nekrasov+Pestun

6d $\mathcal{N}$=(2,0) theory of type SU(2)
on S^4

↓

2d Liouville theory

6d $\mathcal{N}$=(2,0) theory of type SU(2)
on S^4

↓

2d Liouville theory

6d $\mathcal{N}$=(2,0) theory of type SU(2) : supersymmetric.
on S^4

↓

2d Liouville theory : non susy.

4d $\mathcal{N}=2$ gauge theory on S^4 : supersymmetric.

↓

Pestun's 0d matrix model : non susy.

4d N=2 gauge theory on S^4 : supersymmetric.

↓ SUSY Localization ↓

Pestun's 0d matrix model : non susy.

3d $\mathcal{N}$=4 gauge theory on S^3 : supersymmetric.

↓ SUSY Localization ↓

0d matrix model of Kapustin-Yaakov-Willet : non susy.

6d $\mathcal{N}$=(2,0) theory of type SU(2) on S^4 : supersymmetric.

SUSY Localization

2d Liouville theory : non susy.

6d $\mathcal{N}$=(2,0) theory of type SU(2) on $S^4 \times S^1$: supersymmetric.

↓ SUSY Localization ↓

2d Liouville theory on S^1 : non susy.

5d maximal SYM with G=SU(2) on S^4 : supersymmetric.

SUSY Localization

2d Liouville theory on S^1 : non susy.

5d maximal SYM with G=SU(2) on S^4 : supersymmetric.

↓ SUSY Localization ↓

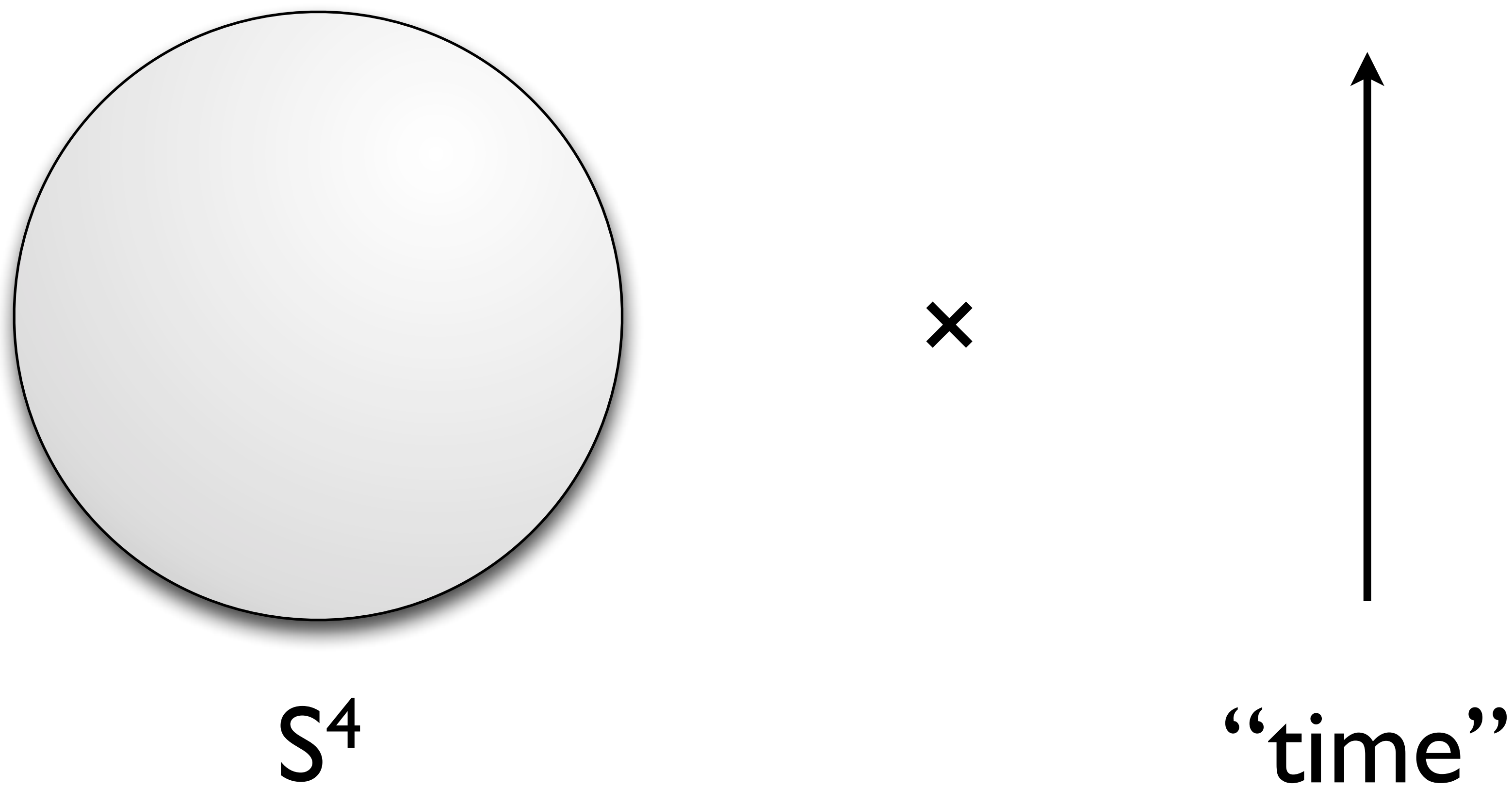
2d Liouville theory on S^1 : non susy.

||

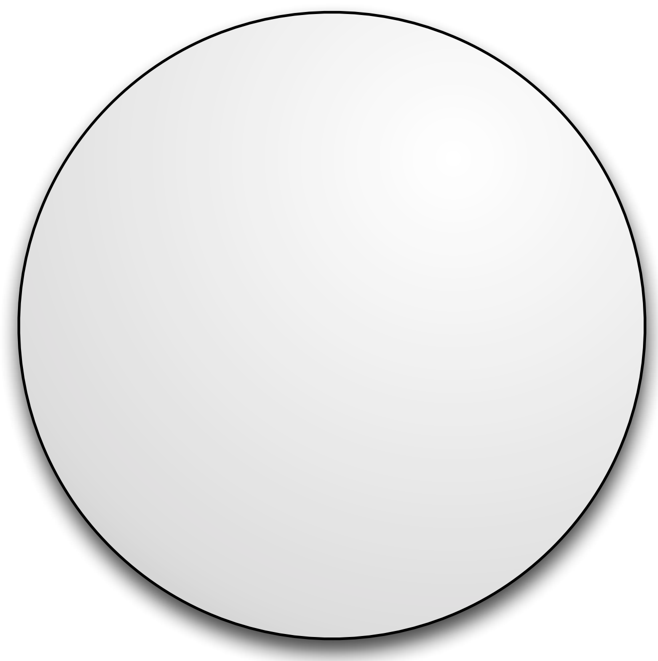
1d Liouville QM

+ Virasoro descendants

5d maximal SYM with G=SU(2)

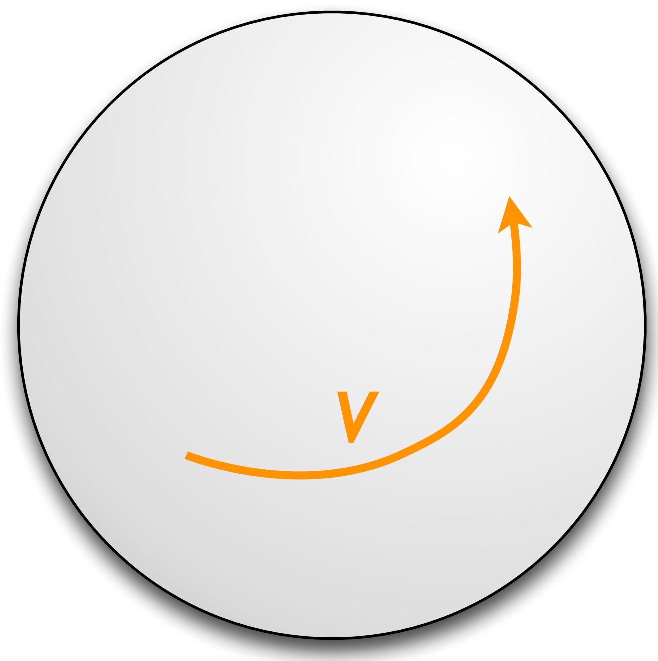


4d *N*=4 SYM with G=SU(2)



S^4

4d $\mathcal{N}$=4 SYM with G=SU(2)



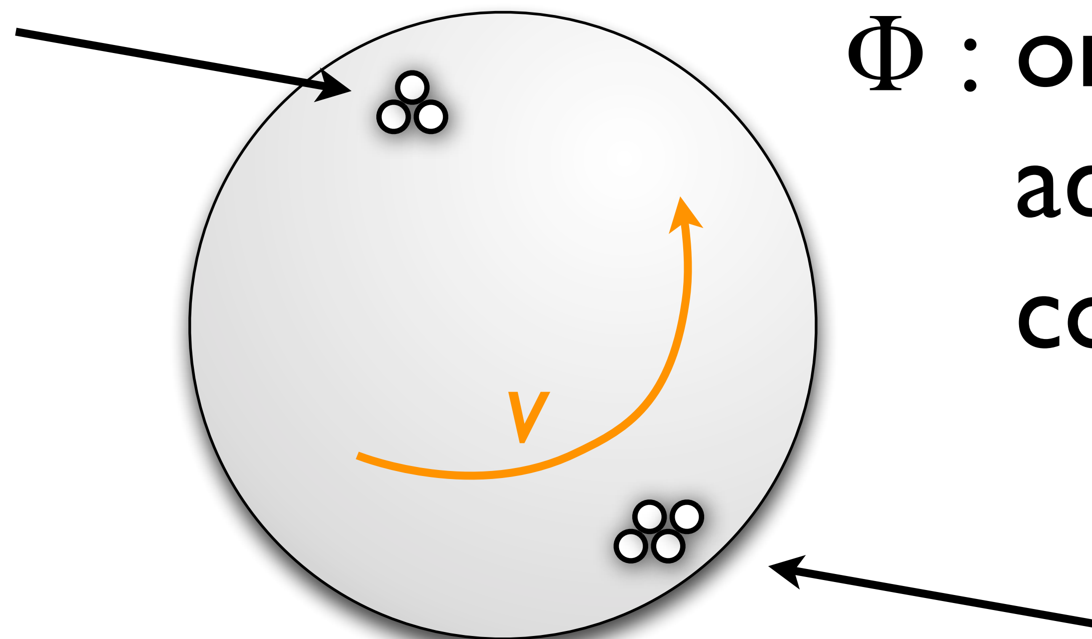
S^4

Localize via the supercharge $Q^2 = v$

4d N=4 SYM with G=SU(2)

Instantons at the north pole

Φ : one hermitean adjoint field constant on S^4



S^4

Instantons at the south pole

Localize via the supercharge $Q^2=v$

4d *N*=4 SYM with G=SU(2)

Instantons at the north pole

Φ : one hermitean adjoint field constant on S^4

$$\Phi = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}, \ a \sim -a$$

Instantons at the south pole

4d $\mathcal{N}$=4 SYM with G=SU(2)

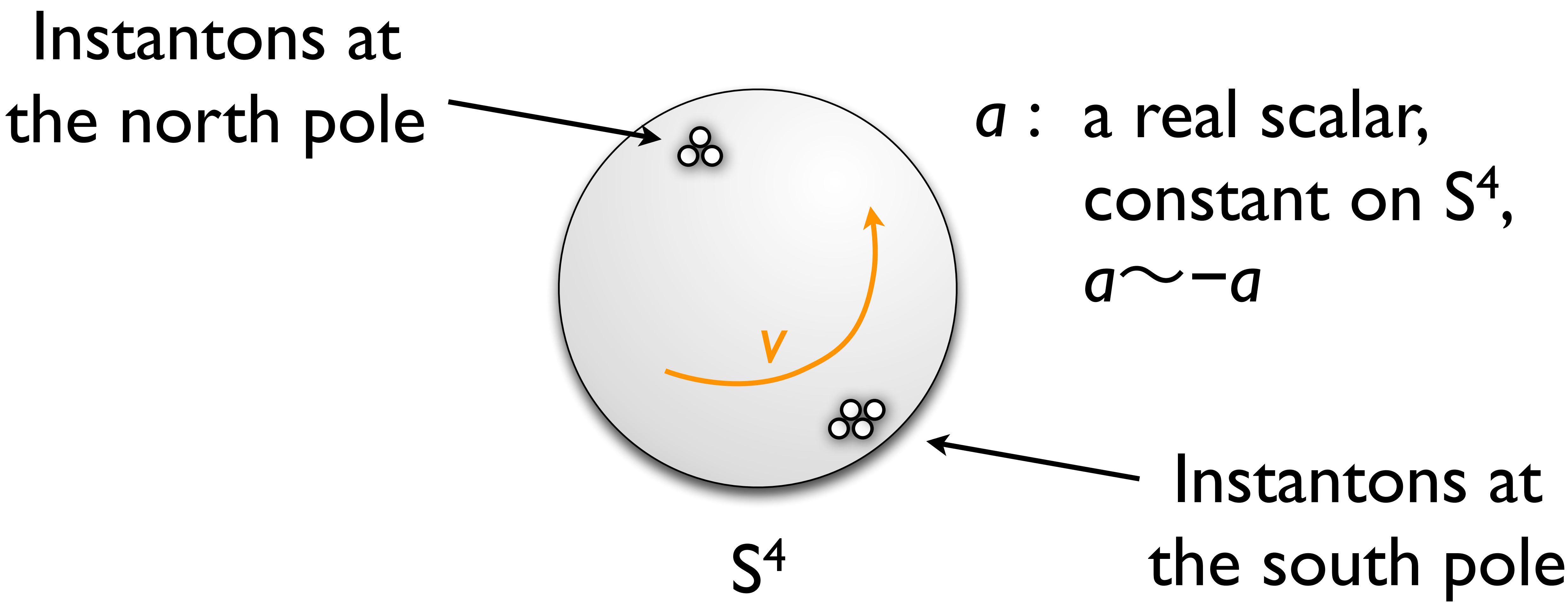
$$\int_0^\infty da\rho(a)Z(a)\overline{Z(a)}$$

constant scalar on S^4 → $\rho(a)$

instantons at the north pole → $Z(a)$

instantons at the south pole → $\overline{Z(a)}$

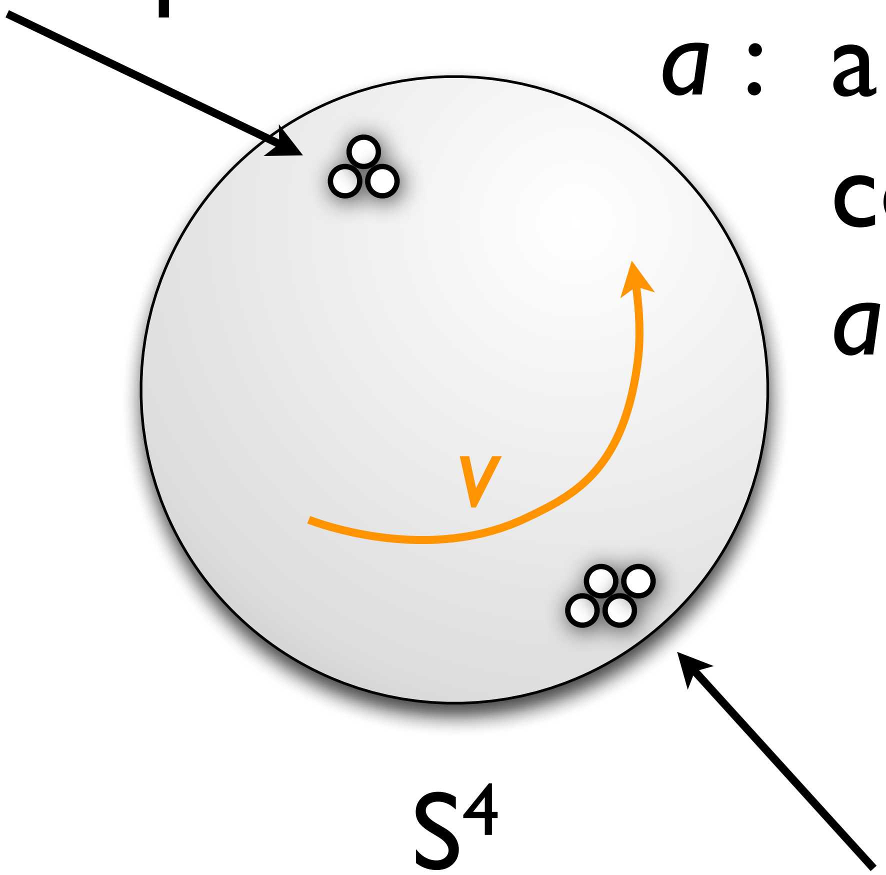
4d N=4 SYM with G=SU(2)



a : a real scalar,
constant on S^4,
$a \sim -a$

5d maximal SYM with G=SU(2) on S^4

Instantons at the north pole

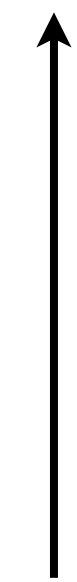


S^4

Instantons at the south pole

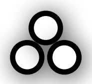
a : a real scalar, constant on S^4, $a \sim -a$

×



“time”

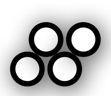
5d maximal SYM with G=SU(2) on S^4



Instantons at
the north pole

a : a real scalar on S^4, $a \sim -a$ ×

↑ "time"



Instantons at
the south pole

5d maximal SYM with G=SU(2) on S^4

= QM of Instantons at the north pole

+ QM of a real scalar on S^4, $a \sim -a$

+ QM of Instantons at the south pole

2d Liouville theory on S^1

= Left-moving Virasoro descendants

+ Liouville QM

+ Right-moving Virasoro descendants

5d maximal SYM with G=SU(2) on S^4

2d Liouville theory on S^1

= QM of Instantons at the north pole

Left-moving Virasoro descendants

+ QM of a real scalar on S^4, $a \sim -a$

Liouville QM

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Right-moving Virasoro descendants

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QM of a real scalar on S^4, $a \sim -a$

Liouville QM

QM of Instantons at the south pole

Right-moving Virasoro descendants

QM of

Instantons at
the north pole

Left-moving Virasoro descendants

QM of

Instantons at
the north pole

Left-moving Virasoro descendants

Mathematically proved !

QM of

Instantons at the north pole

Left-moving Virasoro descendants

Mathematics > Quantum Algebra

Chered nik algebras, W algebras and the equivariant cohomology of the moduli space of instantons on A^2

Olivier Schiffmann, Eric Vasserot

(Submitted on 13 Feb 2012)

We construct a representation of the affine W-algebra of gl_r on the equivariant homology space of the moduli space of U_r-instantons on A^2, and identify the corresponding module. As a corollary we prove the AGT conjecture (in the massless case). Our approach uses a suitable deformation of the universal enveloping algebra of the Witt algebra $W_{1+\infty}$, which is shown to act on the above homology spaces (for any r) and which specializes to all $W(gl_r)$. This deformation is in turn constructed from a limit, as n tends to infinity, of the spherical degenerate double affine Hecke algebra of GL_n, or equivalently as a degeneration of the (spherical) Hall algebra of an elliptic curve.

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QM of a real scalar on S^4, $a \sim -a$

Liouville QM

? ? ?

QM of a real scalar on S^4, $a \sim -a$

Liouville QM

Teschner proved some time ago that a QM on a half-line which can be combined with the Virasoro to form a 2d CFT is necessarily Liouville.

So, in a very indirect way, it's guaranteed that this should give Liouville...

QM of a real scalar on S^4, $a \sim -a$

Liouville QM

? ? ?

Instanton partition function of 4d pure N=2 SU(2)

$= \langle G|\exp(-\,t\,L_0)|G\rangle$

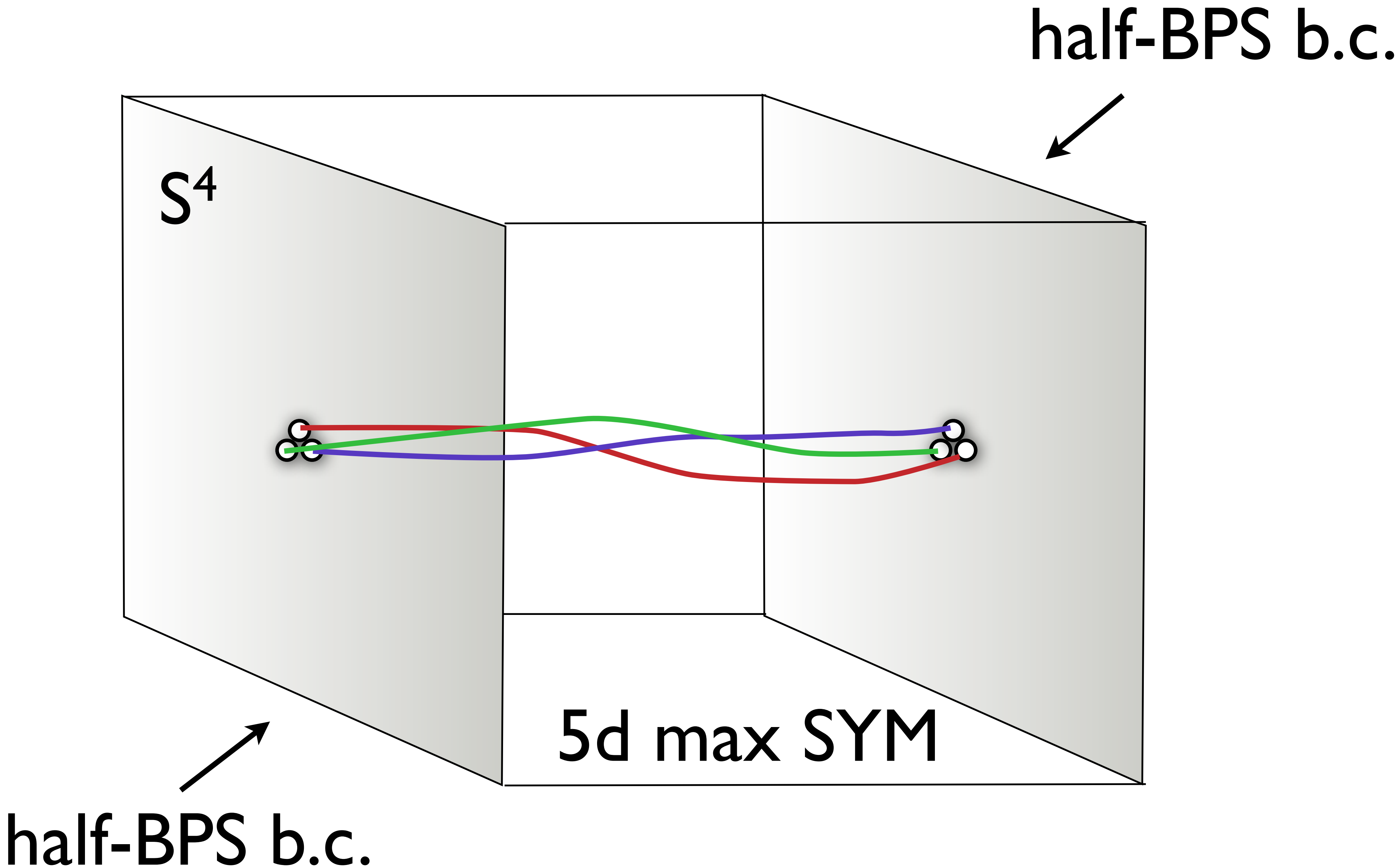
where $|G\rangle$ is a coherent state
in the Verma module of the Virasoro algebra,
and t is the inverse coupling constant.

[Gaiotto]

Why?

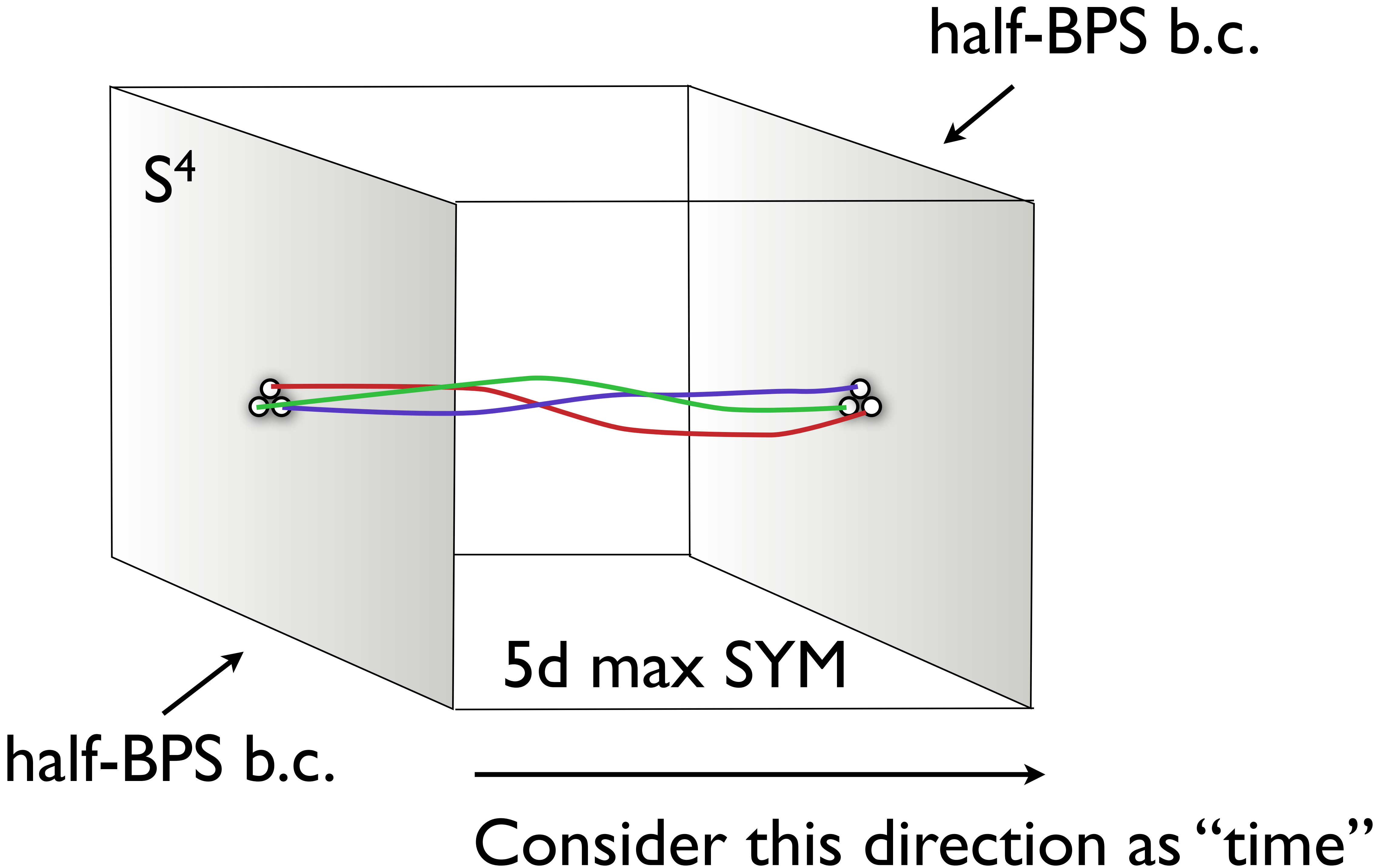
4d pure $\mathcal{N}$=2 SU(2)

≃ 5d max SUSY SU(2) on a segment



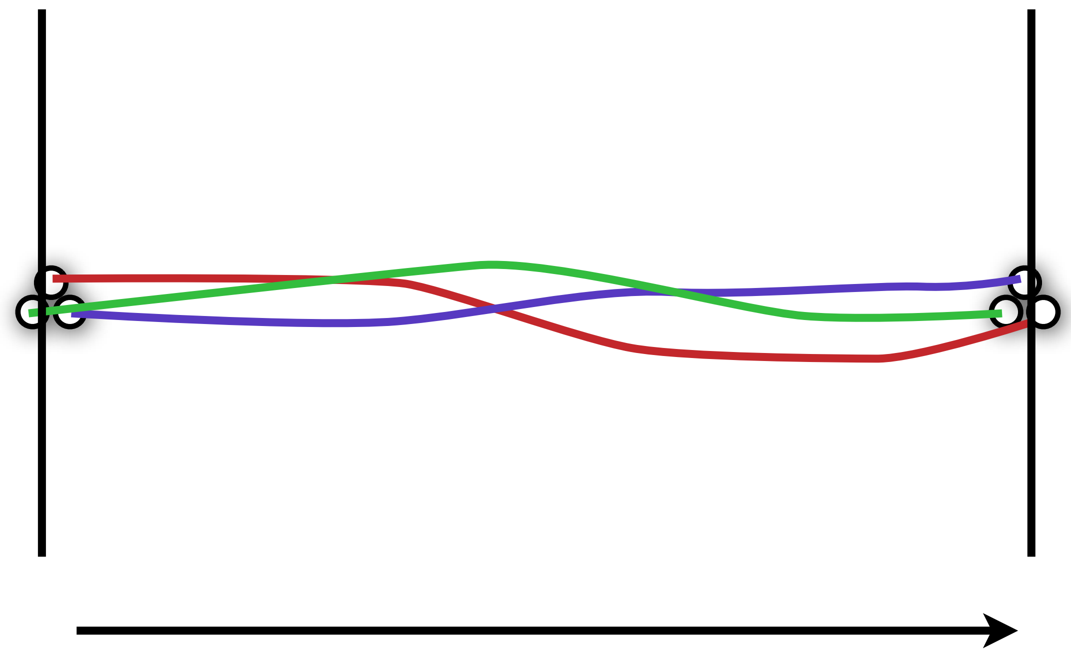
4d pure $\mathcal{N}$=2 SU(2)

≃ 5d max SUSY SU(2) on a segment



4d pure $\mathcal{N}$=2 SU(2)

$\simeq$ 5d max SUSY SU(2) on a segment

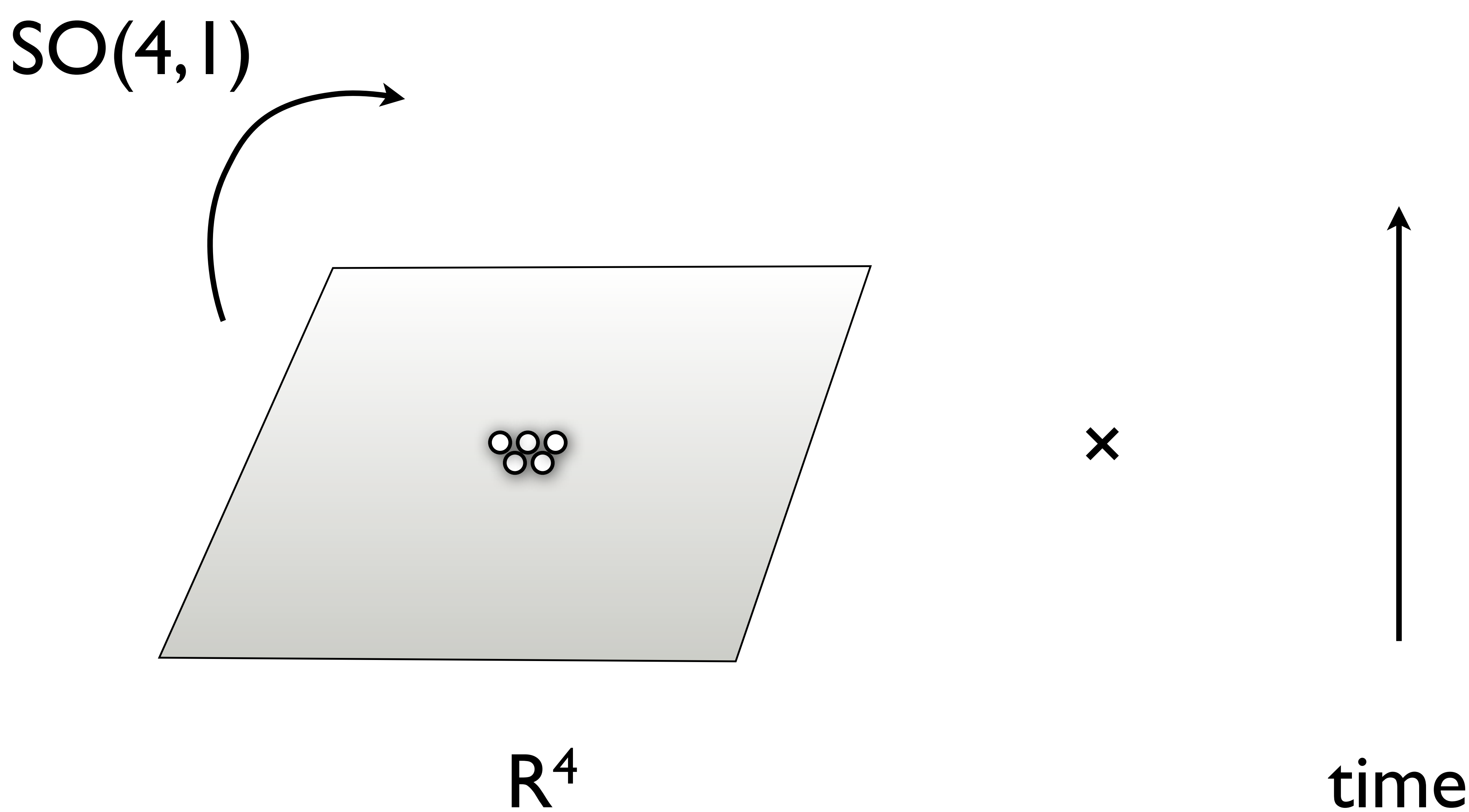


$\simeq$ a state <G| of Virasoro, created by the left b.c., propagating for a while t, then annihilated by the right b.c. ,|G>

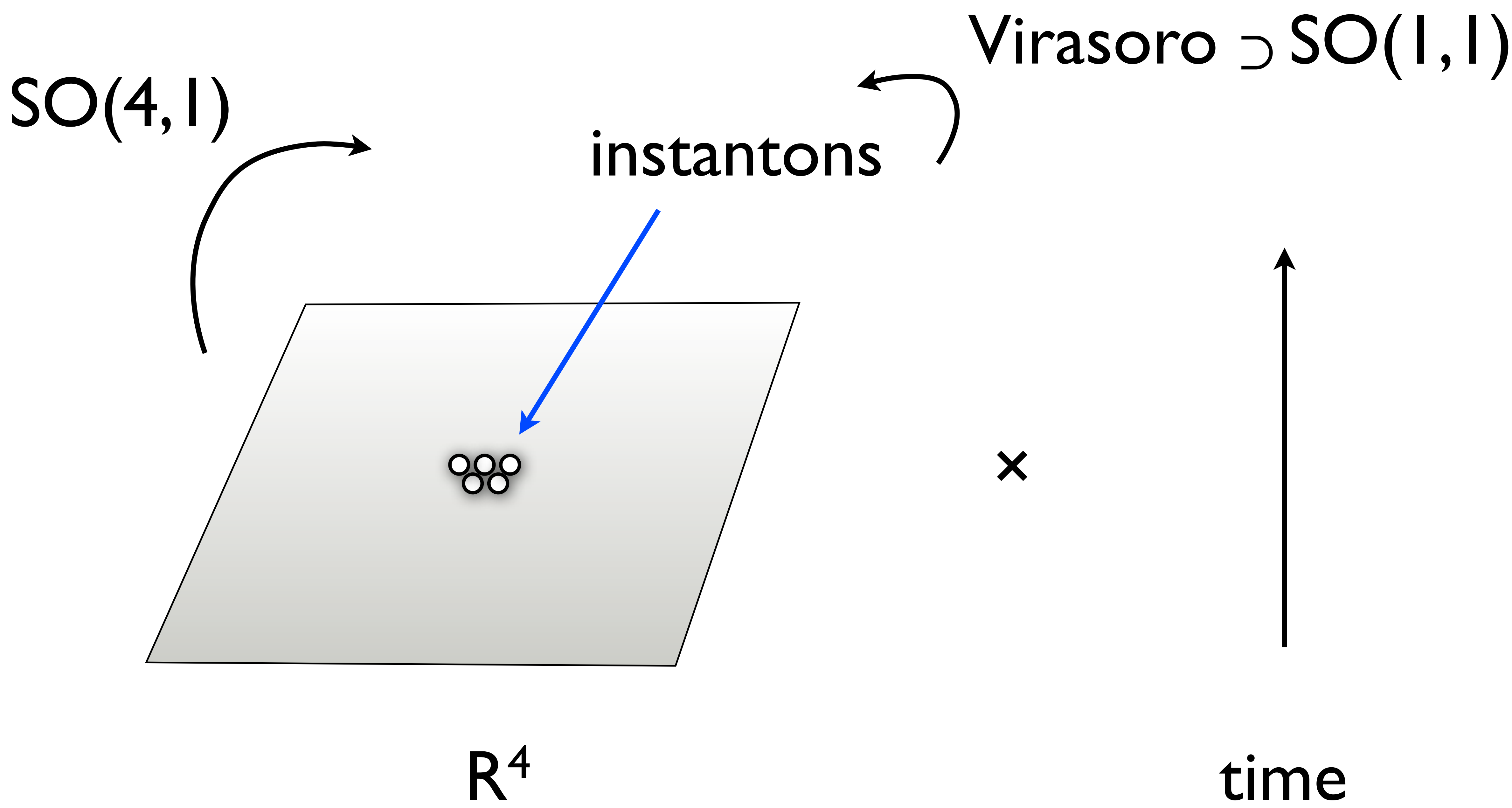
$= \langle G|\, \exp(-\, t\, L_0)\, |G\rangle$

What does it mean?

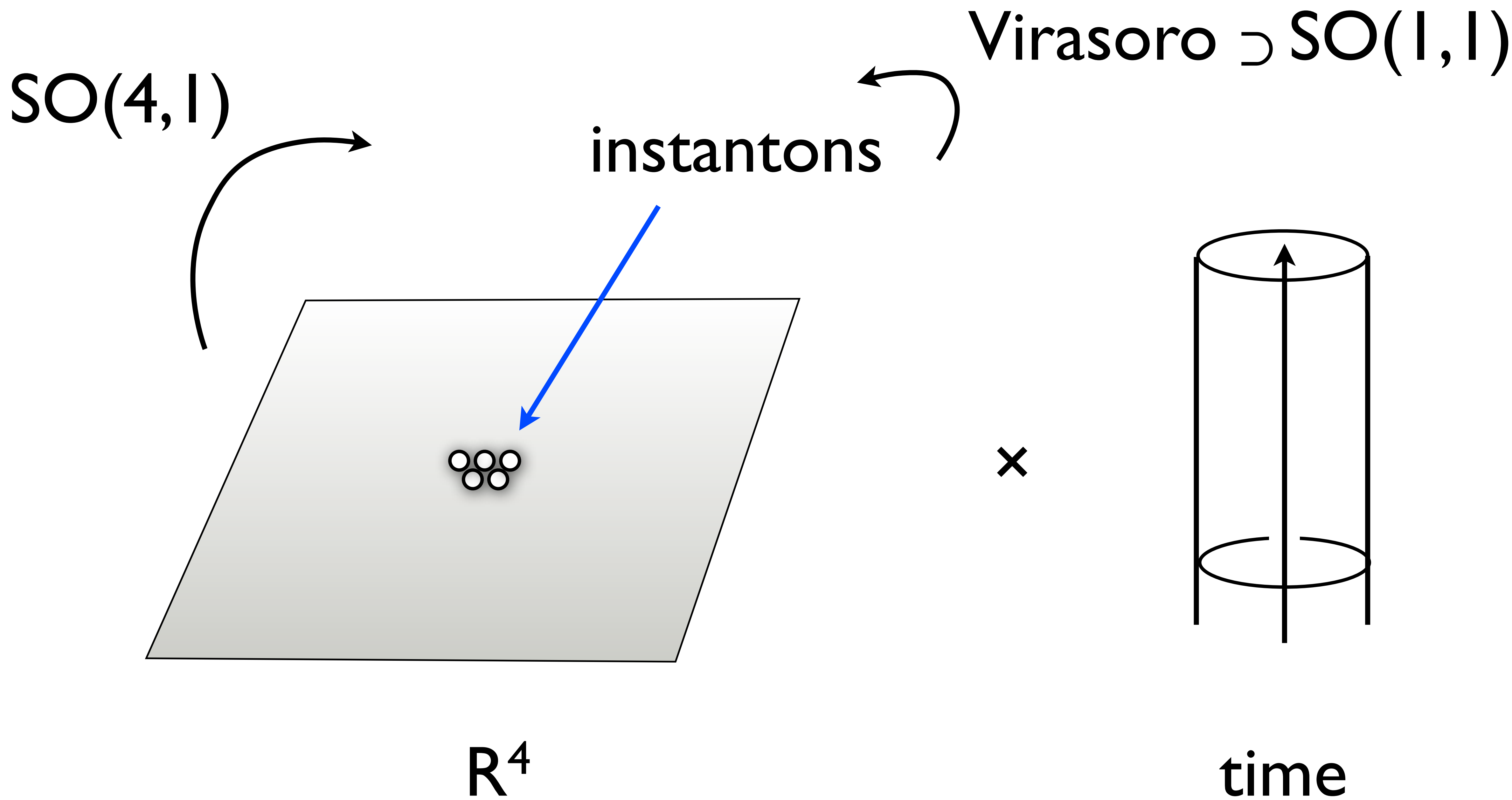
5d maximal SYM



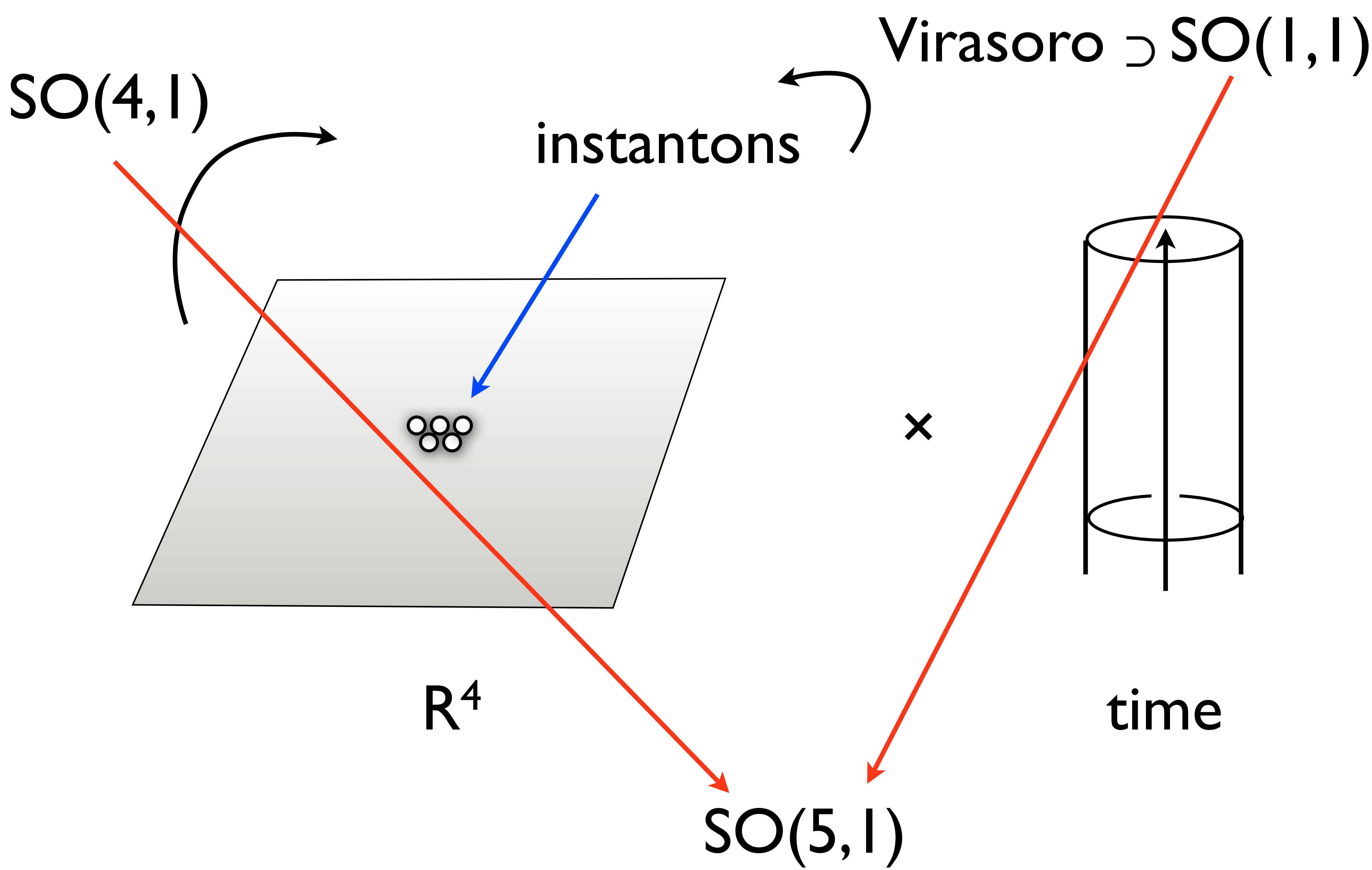
5d maximal SYM



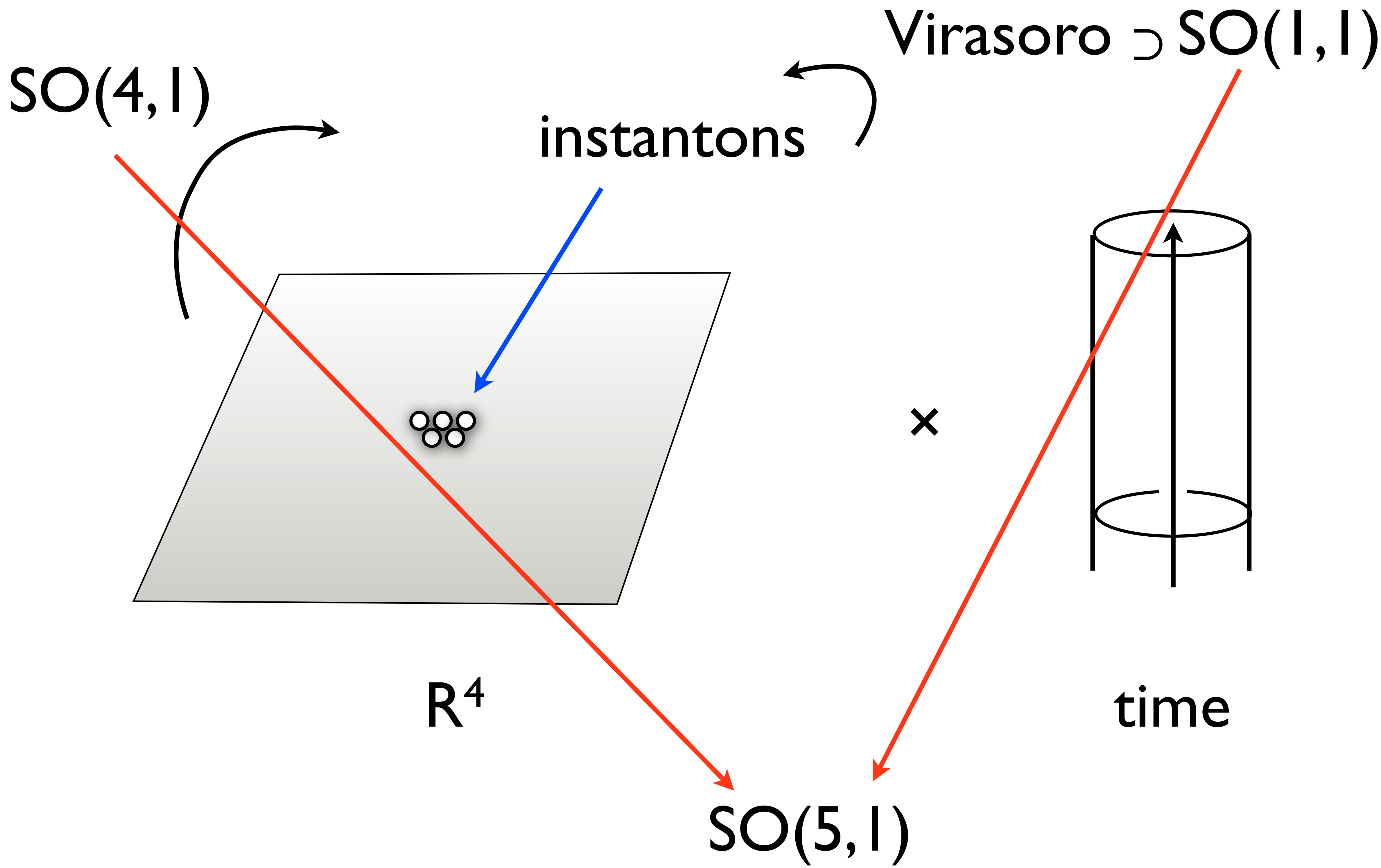
5d maximal SYM



5d maximal SYM



6d N=(2,0) theory



- Instantons generate an additional S^1.
- Instanton number = L_0 = KK momentum.
- Nonperturbative physics can generate a dimension. No large N.

- This should not surprise us.
- Consider Type IIA.
- Nonperturbative excitation: D0s.
- They generate the M-theory circle.

- In a sense, I just repeated the same thing:
- 5d SYM ~ D4 branes
- instantons ~ D0 branes on D4 branes
- instantons can generate a dimension
  ~ D0 branes can generate a dimension

- So, it's not that we learned something very new. But we are slowly getting solid results.
- Recall that there should be one threshold bound state of k D0-branes for each k.
- Not proven yet.
- But now mathematicians proved that there are exactly the right number of bound states of k D0-branes within D4s.

- Looking back at “prehistoric” papers around 1997~2000, you realize that they came very close.
- e.g. the DLCQ of N M5-branes as the QM of instantons on T^4. [Ganor-Sethi]
- Many great papers from those days remain almost forgotten, due to the rise of AdS/CFT.
- Maybe some 温故知新 is useful.