AdS/CFT correspondence

with

Eight Supercharges

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1. Introduction

AdS$_{d+1}$/CFT$_d$ correspondence

Equivalence of

- Conformal Field Theory in d dimensions to
- Gravitational theory in $(d+1)$ dimensional Anti de Sitter spacetime

- Many kinds of CFT $\Leftrightarrow$ Many kinds of grav. theory

- Theme of the thesis : AdS$_5$/CFT$_4$

Why AdS/CFT ?

Conformal Field Theory

- Looks the same in every scale, has **scale invariance**
- Nontrivial dynamics in low energy limit

CFT$_2$:

- Models of critical phenomena
- Describes string worldsheet

CFT$_4$?

Conformal Field Theory in 4D

- ◇ Strongly coupled gauge theory
- ◇ Conformal symmetry makes it a bit more tractable

$\Rightarrow$ Interesting as toy model for other strongly-coupled phenomena

My main interest:

$$\mathcal{N} = 1, \text{ \textit{4D superconformal field theory}}$$

- ◇ Supersymmetry: exchange bosons and fermions

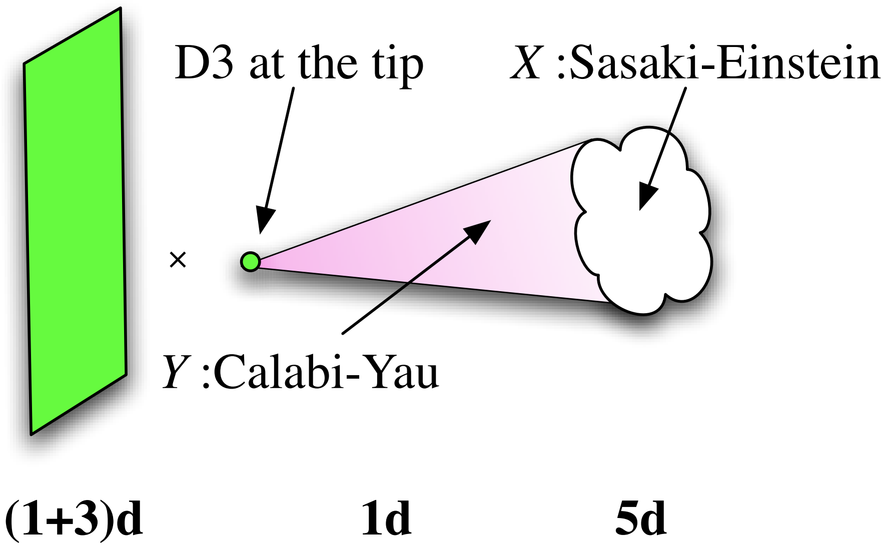
$\Rightarrow$ partial cancellation of quantum corrections

- ◇ SUSY + scaling symmetry $\Rightarrow$ much more tractable !
- ◇ has 8 supercharges = 8 fermionic sym. generators

String theory and CFT$_4$

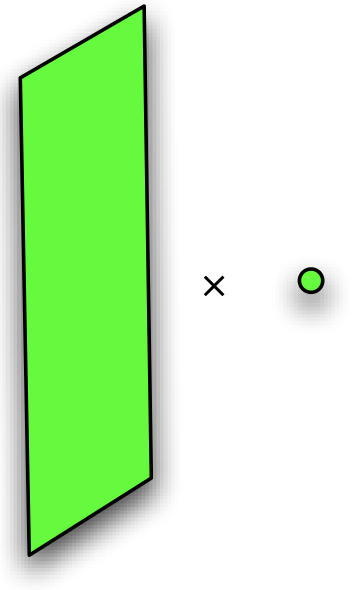
Type IIB string theory:

- **gravity** from closed strings
- **gauge fields** from open strings attached to D-branes



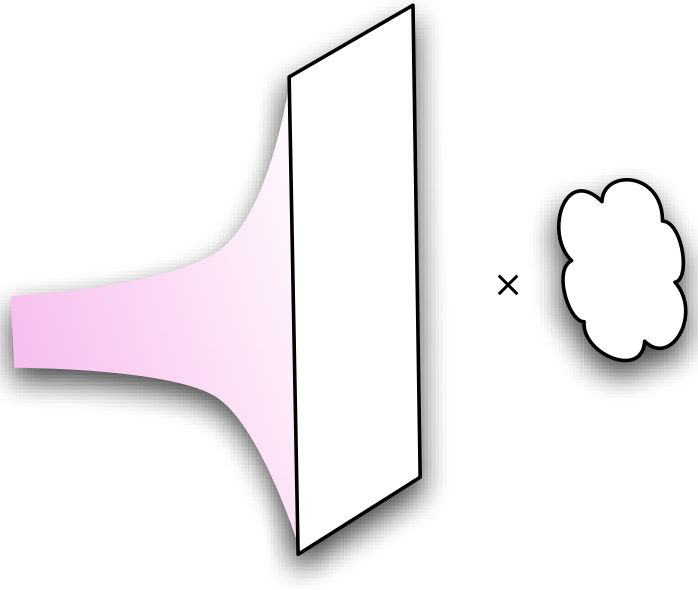
- Introduce $N \gg 1$ branes and take low energy limit in two ways:

Field theory on the D3



Some gauge theory

Near Horizon Limit of the D3



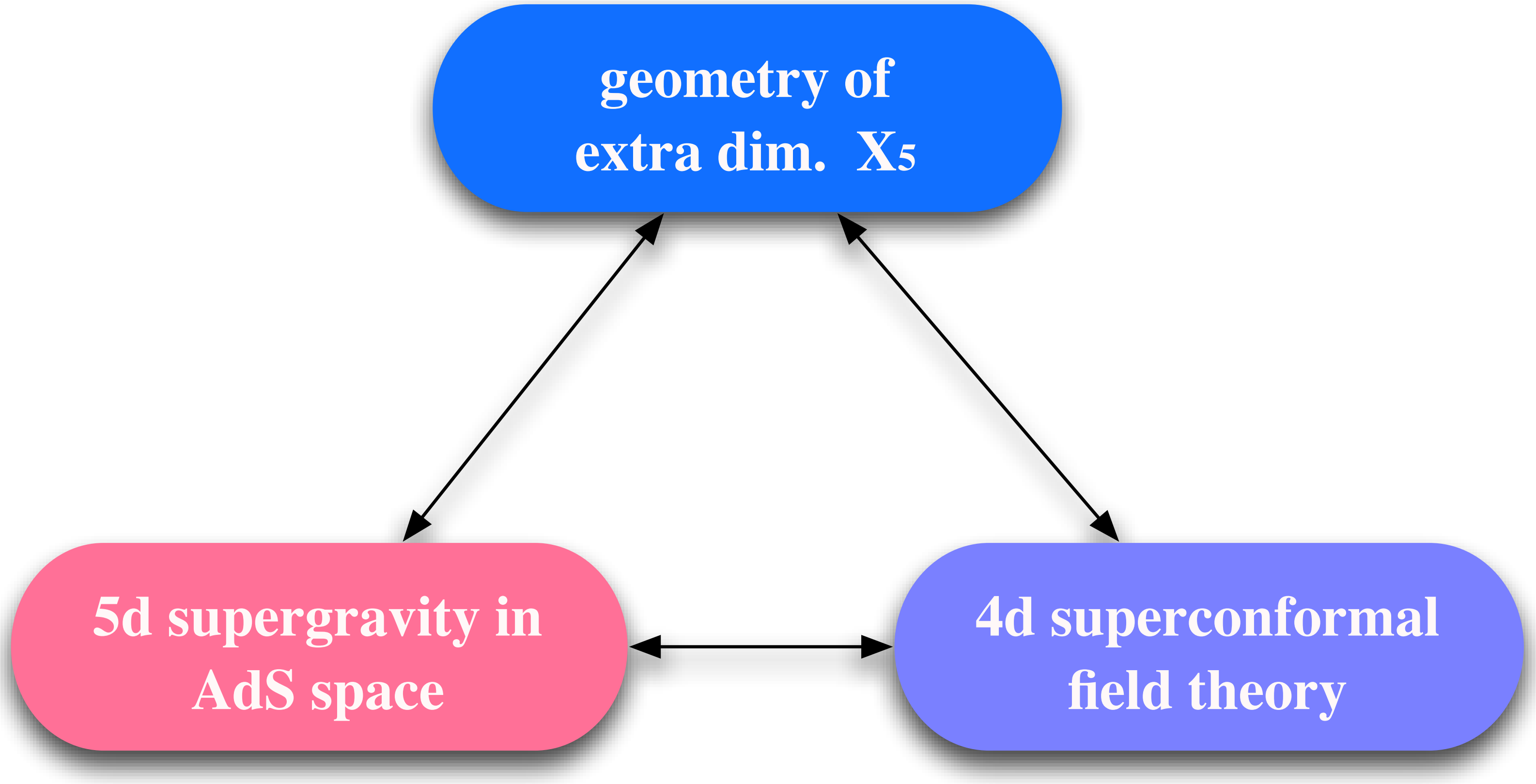
$$\mathrm{AdS}_5 \times X_5$$

- ◇ **Should describe the same physics.** (Maldacena)

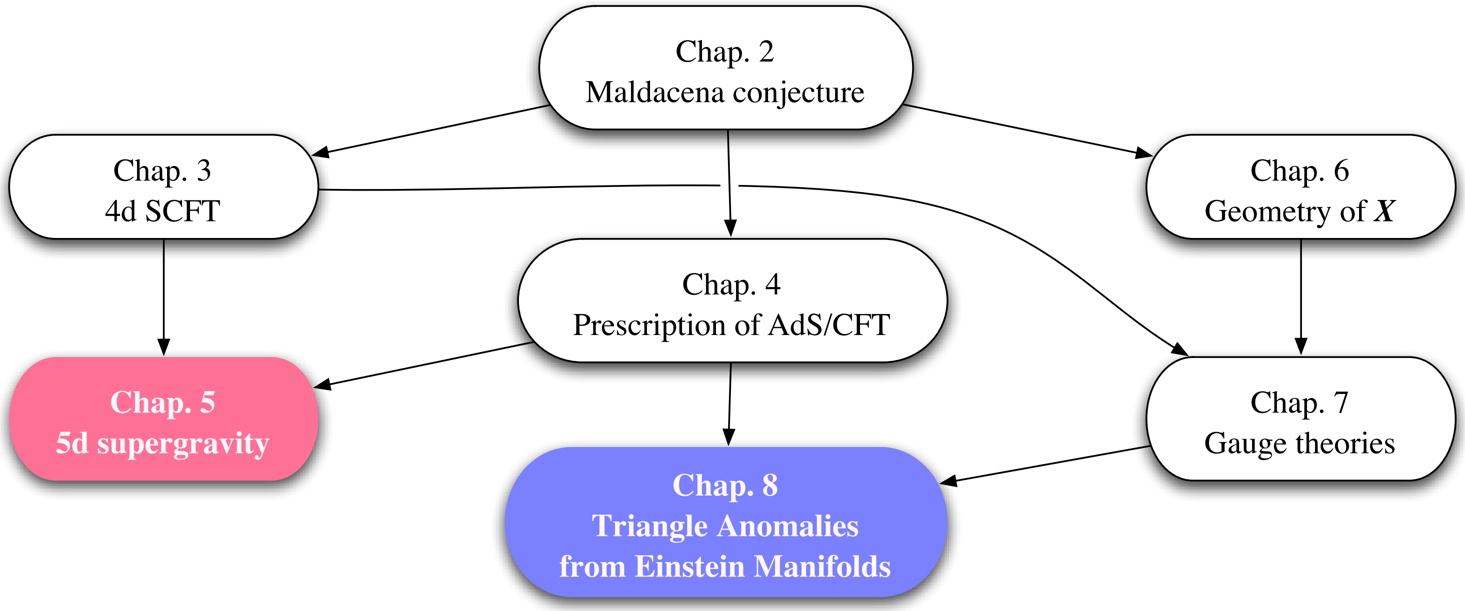
AdS/CFT correspondence

Field theory on $\mathbb{R}^{3,1}$		Gravity on $\mathrm{AdS}_5 \times X_5$
Conformal symmetry	$=$	Isometry of AdS
Global symmetry	$=$	Isometry of X
central charge a	$=$	$1/\,\mathrm{Vol}(X)$
weak coupling	$\leftrightarrow$	strong coupling
strong coupling	$\leftrightarrow$	weak coupling

AdS/CFT with eight supercharges



organization of the thesis



- YT, "Five-dimensional supergravity dual of a-maximization," Nucl. Phys. B733 (2006) 188. (hep-th/0507057) (Chap. 5)
- S. Benvenuti, L. A. Pando Zayas and YT, "Triangle anomalies from Einstein manifolds," Adv. Theor. Math. Phys. 10 (2006) 395. (hep-th/0601054) (Chap. 8)

CONTENTS

2. Basics: a-maximizaion

$\mathcal{N}=1$ Susy Algebra in 4d

$$\{Q_\alpha, Q^\dagger_{\dot\beta}\} = \sigma^\mu_{\alpha\dot\beta} P_\mu$$

Conformal Algebra in 4d

$$[K_\mu, P_\nu] = \Delta \delta_{\mu\nu} + \cdots$$

$\mathcal{N}=1$ Superconformal Algebra in 4d

$$\{Q_\alpha, S^\alpha\} \sim R_{SC} + \cdots$$
$$[R_{SC}, Q_\alpha] = -Q_\alpha$$
$$[R_{SC}, S_\alpha] = \quad S_\alpha$$

◇ **Importance of R_{SC} :**

- $\Delta \geq \frac{3}{2} R_{SC}$, equal for chiral primary op.

- $a = \frac{3}{32}(3\,\mathrm{tr}\, R_{SC}^3 - \mathrm{tr}\, R_{SC})$

where, for 4d CFT,

$$\langle T^\mu_\mu \rangle = \frac{1}{16\pi^2}\left(c\, W^2_{\mu\nu\rho\sigma} - a\, R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} \right)$$

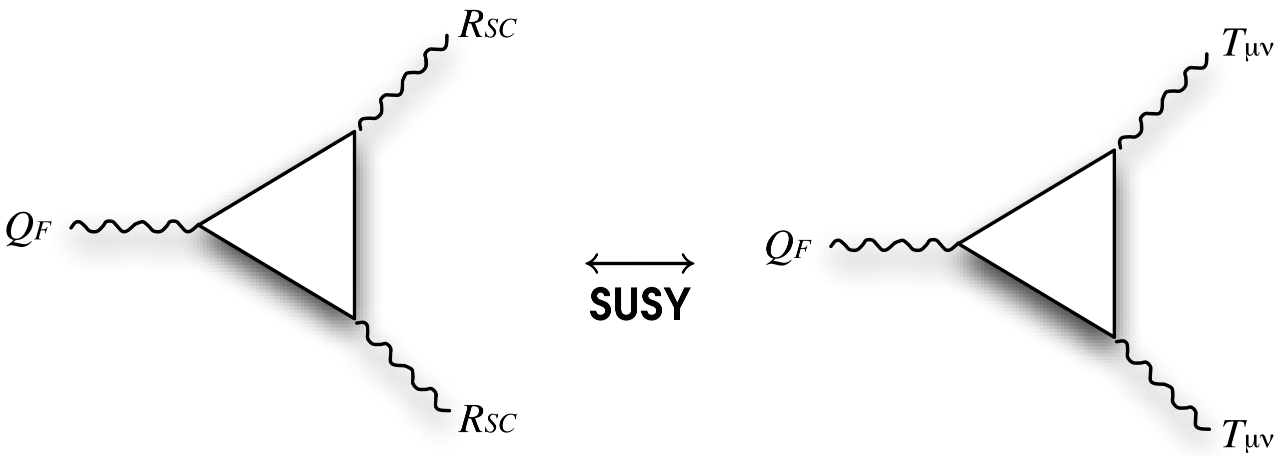
cf. for 2d CFT, $\langle T^\mu_\mu \rangle \propto cR$.

◇ Many global symmetries Q_I with $[Q_I, Q_\alpha] = -\hat{P}_I Q_\alpha$.

◇ R_{SC} is a linear combination : $R_{SC} = r^I Q_I$.

How can we find R_{SC} ?

◇ Call $Q_F = f^I Q_I$ with $[Q_F, Q_\alpha] = 0$ a flavor symmetry.



$$9\,\text{tr}\, Q_F R_{SC} R_{SC} \quad = \quad \text{tr}\, Q_F$$

- $[r^I Q_I, Q_\alpha] = -Q_\alpha \Rightarrow r^I \hat{P}_I = 1.$

- Let $a(s) = \frac{3}{32}(3\,\mathrm{tr}\,R(s)^3 - \mathrm{tr}\,R(s))$ where $R(s) = s^I Q_I$.

- $9\,\mathrm{tr}\,Q_F R_{SC} R_{SC} = \mathrm{tr}\,Q_F \Rightarrow r^I$ extremizes $a(s)$ under $s^I \hat{P}_I = 1$.

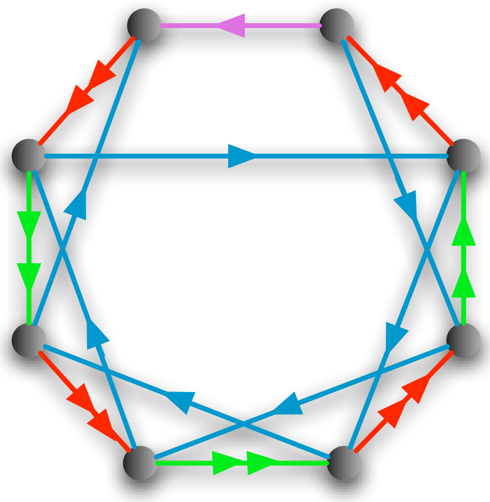
- Unitarity $\Rightarrow$ it's a local maximum.

a -maximization ! (Intriligator-Wecht, 2003)

- $\hat{c}_{IJK} = \mathrm{tr}\,Q_I Q_J Q_K$ and $\hat{c}_I = \mathrm{tr}\,Q_I$:

  Calculable at UV using 't Hooft's anomaly matching.

$Y^{p,q}$ quiver theory

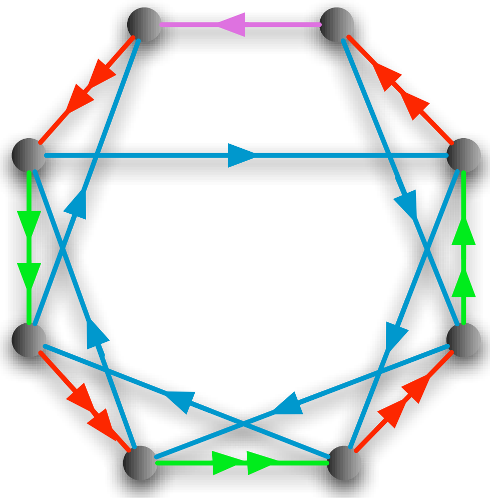


- ◇ Blob : an $SU(N)$ gauge group + gaugino
- ◇ Arrow from a-th to b-th blob: $N \times N$ scalar+fermion fields $\Phi_{i\bar{\jmath}}$,

  i acted by a-th $SU(N)$ and $\bar{\jmath}$ acted by b-th $SU(N)$

cf. Many arrows $\longrightarrow$ quiver (n): 箙(えびら), 矢筒

$Y^{p,q}$ quiver theory



$SU(N)^{2p}$ gauge theory with bifundamentals

	W_I^α	Y	Z	U^i	V^i
multiplicity	$2p$	$p+q$	$p-q$	p	q
R-sym	1	s_1	s_2	$1-(s_1+s_2)/2$	$1+(s_2-s_1)/2$

Maximize $a(s_1, s_2) \Rightarrow$

$$a = \frac{N^2 p^2}{4q^4}\left(-8p^3 + 9pq^2 + \sqrt{4p^2 - 3q^2}^{\,3}\right)$$

and

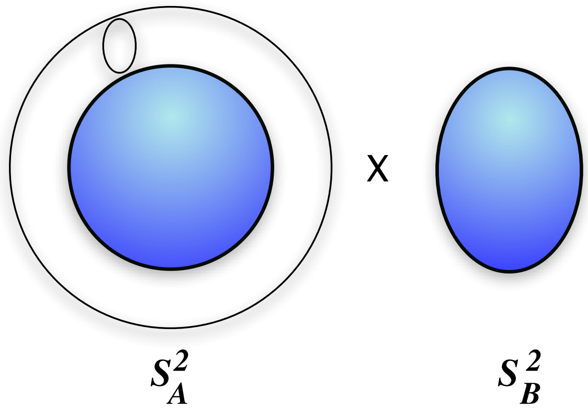
$$s_1 = \frac{1}{3q^2}\left(-4p^2 + 2pq + 3q^2 + (2p - q)\sqrt{4p^2 - 3q^2}\right),$$
$$s_2 = \frac{1}{3q^2}\left(-4p^2 - 2pq + 3q^2 + (2p + q)\sqrt{4p^2 - 3q^2}\right).$$

$\Rightarrow$ R-sym: irrational $\Rightarrow$ Anomalous dimensions: irrational

- $Y^{p,q}$: explicit 5d Sasaki-Einstein metric found in **2004**

- The cone over $Y^{p,q}$ = the Moduli of $Y^{p,q}$ quiver theory

- S^1 'winds' p times on S^2_A and q times on S^2_B



$$\mathrm{Vol}(Y^{p,q}) = \frac{q^2(2p+\sqrt{4p^2-3q^2})}{3p^2(3q^2-2p^2+p\sqrt{4p^2-3q^2})}\pi^3 \quad \Rightarrow \quad a = \frac{N^2}{4}\frac{\pi^3}{\mathrm{Vol}} \quad \text{!!}$$

CONTENTS

3. Supergravity dual

◇ Any CFT phenomena $\Rightarrow$ AdS counterpart

AdS		CFT
ϕ	$\leftrightarrow$	$\mathcal{O}$
$Z[\phi(x)\|_{x_5=\infty} = \hat{\phi}(x)]$	$=$	$\langle e^{-\int \hat{\phi}(x)\mathcal{O}(x)d^4x} \rangle$
A_μ^I	$\leftrightarrow$	J_I^μ : current for Q_I
$Z[A_\mu^I(x)\|_{x_5=\infty} = \hat{A}_\mu^I(x)]$	$=$	$\langle e^{-\int \hat{A}_\mu^I J_I^\mu} \rangle_{SCFT}$

◇ How about a -maximization ? $\Rightarrow$ P -minimization ! (YT)

Q_I has **triangle anomalies** among them :

$$\delta_\chi(\langle e^{-\int \hat{A}^I_\mu J^\mu_I}\rangle_{SCFT}) = \int d^4x \frac{1}{24\pi^2}\hat{c}_{IJK}\,\chi^I\, F^J \wedge F^K$$
$$= \int d^5x \frac{1}{24\pi^2}\hat{c}_{IJK}\, d\chi^I \wedge F^J \wedge F^K$$
$$= \delta_\chi\left(\int d^5x \frac{1}{24\pi^2}\hat{c}_{IJK}\, A^I \wedge F^J \wedge F^K\right)$$

$\Rightarrow$ Presence of **Chern-Simons** terms in AdS

- ◇ $\hat{c}_I = \mathrm{tr}\, Q_I \leftrightarrow \hat{c}_I A^I \wedge \mathrm{tr}\, R \wedge R$, higher derivative effect.
- ◇ $\mathcal{O}(N^{-2})$ in the CFT side. $\Rightarrow$ Neglect them henceforth

◇ 4d $\mathcal{N}=1$ SCFT $\Rightarrow$ 5d $\mathcal{N}=2$ supergravity

Gravity multiplet + Vector multiplet

$$g_{\mu\nu}, \qquad \psi^i_\mu, \qquad A^I_\mu, \qquad \lambda^x_i, \qquad \phi^x$$

where $i=1,2$ for $SU(2)_R$, $\quad I: 0,\ldots,n_V$ and $x: 1,\ldots,n_V$

◇ Presence of the **Chern-Simons** term with c_{IJK} constants :

$$\frac{1}{6\sqrt{6}} c_{IJK}\epsilon^{\mu\nu\rho\sigma\tau} A^I_\mu F^J_{\nu\rho} F^K_{\sigma\tau} \Rightarrow c_{IJK} = \frac{\sqrt{6}}{16\pi^2} \mathrm{tr}\, Q_I Q_J Q_K.$$

◇ ϕ^x parametrize the hypersurface in (n_V+1) dim space of $\{h^I\}$:

$$F = c_{IJK} h^I h^J h^K = 1.$$

Potential

◇ P_I^r : triplet generalization of the Fayet-Iliopoulos term where $r = 1, 2, 3$ label the triplets in $SU(2)_R$.

◇ The potential V is given by ($P^r = P_I^r h^I$)

$$V = 3g^{xy}\partial_x P^r \partial_y P^r - 4P^r P^r$$

◇ Covariant derivative of the gravitino

$$D_\nu \psi_\mu^i = \partial_\nu \psi_\mu^i + A_\mu^I P_I^r \sigma_{rj}^i \psi_I^j$$

◇ SUSY transformation law

$$\delta_\epsilon \phi^x = \frac{i}{2}\bar{\epsilon}^i \lambda_i^x, \qquad \delta_\epsilon \lambda_x^i = -\epsilon_j \sqrt{\frac{2}{3}} \sigma_r^{ij} \partial_x P^r + \cdots$$

Recap.

Field theory	$\Leftrightarrow$	Supergravity
$\hat{c}_{IJK} = \mathrm{tr}\, Q_I Q_J Q_K$		$c_{IJK} h^I h^J h^K = 1$
$[Q_I, Q_\alpha] = -\hat{P}_I Q_\alpha$		$D_\mu \psi^i_\nu = (\partial_\mu + P^r_I \sigma^i_{r\,j} A^I_\mu)\psi_\nu$
a-max		???

SUSY condition for sugra

- $\delta\lambda = \epsilon_j \sigma_r^{ij} \partial_x P^r = 0 \Rightarrow \langle h^I_{,x} \rangle P^r_I = 0$
- $\langle h^*_{,x} \rangle$ and P^r_* is **perpendicular** as $(n_V + 1)$ dim'l vectors.
- $x = 1, \ldots, n_V \Rightarrow P^{r=1,2,3}$ are **parallel** $\Rightarrow P^{r=1,2} = 0$, $P^{r=3} \neq 0$.
- $c_{IJK} h^I h^J h^K = 1 \Rightarrow \underbrace{c_{IJK} h^I h^J}_{h_K} h^K_{,x} = 0$

$$\langle h_I \rangle \propto P_I^{r=3}$$

◇ **Recall**

AdS		CFT
ψ_μ	$\leftrightarrow$	Q_α, S^α
A^I_μ	$\leftrightarrow$	Q_I
$D_\nu \psi^i_\mu = (\partial_\nu \delta^i_j + A^I_\mu P^r_I \sigma^i_{rj})\, \psi^j_I$	$\leftrightarrow$	$[Q_I, Q_\alpha] = -\hat{P}_I Q_\alpha$.

$\Rightarrow P^{r=1,2} = 0 \Rightarrow$ **charge** of Q_α, S^α under Q_I is $\pm P^{r=3}_I$.

$\Rightarrow P^{r=3}_I = \hat{P}_I$.

◇ **SUSY tr. of hypers** $\Rightarrow R_{SC} = r^I Q_I \propto \langle h^I \rangle Q_I$.

◇ **Recall** $r^I P_I = 1. \Rightarrow r^I = \dfrac{\langle h^I \rangle}{\langle h^I \rangle P_I}$.

◇ $R_{\text{trial}} = s^I Q_I$. **Let** $s^I = \dfrac{h^I}{h^I P_I} \Rightarrow$

$$a(s) \propto \text{tr}(s^I Q_I)^3 = \hat{c}_{IJK} s^I s^J s^K = \frac{c_{IJK} h^I h^J h^K}{(h^I P_I)^3} \propto (h^I P_I)^{-3}.$$

◇ **a-max = P-min** = **SUSY vac. condition in** AdS_5 **!**

$$\delta\lambda = h^I_{,x} P_I = (h^I P_I)_{,x} = P_{,x} = 0$$

CONTENTS

4. Triangle Anomalies from Einstein Manifolds

◇ $c_{IJK} = \text{tr}\, Q_I Q_J Q_K$ was important

- Enter in $a(s) \propto \text{tr}\, R(s)^3 = \text{tr}(s^I Q_I)^3$
- Determined the CS term, kinetic term and everything

Q. *How is it determined*
from the geometry of extra dimension X_5 *?*

- We expect an action in 5d

$$S = \frac{1}{2}\int \tau_{IJ} F^I \wedge * F^J + \frac{1}{24\pi^2}\int c_{IJK} A^I \wedge F^J \wedge F^K$$

$$\Downarrow$$

$$\tau_{IJ}\, d * F^I = \frac{1}{8} c_{IJK}\, F^J \wedge F^K$$

- Knowing c_{IJK} is
  - knowing how A^I arise from 10d type IIB supergravity, and
  - knowing how they interact.

- Type IIB on AdS$_5$ times Einstein mfd X_5
- Many fields in 10d: $e^{-\phi} + C$, $B_{\mu\nu}$ and $C_{\mu\nu}$, $g_{\mu\nu}$, F_5 and fermions ...
- Expected: ℓ gauge fields from $g_{\mu\nu}$, b^3 from F_5

where

ℓ: # of isometries k^a ($a = 1, \ldots, \ell$) on X,
b^3: # of harmonic 3-forms on X

◇ After some tedious calculation, we get

$$ds^2 = \sum ds^2_{\text{AdS}} + \sum (e^i + k^i_a A^a)^2$$

$$F_5 = F_5^{(0)} + (1+*) N d(A^I \wedge \omega_I)$$

where gauge fields satisfy $A^a = c^a_I A^I$.

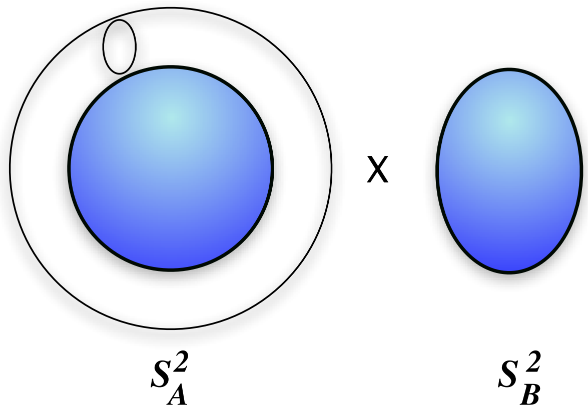
◇ ω_I are 3-forms satisfying

$$d * \omega_I = 0, \qquad d\omega_I + \iota_{k_I} \mathrm{vol}^\circ = 0$$

where $k_I = 2\pi c^a_I k_a$, $\int_X \mathrm{vol}^\circ = 1$ and $I = 1, \ldots, \ell + b^3$, and D3-brane wrapping C has charge $\int_C \omega_I$.

$$c_{IJK} = \frac{N^2}{2} \int_X \omega_{\{I} \wedge \iota_{k_J} \omega_{K\}}$$

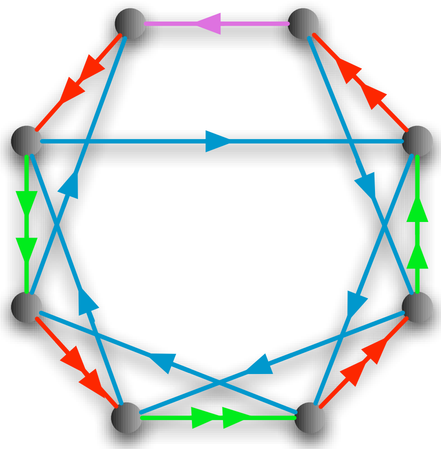
Example: $Y^{p,q}$



Take $k_F = p\partial_\gamma$, and $\omega_B = e_\gamma \wedge (q\omega_A - p\omega_B)$
where $0 \leq \gamma \leq 2\pi$ is the S^1 fiber, $\omega_{A,B}$: volume form of $S^2_{A,B}$.

From $de_\gamma = p\omega_A + q\omega_B,\ d\omega_B = 0 \Rightarrow\ k_B = 0$.

$$c_{BBF} = N^2 \int_{Y^{p,q}} \omega_B \wedge \iota_{k_F}\omega_B = 2p^2 q.$$



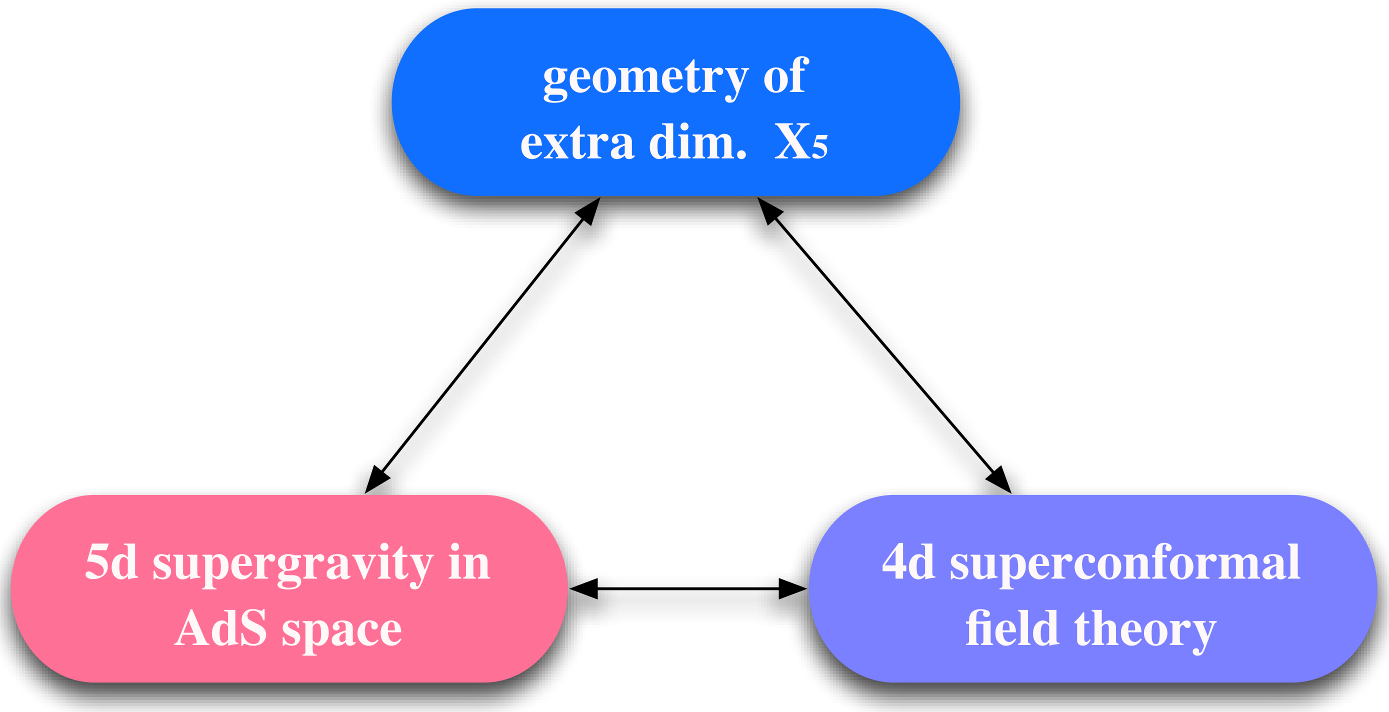
◇ $\det Y \Leftrightarrow$ D3-brane wrapping $Y = 0$ etc. $\Rightarrow$

	W_I^α	Y	Z	U^i	V^i
multiplicity	$2p$	$p+q$	$p-q$	p	q
F	0	-1	$+1$	0	1
B	0	$p-q$	$p+q$	$-p$	q

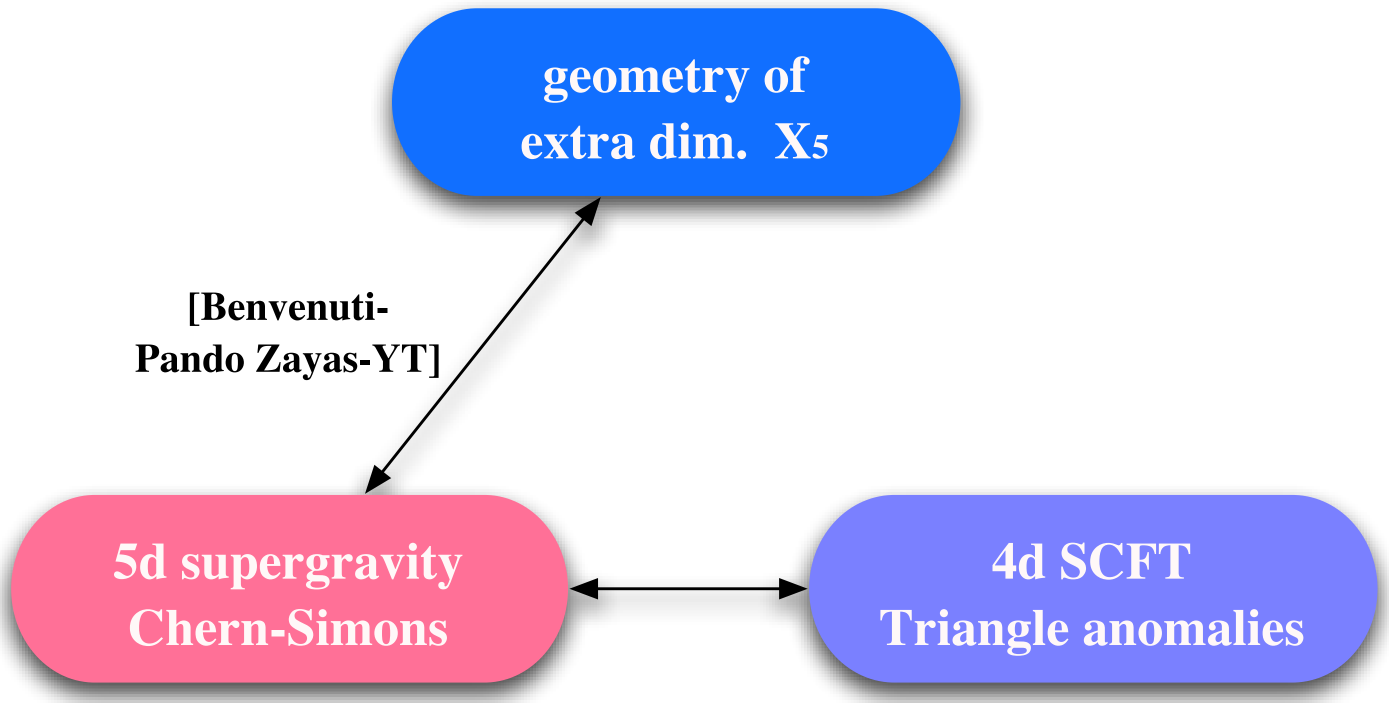
$$\Rightarrow \quad \frac{1}{N^2}\,\mathrm{tr}\,BBF = -(p+q)(p-q)^2 + (p-q)(p+q)^2 + 2q \cdot q^2 = 2p^2 q.$$

CONTENTS

5. Outlook


geometry of
extra dim. X5
5d supergravity in
AdS space
4d superconformal
field theory

5. Outlook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[Benvenuti-
Pando Zayas-YT]
5d supergravity
Chern-Simons
4d SCFT
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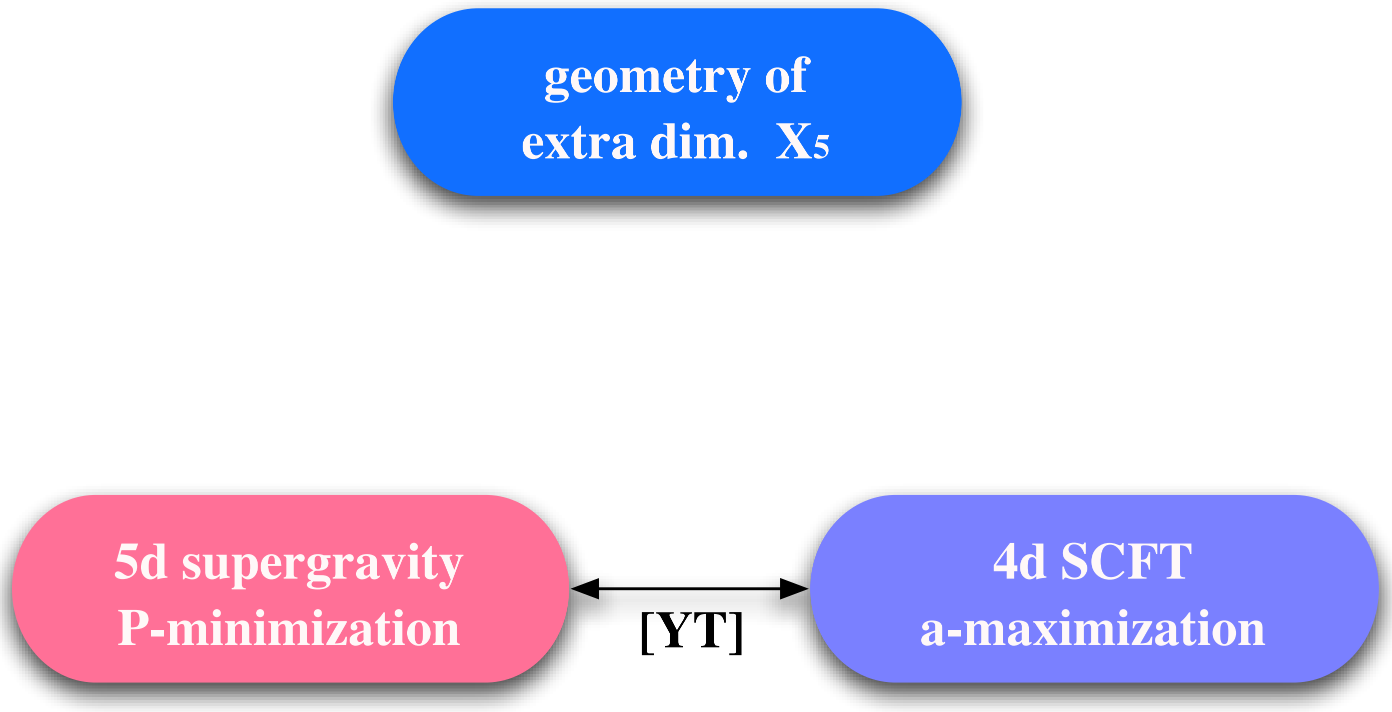
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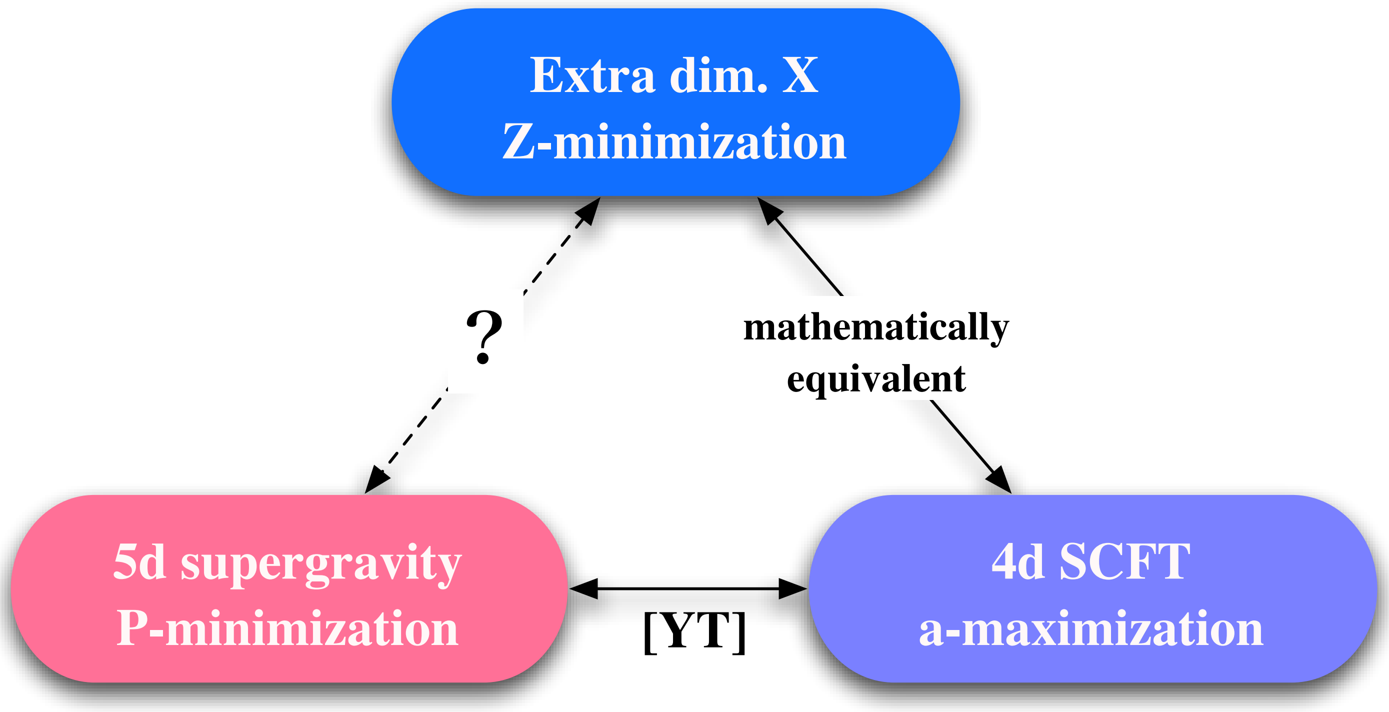

Extra dim. X
Z-minimization
?
mathematically
equivalent
5d supergravity
P-minimization
[YT]
4d SCFT
a-maximization

- ◇ (Benvenuti-Pando Zayas-YT) treated only the gauge fields in AdS ;

- ◇ Extension of (Benvenuti-Pando Zayas-YT)
  - to include scalar fields in AdS
  - should connect physically

    Z-minimization in terms of X to P-minimization in AdS.

- ◇ More supercharges $\Rightarrow$ more tractable, but
- ◇ More than 8 supercharges $\Rightarrow$ too constrained
- ◇ Theories with 8 supercharges : interesting borderline cases !