Vacuum counting in Multi-parameter Models

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to appear

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String Theorists' **Holy Grail:**

Construct the Standard Model+Inflation
inside String/M-theory

However:

Many **Moduli**
(Gauge-neutral massless scalar particles)

Recent Progress!

Type II B on a **Calabi-Yau orientifold**
+ NSNS, RR fluxes

complex structure moduli

$$W = \int \Omega_{(3,0)} \wedge (\tau H_{NSNS} + H_{RR})$$

Gukov-Vafa-Witten

FIXED!

Kahler moduli

$$W = \sum_i \exp(-\langle C_i, \rho \rangle)$$

D-brane instanton/ Gaugino Condensation

FIXED!

[Kachru-Kallosh-Linde-Trivedi, 0301240]

the Discretuum

Moduli is FIXED, but

$$N_{D3} + \int H_{NSNS} \wedge H_{RR} = \frac{\chi(CY_4)}{24}$$

# of 3-cycles ~ 100 χ**/24~1000**

→ **# of 'vacuum' ~ 10^{100}**

Good in a sense:

Tuning on the level of **10^{20}**
is no longer **a fine tuning**

Bad in a sense:

Haven't we believed that

String theory should provide a **unique vacuum,** which is **our world.**

This is not necessarily the speaker's opinion.

More Practically...

Studying each vacua **one-by-one**
is **not feasible !**

Douglas' Copernican Revolution

Douglas *et al.* 0303194,0307049

Study the statistical properties of the ensemble of vacua!

Distribution of ...

- **vacua** on the 'moduli space'
- **the cosmological constant**

Take:

Kahler potential: **given and fixed**

Superpotential: **random**

with correlation $\langle W(\phi_1) W(\phi_2)^* \rangle = e^{-K(\phi_1, \phi_2{}^*)}$

- Consistent with **Kahler transformation**
- Satisfied by the **Gukov-Vafa-Witten** superpotential with **monodromy-invariant flux ensemble**

Distribution function
of AdS supersymmetric vacua:

$$\rho_{vac}(\phi) = \langle \delta(D_i W(\phi)) |\det(\partial_i D_j W)| \rangle$$

Behavior of the index density

Instead, consider

$$\rho_{ind}(\phi) = \langle \delta(D_i W(\phi)) \det(\partial_i D_j W) \rangle$$

This counts the vacua with

+1 if $\prod_{i=1}^{n} |m_i| > |W|^n$

-1 if $\prod_{i=1}^{n} |m_i| < |W|^n$

$$\rho_{ind}(\phi) \prod_i d\phi_i d\phi_i^* = \det(R^i_j + \delta^i_j \omega)$$

where

R^i_j : **curvature 2-form** on the moduli space

ω : **the Kahler form** on the moduli space

Curvature **small** → $\rho_{ind} \sim \omega^n$ (=volume form)

Curvature **large** → $\rho_{ind} \sim \det R$ (=Euler class)

Huge enhancement at **Curvature Singularity!**

Enhancement at the conifold point

Consider: **type IIB on one-modulus CY with flux,**

Denef-Douglas, 0404116

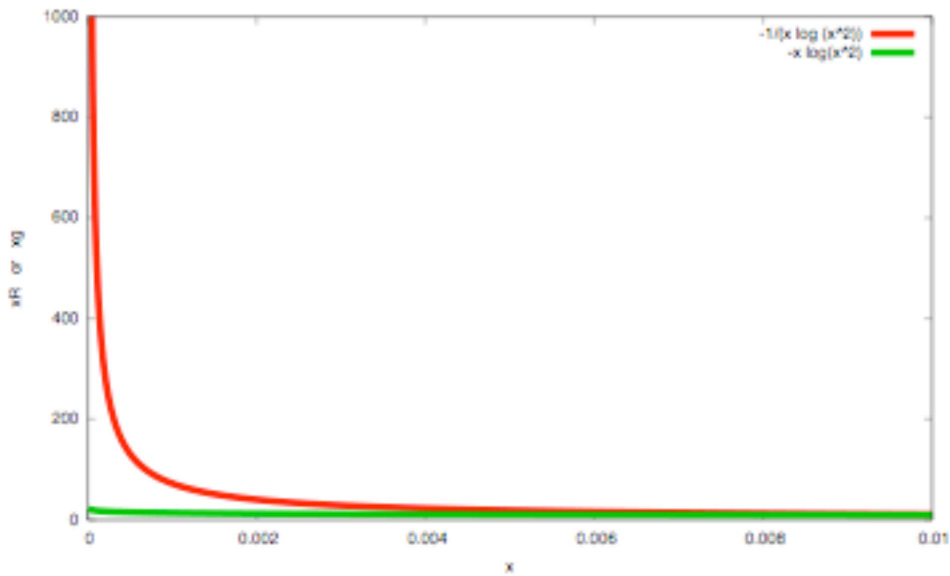
Giryavets-Kachru-Tripathy, 0404243

Near the conifold point, (ϕ=0)

$$g_{\phi\phi^*} \sim \log|\phi|^2$$

$$R_{\phi\phi^*} \sim \frac{1}{\phi\phi^*(\log|\phi|)^2} \gg g_{\phi\phi^*}$$

$\int R$ **is barely convergent !**



Maybe good for stringy inflation...

Our Work

- Extension to **multiple moduli**

  (mirror of) degree 8 hypersurface in $\mathbb{WCP}^4_{1,1,2,2,2}$

- **Different** types of **singularities**

  Large Complex Structure Limit

  Large Radius Limit of the mirror CY

  Geometric Engineering Limit

  Low Energy **Super Yang-Mills** decouples

  Argyres-Douglas Point

  "Electron" and "Monopole" become simultaneously massless

- Is the integral **always convergent ?**

  Around Conifold Locus, YES.

Two-modulus Example

$$\frac{1}{8}x_1^8 + \frac{1}{8}x_2^8 + \frac{1}{4}x_3^4 + \frac{1}{4}x_4^4 + \frac{1}{4}x_5^4 - \psi_0 x_1 x_2 x_3 x_4 x_5 - \frac{1}{4}\psi_s (x_1 x_2)^4 = 0$$

in $\mathbb{WCP}^4_{1,1,2,2,2}$

Denote $\epsilon = \frac{1}{2\psi_s}$, $u = \psi_s + \psi_0^4$

$\epsilon \to 0, u \to \text{finite}$ **Geometric Engineering Limit**

$|\epsilon|^{1/2} \log|1/\epsilon|$: **dynamical scale** of SU(2) SYM

u : Seiberg-Witten's **u**

$\epsilon \to 0, (\epsilon u)^{-1} \to 0$ **Large Complex Structure Limit**

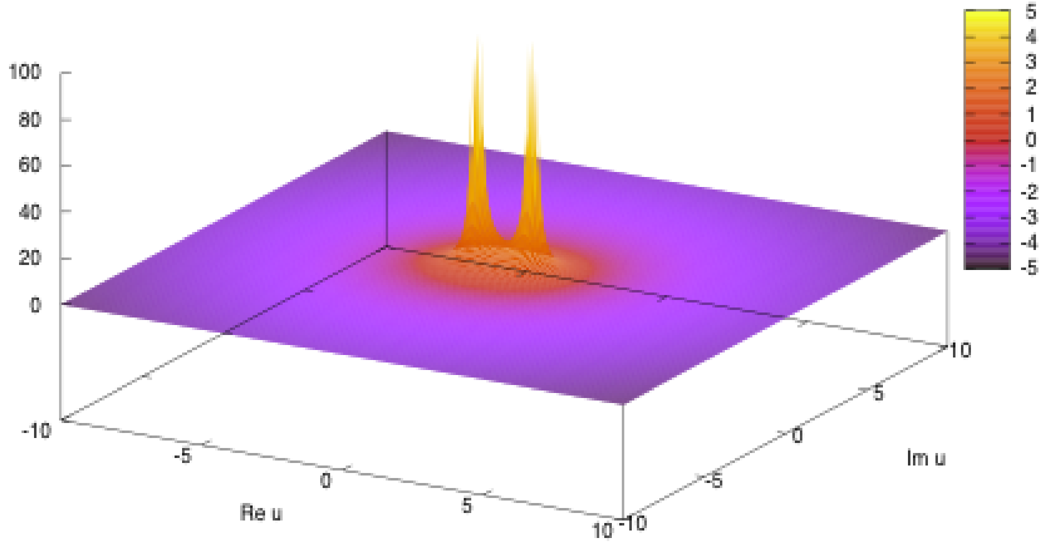
$-\log\epsilon$, $\log(\epsilon u)$

:**area** of two 2-cycles of the **mirror**

For fixed ϵ , u **takes value in** $\mathbb{CP}^1$

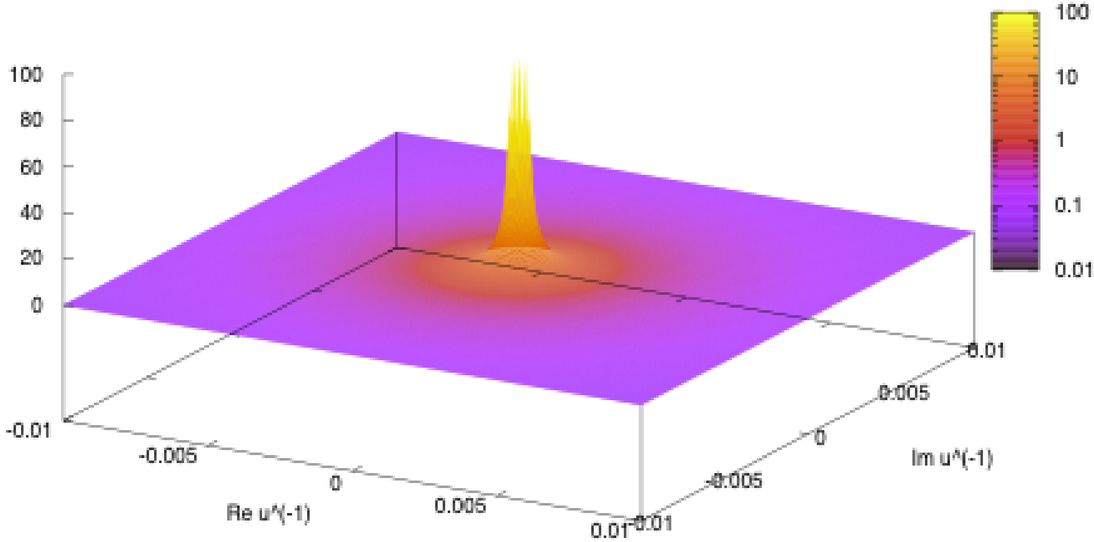
Let us look at u dependence with $\epsilon = 0.001$

$u \sim 0$



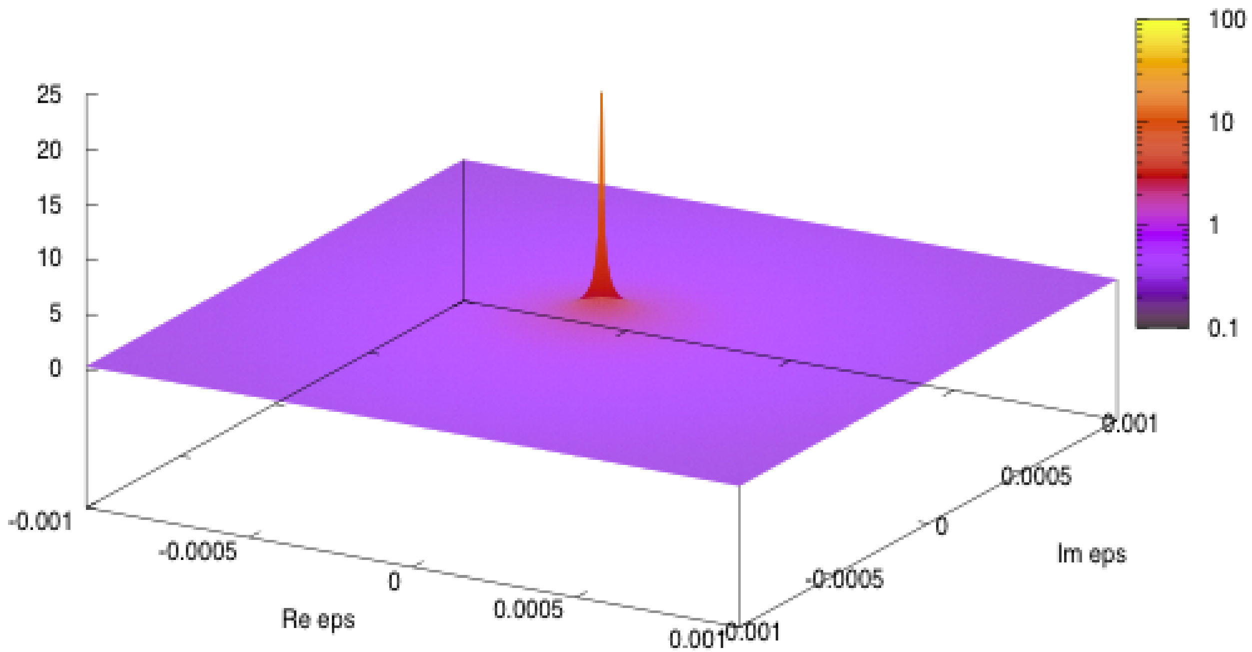
$$\det(R + \omega) \sim \frac{1}{|u-1|^2(\log|u-1|)^2}$$

$u \sim \infty$



$$\det(R + \omega) \sim \frac{1}{|u|^{-2}(\log|u|)^4}$$

Next, let $u = 5$ **and vary** ϵ



$$\det(R + \omega) \sim \frac{1}{|\epsilon|^1 (\log|\epsilon|)^3}$$

if

$$1/\epsilon \gg u \gg 1 \quad .$$

$$\longrightarrow \quad \frac{1}{|\epsilon|^3 (\log|\epsilon|)^3}$$ **behavior?**

Mathematicians are actively investigating these convergence properties !

Summary

- Lots of progress in **moduli fixing** recently
- **Huge number** of vacua

  → **Statistical properties** of vacua
- Studied the behavior of **index density** around **various types of singularities**
- Studied an **explicit example with two moduli**

Outlook

- Taking **the Kahler moduli** into account
- **Phenomenological application**
- **Convergence properties** of the index density
- What's the **correct** *a priori* **probability?**
- What's the correct interpretation of **multi-valued superpotential**