Rigid Limit in $\mathcal{N} = 2$ Supergravity and Weak-gravity Conjecture

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Some Success of String Theory

- Include quantized graviton.
- Many consistent models :
    - 10d flat IIA / IIB
    - 4d $\mathcal{N} = \mathbf{8}$, type II on T^6
    - 4d $\mathcal{N} = \mathbf{2}$, type II on CY = het on $K3 \times T^2$

Some Success of String Theory

- Include quantized graviton.
- Many consistent models :
  - 10d flat IIA / IIB
  - 4d $\mathcal{N} = \mathbf{8}$, type II on T^6
  - 4d $\mathcal{N} = \mathbf{2}$, type II on CY = het on $K3 \times T^2$
- Models with Less SUSY :
  - 4d $\mathcal{N} = \mathbf{1}$ from type II on CY with Fluxes
  - $\mathbf{10^{300}}$ of them ???
  - Non-perturbative effects which select the Real World ?????

Swampland

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If there're really $\mathbf{10^{300}}$ consistent models,
can arbitrary low energy Lagrangian be UV-completed with gravity ?

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- Anomaly cancelation.
- More criteria ?

Weak-gravity Conjecture

Standard Lore

String theory cannot have continuous global symmetry.

- Gauge theory with $g \ll 1$ indistinguishable from global sym.
- there should be lower bound for g.

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Claim[Arkani-Hamed,Motl,Nicolis,Vafa]

Gravity + $U(1)$ gauge field with c.c. g ⟶ Extra physics at gM_{planck}

- $g_{\text{grav}} = E/M_{\text{planck}} < g$ if $E < gM_{\text{planck}}$
- i.e. gravity is weaker than $U(1)$, before new physics comes in.

Motivation behind the Conjecture

- perturbative string compactification → $M_{\text{string}} = g M_{\text{planck}}$
- Simplest monopole should not be black :

$$m_{\text{monopole}} \sim \frac{E_{\text{cutoff}}}{g^2} \quad \text{and} \quad R_{\text{monopole}} \sim \frac{1}{E_{\text{cutoff}}}$$

then $R_{\text{schwarzshild}} < R_{\text{monopole}}$ leads

$$\frac{m_{\text{monopole}}}{M_{\text{planck}}^2} < \frac{1}{E_{\text{cutoff}}} \quad \longrightarrow \quad E_{\text{cutoff}} < g M_{\text{planck}}$$

- see the original article for more.

$\mathcal{N} = 2$ SUGRA and Weak-Gravity Conjecture

Objective

To confirm the scale gM_{planck} in $\mathcal{N} = \mathbf{2}$ Yang-Mills + supergravity.

- Why $\mathcal{N} = \mathbf{2}$?
- because we have a lot more control.
  - Everything comes from Prepotential, which is holomorphic
  - Singularity structure (cuts, poles ...) determines the function
- Holomorphy in SW told us a lot about the dynamics of Yang-Mills

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Another Objective

dynamics of $\mathcal{N} = 2$ gravity from holomorphy + monodromy

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$\mathcal{N} = 2$ Yang-Mills from type IIA

- type IIA on Calabi-Yau $\longrightarrow$ $\mathcal{N} = 2$ sugra in 4d, a lot of $U(1)^n$

$$C_3 = A^I_{4\text{d}} \wedge \omega_I^{\text{CY}}$$

- D2-brane couples to C_3 $\longrightarrow$ charged solitons in 4d

$$\int_S C_3 = A^I \cdot \underbrace{\int_S \omega_I}_{\text{charge of D2 wrapping } S}$$

- S^2 shrinks $\longrightarrow$ nearly massless charged particles = W-boson
- $SU(2)$ gauge theory appears.

$\mathcal{N} = 2$ Yang-Mills from type IIB

- type IIA : worldsheet instantons of order $e^{-\text{Area of } S^2}$
  → large corrections for S^2 small

	IIA	IIB
α'/R^2	vector	hyper
g_{string}	hyper	hyper

- Mirror symmetry : IIA on M = IIB on W
- No correction to vector kinetic terms in IIB
- Classical geometry of W should capture the $SU(2)$ gauge dynamics

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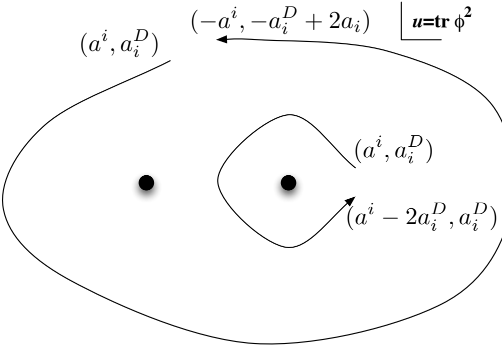
- No correction to vector kinetic terms in IIB
- Classical geometry of W should capture the $SU(2)$ gauge dynamics coupled to $\mathcal{N} = 2$ gravity !

Recap: $\mathcal{N} = 2$ rigid SUSY

- $\mathcal{N} = 2$ pure $SU(n+1)$ gauge theory : complex adjoint scalar ϕ
- $\langle\phi\rangle \neq 0$ ⟶ Higgsed to $U(1)^n$
- special coordinates a^i, $i = 1, \ldots, n$
- dual special coordinates

$$a_i^D = \frac{\partial \mathcal{F}_{\text{gauge}}}{\partial a^i}$$

- $K = \text{Im}(a_i^D)^* a^i$
- mass of BPS particles : $|q_i a^i + m^i a_i^D|$



- Monodromy determines a, a^D
- SW curve

$$w + \frac{\Lambda^2}{w} = x^2 - u$$

and then

$$a = \int_A x \frac{dw}{w}, a^D = \int_B x \frac{dw}{w};$$

$$\tau = \frac{\partial a^D}{\partial a}$$

- Curve singular at the monopole/dyon points

Recap: $\mathcal{N} = 2$ Supergravity

- N $U(1)$ vector multiplets → $N + 1$ gauge fields (remember graviphoton !)
- N scalars Φ^I ($I = 1, \ldots, N$)
- special coordinates, or periods X^a and F_a ($a = 1, \ldots, N + 1$)

$$e^{-K} = \mathrm{Im}\, F_a^* X^a$$

- Kähler transformation: $K \to K - f - f^*$ and $X^a, F_a \to e^f X^a, e^f F_a$
- mass of BPS particle $m^2 = e^K |q_a X^a + m^a F_a|^2$

Recap: $\mathcal{N} = 2$ supergravity from IIB on CY

- $(2N+2)$ 3-cycles on CY $\longrightarrow$ Canonical basis A^a, B_a s.t. $A^a \cap A^b = B^a \cap B^b = 0$, $A^a \cap B_b = \delta^a_b$
- Covariantly constant $(3,0)$-form Ω

$$X^a = \int_{A^a} \Omega, \qquad F_a = \int_{B_b} \Omega$$

- Kähler tr. = overall factor in Ω
- $\delta\Omega$ is $(3,0)+(2,1)$ $\longrightarrow$ $\int_{CY} \Omega \wedge \partial_{\Phi^I} \Omega = 0$

$$\longrightarrow \sum_a X^a \frac{\partial F_a}{\partial \Phi^i} - \sum_a \frac{\partial X^a}{\partial \Phi^i} F_a = 0$$

Embedding $\mathcal{N} = 2$ pure $SU(2)$ in supergravity

- u: moduli for $SU(2)$, normalized to have monopole pt. at $u = 1$
- ϵ: hierarchy between gauge theory and gravity
- two moduli → three gauge fields → six periods
- $SU(2)$ theory has two periods → need extra four periods

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- $\epsilon \to 0$: gravity decouples, susy becomes rigid.
- does new scale gM_{planck} appear ? if so, how ?

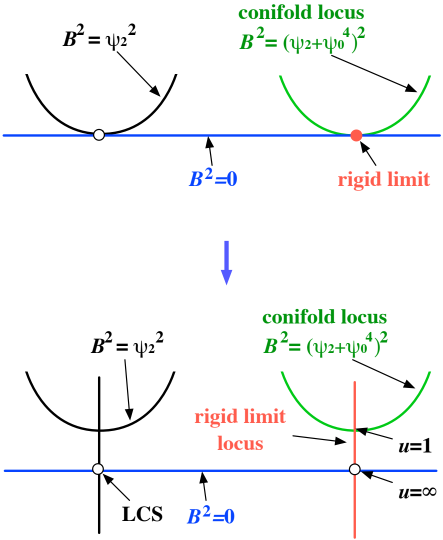
$$\frac{B}{8}x_1^8 + \frac{B}{8}x_2^8 + \frac{1}{4}x_3^4 + \frac{1}{4}x_4^4 + \frac{1}{4}x_5^4 - \psi_0 x_1 x_2 x_3 x_4 x_5 - \frac{1}{4}\psi_2 (x_1 x_2)^4 = 0$$

in $\mathbb{WCP}_{1,1,2,2,2}$; $[B : \psi_0 : \psi_2]$ gives two parameters; singular when

$$B^2(B^2 - \psi_2^2)(B^2 - (\psi_2 + \psi_0^4)^2) = 0$$

[Candelas, de la Ossa, Font, Katz, Morrison]
[Hosono, Klemm, Theisen, Yau]
[Billó, Denef, Frè, Pesando, Troost, Van Proeyen, Zanon]

Moduli of the Calabi-Yau



- $\epsilon = B$, $\epsilon u = \psi_2 + \psi_0^4$
- $\epsilon = 0$: rigid limit locus,
- CY becomes

$$w + \frac{1}{w} + x^2 + y^2 + z^2 - u + O(\epsilon) = 0$$

- the SW curve !
- u-plane embedded !

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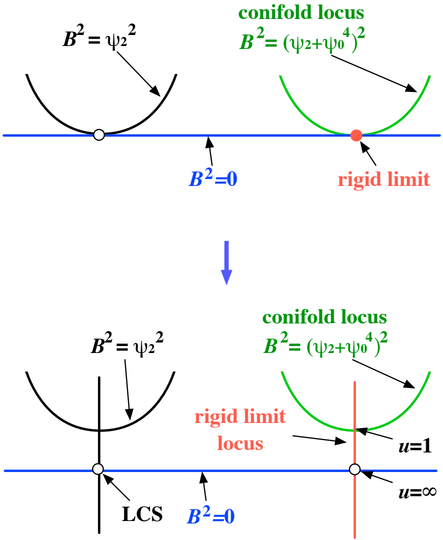

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- the SW curve !
- u-plane embedded !
- one-year before SW !

Behavior of the periods

- $X^1 \sim \epsilon^{1/2} a, \qquad F_1 \sim \epsilon^{1/2} a^D$
- $X^{2,3} \sim O(1)\ , \qquad F_{2,3} \sim \log \epsilon$
- the Kähler potential is then

$$e^{-K} \sim (\log 1/|\epsilon|) + |\epsilon|\, \mathrm{Im}(a^D)^* a$$

that is

$$K \sim \log(\log 1/|\epsilon|) + \frac{|\epsilon|}{\log 1/|\epsilon|}\, \mathrm{Im}(a^D)^* a$$

The mass scale of gauge theory I

- $X^1 \sim \epsilon^{1/2} a, \qquad F_1 \sim \epsilon^{1/2} a^D$ where a, a^D determined from

$$w + \frac{1}{w} + x^2 + y^2 + z^2 - u = 0$$

- Usually the curve is

$$w + \frac{\Lambda^2}{w} + x^2 + y^2 + z^2 - u = 0$$

$\longrightarrow$ $\epsilon^{1/2} \sim \Lambda = M_{\text{cutoff}} \exp \frac{1}{4}\left(\theta i - \frac{8\pi^2}{g^2}\right)$ [Vafa et al.]

- Let us define $S = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2} = \frac{1}{\pi i} \log \epsilon$

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- Let us define $S = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2} = \frac{1}{\pi i} \log \epsilon$
- At which scale g is defined ??

The mass scale of gauge theory II

- Low energy coupling is determined by

$$\tau = \frac{\partial a^D}{\partial a} = \frac{\partial^2 \mathcal{F}}{\partial a^2} = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2(m_W)}$$

- in the weak coupling regime $u \gg \Lambda^2$,

$$a \sim \sqrt{u}, \qquad a^D \sim \sqrt{u} \log u \longrightarrow e^{-2\pi^2/g^2(m_W)} = u^{-1/2}$$

- $g^2(m_W)$ is defined at the scale of W-boson, of mass

$$m_W^2 = e^{-K}|X^1|^2 = \frac{1}{\log 1/|\epsilon|}|\epsilon u|.$$

- Thus, the RG-invariant scale is

$$\Lambda_{\text{gauge}} = m_W e^{-2\pi^2/g^2(m_W)} = \frac{|\epsilon|^{1/2}}{(\log 1/|\epsilon|)^{1/2}} M_{\text{planck}}$$

The mass scale of gauge theory III

Result.

$$\Lambda_{\text{gauge}} = \frac{|\epsilon|^{1/2}}{(\log 1/|\epsilon|)^{1/2}} M_{\text{planck}} = e^{-2\pi^2/g^2} g M_{\text{planck}}$$

- recall $S = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2} = \frac{1}{\pi i} \log \epsilon$ is the natural UV coupling.
- it is determined at $g M_{\text{planck}}$!

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- it is determined at $g M_{\text{planck}}$!
- came essentially from $m_W^2 = e^K |X^1|^2 = e^K |\epsilon u|$,
- $e^{-K} = \text{Im}\, F_a^* X^a$ is dominated by $a = 3, 4$ with $F \sim \log \epsilon$.

kinetic term for S

- $S = \frac{1}{\pi i} \log \epsilon$
- $e^{-K} \sim \log |\epsilon| = \frac{1}{\pi i}(S - S^*)$ → $K = \log \operatorname{Im} S$

$$\rightarrow \quad g_{SS^*} \partial_\mu S \partial_\mu S^* = \frac{\partial_\mu S \partial_\mu S^*}{(\operatorname{Im} S)^2}$$

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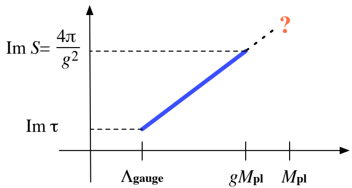
$$\longrightarrow \quad g_{SS^*} \partial_\mu S \partial_\mu S^* = \frac{\partial_\mu S \partial_\mu S^*}{(\operatorname{Im} S)^2}$$

- Looks like a dilaton !
- if the CY has a heterotic dual, it is the dilaton.

Summary



- $S = \frac{1}{\pi i} \log \epsilon$
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- New scale at gM_{planck} !

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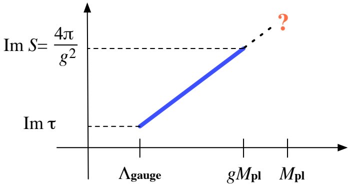

- $S = \frac{1}{\pi i} \log \epsilon$
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- New scale at $g M_{\text{planck}}$!

- Gauge theory periods $X^1, F_1 \sim \epsilon^{1/2}$
- Extra periods
  $X^{3,4} \sim O(1)$, $F_{3,4} \sim \log \epsilon$
- which is the source of the new scale !

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Periods of $\mathcal{N} = 2$ pure SYM + sugra

- special coordinates of pure SYM a^i, a_i^D $(i = 1, \ldots, r)$
- embedding into sugra: $S = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$ is also a field
- $\longrightarrow$ $r + 1$ parameters
- $\longrightarrow$ $2r + 4$ periods, i.e. FOUR extra periods
- gauge theory periods $X^i \sim \epsilon^{1/h} a^i$ and $F_i \sim \epsilon^{1/h} a_i^D$
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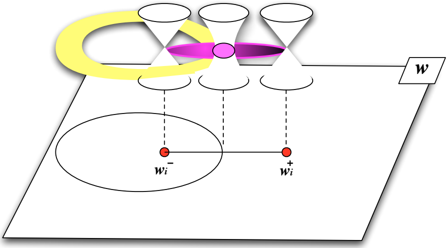
- we'll show two behave as $O(1)$, the other two $\log \epsilon$
- $\longrightarrow$ New Scale $g M_{\text{planck}}$

CY which is K3 fibration

$$w + \frac{\mu^2}{w} + W_{K3}(x, y, z; u_i, v_a) = 0$$



- K3 changes as w changes
- Suppose a two-cycle S shrinks at $w_i + \mu^2/w_i = k_i$
- → two three-cycles in CY !
- $X, F = \int_C \frac{dw}{w} \int_S \Omega_{K3}$

A_r singularity

- $w + \mu^2/w = x^2 + y^2 + z^n$
  → $SU(n)$ gauge group; $r = (n-1)$ two-cycles

$$w + \frac{\mu^2}{w} = x^2 + y^2 + z^n + u_2 z^{n-2} + u_3 z^{n-3} + \cdots + u_n + O(\mu^{1+1/n})$$

- Rescale according to the mass dimension :

$$\mu^2 = \epsilon^2, \quad x = \epsilon^{1/2}\tilde{x}, \quad y = \epsilon^{1/2}\tilde{y}, \quad z = \epsilon^{1/n}\tilde{z},$$
$$w = \epsilon\tilde{w}, \quad u_i = \epsilon^{i/n}\tilde{u}_i \quad \rightarrow$$

$$\tilde{w} + \frac{1}{\tilde{w}} = \tilde{x}^2 + \tilde{y}^2 + \tilde{z}^n + u_2 \tilde{z}^{n-2} + \ldots + u_n + O(\epsilon^{1/n})$$

- $(n-1)$ 2-cycles are at finite values of $\tilde{x}, \tilde{y}, \tilde{z}$ → $x, y, z \ll 1$

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- $(n-1)$ 2-cycles are at finite values of $\tilde{x}, \tilde{y}, \tilde{z}$ $\longrightarrow$ $x, y, z \ll 1$

- do they exhaust all the K3 cycles ? NO !

Recap: K3

- CY manifold of real dimension 4, i.e. $SU(2)$ holonomy
- 22 2-cycles
- two 2-cycles S, S' ⟶ intersection number $(S, S') = \#(S \cap S')$
- Signature is $(3, 19)$
- some 2-cycles C are holomorphically embedded ⟶ $\int_C \Omega_{2,0} = 0$
- other 2-cycles S are called transcendental; has signature $(2, x)$
- 2-cycles collapsible at a point (e.g. ADE singl.) have negative self-intersection ⟶ rank r singularity : $x \geq r$.

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  ⟶ FOUR extra CY periods.

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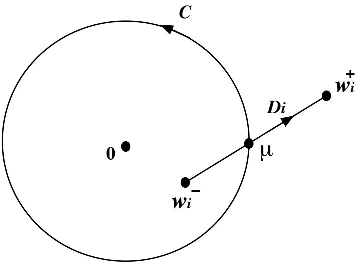
  ⟶ FOUR extra CY periods.
- Another nice matching of pure math and supergravity.

Logarithmic periods



- In the $\epsilon \to 0$ limit,

$$w+\frac{\epsilon^2}{w}+W_{K3}(x,y,z;\epsilon^{i/h}u_i,v_a)=0$$

becomes

$$w+\frac{\epsilon^2}{w}+W_{K3}(x,y,z;0,v_a)=0$$

- Suppose T_a shrinks at $w+\epsilon^2/w=k_a$
  → $w_a^+\sim k_a$, $w_a^-\sim\epsilon^2/k_a$ →

$$\int\Omega=\int_{w_a^-}^{w_a^+}\frac{dw}{w}\int_{T_a}\Omega_{K3}\sim 2c\log\epsilon$$

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RG equation

$$\frac{\partial \mathcal{F}_{\text{gauge}}}{\partial \log \Lambda} = u_2 \quad \text{where} \quad u_2 = \langle \operatorname{tr} \phi^2 \rangle$$

- First noticed by [Matone] for $SU(2)$ using SW curve
- later generalized to classical gauge groups by [Sonnenshein,Theisen,Yankielowicz] and [Eguchi,Yang], using SW curve
- it is built in the instanton calculation [Dorey,Khoze,Mattis] :

$$\langle \partial_\lambda L_0 \rangle = \partial_\lambda L_{\text{eff}}, \quad \langle \partial_\lambda W_0 \rangle = \partial_\lambda W_{\text{eff}}, \quad \langle \partial_\lambda \mathcal{F}_0 \rangle = \partial_\lambda \mathcal{F}_{\text{eff}}.$$

  while $F_0 = \tau_0 \operatorname{tr} \phi^2$, $\log \Lambda \propto \tau_0$
- for $E_{6,7,8}$, curves are known; RG relation not proven

Embedding into compact CY

- a^i, a_i^D : periods of gauge theory
- $X^i = \epsilon^{1/h} a^i$, $F_i = \epsilon^{1/h} a_i^D$
- extra periods X^a, F_a : Recall

$$w + \frac{\mu^2}{w} = W_{K3}(x, y, z; \epsilon^{i/h} u_i, v_a)$$

for small ϵ, so that extra cycles are at finite values of x, y, z
- one can calculate X^a, F_a perturbatively in $\epsilon^{i/h} u_i$.

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for small ϵ, so that extra cycles are at finite values of x, y, z

- one can calculate X^a, F_a perturbatively in $\epsilon^{i/h} u_i$.

- they have **log** ϵ,
- but all u_i dependence is analytic in $\epsilon^{i/h} u_i$.

Proof.

Let us prove

$$\frac{\partial \mathcal{F}_{\text{gauge}}}{\partial \log \Lambda} = u_2 \longrightarrow \frac{\partial^2 \mathcal{F}_{\text{gauge}}}{\partial u_j \partial \log \Lambda} = \delta_2^j.$$

First, rewrite LHS :

$$= \frac{\partial}{\partial u_j}\left(2\mathcal{F}_{\text{gauge}} - \sum_i a^i \frac{\partial \mathcal{F}_{\text{gauge}}}{\partial a^i}\right) = \sum_i \frac{\partial a^i}{\partial u_j} a_i^D - \sum_i a^i \frac{\partial a_i^D}{\partial u_j}.$$

i.e.

$$\sum_i F_i \frac{\partial X^i}{\partial u_j} - \sum_i X^i \frac{\partial F_i}{\partial u_j} = \epsilon^{2/h} \frac{\partial^2 F_{\text{gauge}}}{\partial u_j \partial \log \Lambda} + O(\epsilon^{3/h})$$

Proof.

- Consider the transversality condition

$$\sum_i F_i \frac{\partial X^i}{\partial u_j} - \sum_i X^i \frac{\partial F_i}{\partial u_j} = -\sum_a F_a \frac{\partial X^a}{\partial u_j} + \sum_a X^a \frac{\partial F_a}{\partial u_j}.$$

  which came from $\int_{\mathrm{CY}} \Omega \wedge \partial_{u_j} \Omega = 0$.
- X^a, F_a analytic in $\epsilon^{i/h} u_i$ $\longrightarrow$ RHS $=$ const $\cdot\, \delta_j^2 \cdot \epsilon^{2/h} + O(\epsilon^{3/h})$
- F_a contain $\log \epsilon$, but they should cancel in RHS
- DONE !
- const fixes the proportionality factor between u_2 in CY and $\langle \mathrm{tr}\, \phi^2 \rangle$

Comments

- First proof for E-type gauge groups.
- Known proofs for classical gauge groups similar :
  there, SW differential has a pole at infinity of the SW curve.
  $\longrightarrow$ Riemann bilinear id. leads to

$$\sum_i \frac{\partial a^i}{\partial u_j} a_i^D - \sum_i a^i \frac{\partial a_i^D}{\partial u_j} = \text{contribution from the pole}$$

- cycles at finite $\tilde{x}, \tilde{y}, \tilde{z}$; poles at infinite $\tilde{x}, \tilde{y}, \tilde{z}$
  $\Longleftrightarrow$ gauge-theory cycles at infinitesimal x, y, z ;
  extra cycles at finite x, y, z
- for exceptional gauge groups, the curve is not hyperelliptic
  $\longrightarrow$ horrible poles ! which prevented the proof.

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$$\frac{\partial_\mu S \partial_\mu S}{(\mathrm{Im}\, S)^2}.$$

- Byproduct: the RG equation

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understood from the embedding into supergravity,

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understood from the embedding into supergravity,

- Holomorphy was the key.
- Holomorphy will tell us more about quantum gravity !