Five-dimensional Chern-Simons terms and Nekrasov's instanton counting

東大本郷 **立川裕二**

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[hep-th/0401184]

X: Calabi-Yau 3-fold

IIA on X	A-model on X

Consider:

Graviphoton-corrected Prepotential	Free energy

Correspondence:

$$\mathcal{F} = \log Z$$

[Bershadsky-Cecotti-Ooguri-Vafa]
hep-th/9309140
[Antoniadis-Gava-Narain-Taylor]
hep-th/9307158

Exactly calculable by

Nekrasov's Instanton Counting (as 4-dim N=2 SYM)	Topological Vertex
hep-th/0206161	hep-th/0305132

Where

$$\hbar \longleftrightarrow g_s$$

Graviphoton Field Strength	String Coupling

Instanton Counting

For pure U(N)

Method: **4+1 dim'l SUSY field theory**

Reduction

0+1 SUSY QM on the Instanton Moduli

Hence:

$e^{\mathscr{F}/\hbar^2}$ **= (generalized) Witten index of the Moduli !**

Localizes onto

Lorentz-invariant Anti-Self-Dual Config.

They're labeled by

N-tuples of Young tableaux.

#boxes=#instantons

4D formula

$$e^{\mathscr{F}/\hbar^2} = \sum_{Y_1,\dots,Y_N} \Lambda^{\sum \ell_{Y_i}} \prod_{l,n=1}^{N} \prod_{i,j=1}^{\infty} \frac{a_{ln}+\hbar(y_{l,j}-y_{n,i}+j-i)}{a_{ln}+\hbar(j-i)}$$

5D formula

β : circumference of the fifth direction

$$e^{\mathscr{F}/\hbar^2} =: \sum_{Y_1,\dots,Y_N} \left(\frac{q}{4^N}\right)^{\sum \ell_{Y_i}} \prod_{l,n=1}^{N} \prod_{i,j=1}^{\infty} \frac{\sinh\frac{\beta}{2}(a_{ln}+\hbar(y_{l,j}-y_{n,i}+j-i))}{\sinh\frac{\beta}{2}(a_{ln}+\hbar(y_{l,j}-y_{n,i}))}$$

Topological Vertex

A-Model Partition function on

local toric CY ← **Several $\mathbb{C}^3$ Sewn**

Feynman-like Diagrammatic Rule

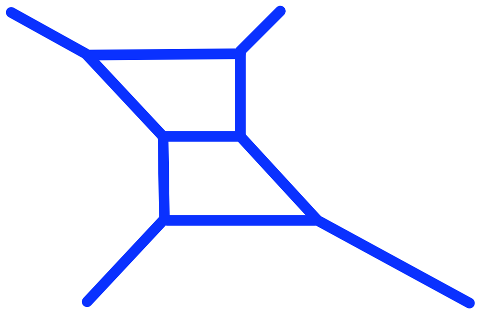
$\mathbb{C}^3$ - - - - - - - - - - - - - **vertex**

sewing - - - - - - - - - - **propagator**

For the local toric CY

X_N^0 **: Blown up** $\mathbb{C}^2/\mathbb{Z}_N$

fibered over S^2



ex.) N=3

$$\mathbf{Z}= \sum_{Y_1,\ldots,Y_N} \left(\frac{q}{4^N}\right)^{\sum \ell_{Y_i}} \prod_{l,n=1}^{N} \prod_{i,j=1}^{\infty} \frac{\sinh \frac{\beta}{2}(a_{ln} + \hbar(y_{l,j} - y_{n,i} + j - i))}{\sinh \frac{\beta}{2}(a_{ln} + \hbar(y_{l,j} - y_{n,i}))}$$

[Iqbal-KashaniPoor] **hep-th/0305132**

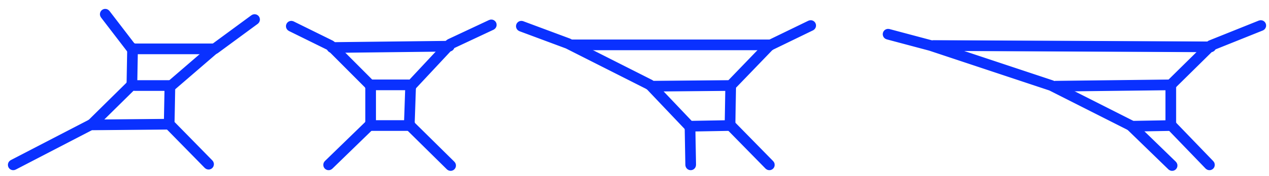
Exactly Matches and

Captures M-theory!

However...

Many other local toric CY for pure U(N)

ex.) $X^k_{N=3}$



$k =$ **0** **1** **2** **3**

Distinct partition functions for each!

How can they match
with the Instanton Counting?

Clues:

IIA, A-model $\downarrow$ **triple intersection of CY**

"M-theoretic" $\downarrow$ **Five Dimensional**

5-dim Chern-Simons term !

Indeed, [Intriligator-Morrison-Seiberg]

hep-th/9702198

have shown M-theory on X_N^k

gives tree level Lagrangian

Determined by Geometry!

$$\boxed{\int d^5x \frac{1}{g^2} F_{\mu\nu} F_{\mu\nu} + ik \int CS(A,F)}$$

where

$$\boxed{dCS(A,F) = \mathrm{tr}(F \wedge F \wedge F)}$$

How does this affect the Instanton Counting?

Dim'l reduction

4+1 SYM $\longrightarrow$ **0+1 SUSY QM**

kinetic term $\longrightarrow$ **kinetic term**

non-Abelian Chern-Simons $\longrightarrow$ **U(1) external field on the Moduli**

$\Downarrow$

Determinant Line Bundle!

- ☆ **Follows from the Classic works of Alvarez-Gaumé et al.**
- ☆ **Explicitly constructible using ADHM**
- ☆ **Needs NC to tame the small-instanton singularity**
- ☆ **Needs NC versions of [Alvarez-Gaumé et al.]**

$$e^{\mathscr{F}/\hbar^2} = \sum_{Y_1,\ldots,Y_N} \left(\frac{q}{4^N}\right)^{\sum_i \ell_{Y_i}} e^{-k\beta\sum(\ell_{Y_i}a_i+\hbar\kappa_{Y_i})} \times \prod_{l,n=1}^{N}\prod_{i,j=1}^{\infty} \frac{\sinh\frac{\beta}{2}(a_{ln}+\hbar(y_{l,j}-y_{n,i}+j-i))}{\sinh\frac{\beta}{2}(a_{ln}+\hbar(y_{l,j}-y_{n,i}))}$$

Summary

5 dim'l N=1 pure U(N) SYM

Top. Vertex $\longleftarrow$ **BCOV** $\longrightarrow$ **Inst. Counting**

X_N^0 — **ok** [Iqbal,Kashani-Poor]

X_N^1 — **?**

X_N^2 — **?**

Inclusion of 5-dim non-Abelian Chern-Simons
Solves the discrepancy!

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Outlook

Proof of the equivalence of the both sides
for general local toric compactification???