5d Supergravity Dual

of

a-maximization

Yuji Tachikawa
(Univ. of Tokyo, Hongo)

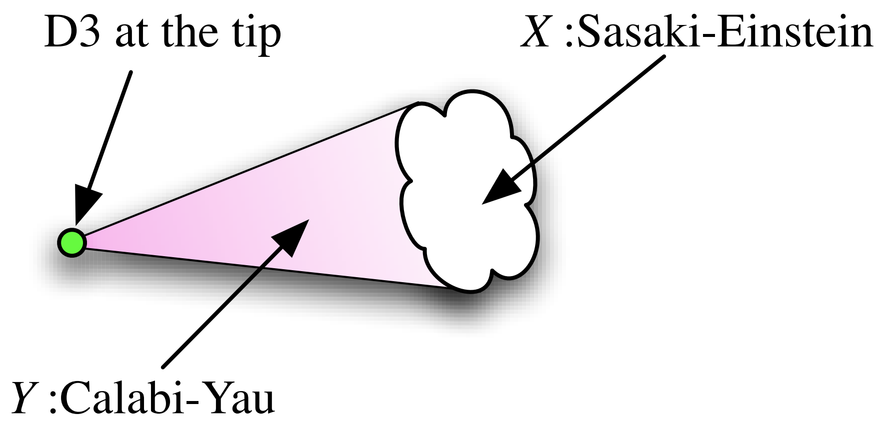
based on (YT, hep-th/0507057)

October, 2005 @ Caltech

1. Introduction

◇ Consider a D3 brane probing the tip of the CY cone Y

$$ds_Y^2 = dr^2 + r^2 ds_X^2$$



◇ Two way of analysis

- Field theory on the D3 $\Rightarrow$ Some quiver gauge theory
- Near Horizon Limit of the D3 $\Rightarrow$ $\mathrm{AdS}_5 \times X_5$

Do the properties match ?

- Both give a theory with $SU(2,2|1)$ symmetry.
  - 4d $\mathcal{N}=1$ SCFT from the field theory p.o.v.
  - Isometry of spacetime from the gravity p.o.v.
- Basic quantity : Central charge a and c

  $a-c$: $1/N$ correction / higher-derivative correction
- How can we calculate a ?

Field Theory Side

CY cone as toric singularity

$\Rightarrow$ quiver gauge theory (Hanany et al.)

$\Rightarrow$ a-maximization (Intriligator-Wecht)

Sasaki-Einstein Side

CY cone as toric singularity

$\Rightarrow$ Need to find Einstein metric on Sasaki-manifold

$\Rightarrow$ Z-minimization (Martelli-Sparks-Yau)

They match !

- A great check of AdS/CFT,
- Why do they match ?

cf. (Butti-Zaffaroni 0506232) showed by brute force that $Z = 1/a$ after maximizing for baryonic symmetry, but no clear physical explanation yet.

cf. (Barnes-Gorbatov-Intriligator-Wright 0507146) showed the equivalence $Z = 1/a$ on the vacua by considering two-point current correlators.

Divide and Conquer ...

$$\begin{array}{rl} \text{type IIB on } \mathrm{AdS}_5 \times X_5 & \\ \downarrow & \longleftarrow \text{ in a slow progress} \\ \text{5d gauged sugra on } \mathrm{AdS}_5 & \\ \updownarrow & \longleftarrow \text{ my work} \\ \text{4d SCFT} & \end{array}$$

◇ 5d gauged sugra is very **constrained**,
just as the Seiberg-Witten theory.

◇ Some insight might be expected from this point of view ?

◇ My message today :

a-maximization in 4d is P-minimization in 5d.

◇ More common name for P-maximization is the Attractor Eq.

Historical Curiosity

In the penultimate paragraph in

(Ferrara-Zaffaroni 'N=1,2 SCFT and Supergravity in 5d' 9803060):

> The presence of a scalar potential for supergravities in AdS5 allows to study critical points for different possible vacua in the bulk theory. It is natural to conjecture that these critical points should have a dual interpretation in the boundary superconformal field theory side.

They could have discovered a-maximization

before (Intriligator-Wecht, 0304128) !

CONTENTS

2. a-maximization

$\mathcal{N} = 1$ Superconformal Algebra in 4d

$$\{Q_\alpha, S^\alpha\} \sim R_{SC} + \cdots$$
$$[R_{SC}, Q_\alpha] = -Q_\alpha$$

◇ R_{SC} carries a lot of info:

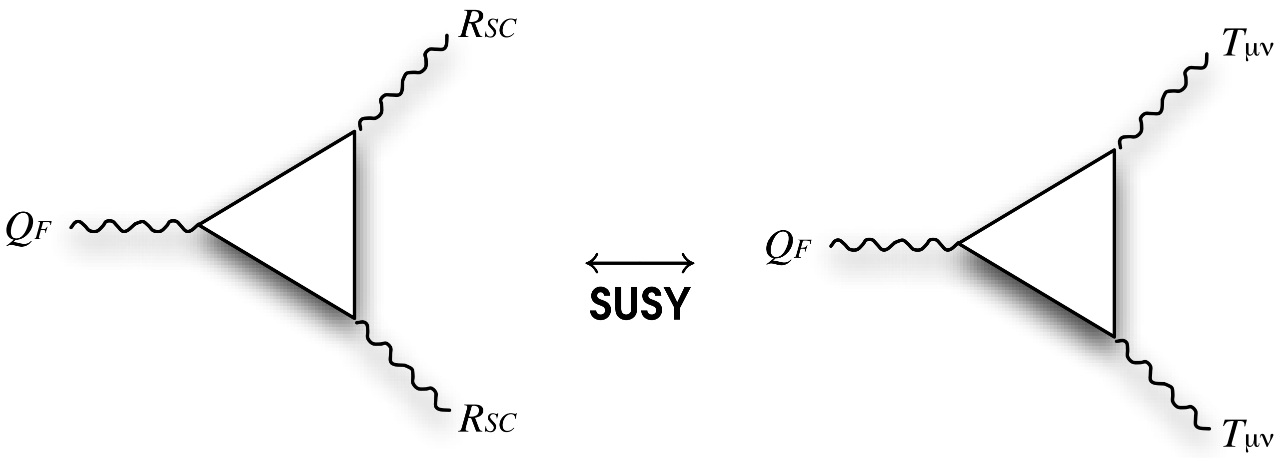
- $\Delta \geq \frac{3}{2} R_{SC}$
- $a = \frac{3}{32}(3 \,\text{tr}\, R_{SC}^3 - \text{tr}\, R_{SC})$

cf. $\langle T^\mu_\mu \rangle = a \times \text{Euler} + c \times \text{Weyl}^2$

◇ Let Q_I be integral $U(1)$ charges. $\Rightarrow R_{SC} = \tilde{s}^I Q_I$.

How can we find $\tilde{s}^I$?

◇ Let $[Q_I, Q_\alpha] = -\hat{P}_I Q_\alpha$.
◇ Call $Q_F = f^I Q_I$ with $[Q_F, Q_\alpha] = 0$ a flavor symmetry.



$$9 \,\mathrm{tr}\, Q_F R_{SC} R_{SC} \quad = \quad \mathrm{tr}\, Q_F$$

- $[\tilde{s}^I Q_I, Q_\alpha] = -Q_\alpha \Rightarrow \tilde{s}^I \hat{P}_I = 1$.
- Let $a(s) = \frac{3}{32}(3\,\mathrm{tr}\, R(s)^3 - \mathrm{tr}\, R(s))$ where $R(s) = s^I Q_I$.
- $9\,\mathrm{tr}\, Q_F R_{SC} R_{SC} = \mathrm{tr}\, Q_F \Rightarrow \tilde{s}^I$ extremizes $a(s)$ under $s^I \hat{P}_I = 1$.
- Unitarity $\Rightarrow$ it's a local maximum.

a-maximization !

- Let $\hat{c}_{IJK} = \mathrm{tr}\, Q_I Q_J Q_K$ and $\hat{c}_I = \mathrm{tr}\, Q_I$.

cf. Both calculable at UV using 't Hooft's anomaly matching.

CONTENTS

3. 5d gauged supergravity

Multiplet structure

- Minimal number of supercharges is 8, called $\mathcal{N} = 2$
- Gravity multiplet, Vector multiplet, Hypermultiplet

Gravity multiplet

$$g_{\mu\nu}, \qquad \psi^i_\mu, \qquad A^I_\mu$$

- $i = 1, 2$: index for $SU(2)_R$.
- I: explained in a second

Vector Multiplet

$$A^I_\mu, \qquad \lambda^x_i, \qquad \phi^x$$

◇ $I : 0, \ldots, n_V$ and $x : 1, \ldots, n_V$

cf. A^I_μ constitutes integral basis; graviphoton is a mixture.

◇ ϕ^x parametrize a hypersurface

$$F = c_{IJK} h^I h^J h^K = 1$$

in $(n_V + 1)$ dim space $\{h^I\} \Rightarrow h^I = h^I(\phi^x)$: sp. coordinates

◇ Kinetic terms are

$$-\frac{1}{2}g_{xy}\partial_\mu\phi^x\partial_\mu\phi^y - \frac{1}{4}a_{IJ}F^I_{\mu\nu}F^J_{\mu\nu}$$

where

$$\begin{aligned} g_{xy} &= -3_{IJK}h^I_{,x}h^J_{,y}h^K \\ a_{IJ} &= h_I h_J + \frac{3}{2}g^{xy}h_{I,x}h_{J,y} \\ h_I &\equiv c_{IJK}h^J h^K \qquad \text{(dual sp. coordinates)} \end{aligned}$$

◇ There is the Chern-Simons term

$$\frac{1}{6\sqrt{6}}c_{IJK}\epsilon^{\mu\nu\rho\sigma\tau}A^I_\mu F^J_{\nu\rho}F^K_{\sigma\tau}$$

Hypermultiplet

$$q^X, \qquad \zeta^A$$

- $X : 1, \ldots, 4n_H$, holonomy $SO(4n_H) \supset Sp(n_H) \otimes Sp(1)_R$
- $A : 1, \ldots, 2n_H$ labels the fundamental of $Sp(n_H)$
- Vierbein f^X_{iA}, $i = 1, 2$
- $Sp(1)_R$ part of the curvature is fixed:

$$R_{XYij} = -(f_{XiA} f^A_{Yj} - f_{YiA} f^A_{Xj})$$

We use $\{ij\} \longleftrightarrow r = 1, 2, 3$

Gauging

◇ Potential is associated with the gauging of the hypers:

$$\partial_\mu q^X \longrightarrow D_\mu q^X = \partial_\mu q^X + A_\mu^I K_I^X$$

where K_I^X is triholomorphic:

$$K_I^X R^r_{XY} = D_Y P_I^r$$

cf. P_I^r is a triplet generalization of the D-term.

◇ The potential V is given by

$$V = 3g^{xy}\partial_x P^r \partial_y P^r + g^{XY} D_X P^r D_Y P^r - 4P^r P^r$$

where $P^r = h^I P_I^r$

cf. Gukov-Vafa-Witten type superpotential.

- P^r **appears everywhere:**

  - **Covariant derivative of the gravitino**

$$D_\nu \psi^i_\mu = \partial_\nu \psi^i_\mu + A^I_\mu P^i{}_{jI} \psi^j_I$$

  - **SUSY transformation law**

$$\delta_\epsilon \phi^x = \frac{i}{2} \bar{\epsilon}^i \lambda^x_i$$

$$\delta_\epsilon \lambda^i_x = -\epsilon_j \sqrt{\frac{2}{3}} \partial_x P^{ij} + \cdots$$

$$\delta_\epsilon q^X = -i \bar{\epsilon}_i f^{XiA} \zeta_A$$

$$\delta_\epsilon \zeta_A = \frac{\sqrt{6}}{4} \bar{\epsilon}^i f_{XiA} K^X_I h^I + \cdots$$

◇ Suppose $K_I^X = Q_{IY}^X q^Y + \cdots$

where Q_{IY}^X is the charge matrix of the hypers.

◇ Recall $K_I^X R_{XY}^{ij} = D_Y P_I^{ij}$

$\Rightarrow$ some calculation $\Rightarrow$

$$\text{projection} \quad \begin{array}{ccc} Q_{I}{}^{X}_{Y} & \in & so(4n_H) \\ \downarrow & & \cup \\ P_I^{ij} & \in & sp(1)_R \end{array}$$

at $q^X = 0$.

CONTENTS

4. How do they match?

Recap.

Field theory	$\Leftrightarrow$	Supergravity
$\hat{c}_{IJK} = \mathrm{tr}\, Q_I Q_J Q_K$		$c_{IJK} h^I h^J h^K = 1$
$[Q_I, Q_\alpha] = -\hat{P}^I Q_\alpha$		$D_\mu \psi^i_\nu = (\partial_\mu + P^i_{jI} A^I_\mu)\psi_\nu$
a-max		???

AdS/CFT correspondence

AdS		CFT
$\hat{\phi}$		$\mathcal{O}$
$Z[\hat{\phi}(x)\big\vert_{x_5=\infty} = \phi(x)]$	$=$	$\langle e^{-\int \phi(x)\mathcal{O}(x)d^4x}\rangle$
A^I_μ	$\leftrightarrow$	J^μ_I : current for Q_I

Thus, we need to introduce $\langle e^{-\int A^I_\mu J^\mu_I}\rangle_{SCFT}$.

Q_I has triangle anomalies among them

$\Rightarrow \langle e^{-\int A_\mu^I J_I^\mu} \rangle_{SCFT}$ depends on the gauge !

$$\delta_g(\cdots) = \int d^4x \frac{1}{24\pi^2} \hat{c}_{IJK} g^I F^J \wedge F^K$$
$$= \int d^5x \frac{1}{24\pi^2} \hat{c}_{IJK} \delta_g(A^I) \wedge F^J \wedge F^K$$

$$c_{IJK} = \frac{\sqrt{6}}{16\pi^2} \hat{c}_{IJK}.$$

- ◇ $\hat{c}_I = \mathrm{tr}\, Q_I$ is related to $\hat{c}_I A^I \wedge \mathrm{tr}\, R \wedge R$ in the same way.
- ◇ In the following, we set $\hat{c}_I \ll \hat{c}_{IJK}$.

SUSY condition for sugra

- Assume $q^X = 0$ and $\delta\zeta^A = 0$.
- Recall $\delta\lambda_x^i \propto \epsilon_j \partial_x P^{ij}$. $\Rightarrow$ $\delta\lambda = 0 \Rightarrow \tilde{h}^I_{,x} P_I^{ij} = 0$

- $h^*_{,x}$ and P^r_* is **perpendicular** as $(n_V + 1)$ dim'l vectors.
- $x = 1, \ldots, n_V \Rightarrow P^{r=1,2,3}$ are **parallel**.
- Rotate so that $P^{r=1,2} = 0$, $P^{r=3} \neq 0$.

- $c_{IJK} h^I h^J h^K = 1 \Rightarrow \underbrace{c_{IJK} h^I h^J}_{h_K} h^K_{,x} = 0$

$$\tilde{h}_I = k P_I^{r=3}$$: **Attractor Eq.**

◇ Recall

AdS		CFT
ψ_μ	$\leftrightarrow$	$Q_\alpha,\ S^\alpha$
A_μ^I	$\leftrightarrow$	Q_I

and

$$D_\nu \psi_\mu^i = \partial_\nu \psi_\mu^i + A_\mu^I P^i_{jI} \psi_I^j,$$

$\Rightarrow$ The **charge** of Q_α, S^α under Q_I is $\pm P_I^{r=3}$.

◇ Recall $[Q_I, Q_\alpha] = -\hat{P}_I Q_\alpha$

$$\Rightarrow P_I^{r=3} = \hat{P}_I.$$

◇ **From the susy tr,**

$$\{\delta_\epsilon, \delta_{\epsilon'}\} q^X \propto (\bar{\epsilon}\epsilon') h^I K_I^X$$

$\Rightarrow R_{SC} = \tilde{s}^I Q_I \propto \tilde{h}^I Q_I$.

◇ **Recall** $\tilde{s}^I P_I = 1. \Rightarrow \tilde{s}^I = \tilde{h}^I / (\tilde{h}^I P_I)$.

◇ **Extend the relation to** $s^I = h^I / (h^I P_I) \Rightarrow$

$$a(s) \propto \mathrm{tr}(s^I Q_I)^3 = \hat{c}_{IJK} s^I s^J s^K = \frac{c_{IJK} h^I h^J h^K}{(h^I P_I)^3} \propto (h^I P_I)^{-3}.$$

◇ a**-max =** P**-min !**

$$\delta\lambda = h^I_{,x} P_I = (h^I P_I)_{,x} = P_{,x} = 0$$

E.g. Exactly marginal deformation

AdS		CFT
M_c	$\leftrightarrow$	'Conformal manifolds'

◇ A puzzle

- M_c should have **Kähler structure** from CFT p.o.v.

- M_c must corresponds to **hyper**scalars,
since vectors are fixed by a-max = P-min.

- Hypers are **quaternionic**, which is **not** Kähler. **???**

- ◇ $\delta\zeta^A = 0 \Rightarrow K^X = 0$ where $K^X \equiv \tilde{h}^I K_I^X$.
- ◇ Superconformal deformation $\leftrightarrow M_c = \{q^X \in M_{\text{hyper}} \mid K^X = 0\}$
- ◇ From $D_X P^r = R^r_{XY} K^X$, P^r is covariantly constant on M_c.
- ◇ Then M_c is Kähler, because

$$J_Y^X \equiv R^r_{YZ} g^{XZ} P_r / |P_r|$$

is a covariantly constant matrix with $J^2 = -1$.

a-max with Lagrange multipliers

- SCFT also has anomalous symmetries

$$\partial_\mu J_I^\mu = \sum_a m_I^a \, \mathrm{tr}\, F^a \wedge F^a$$

- (Kutasov) extended the a-max to include them,

$$a(s^I, \lambda_a) = a(s) + \lambda_a m_I^a s^I$$

where λ_a are Lagrange multipliers enforcing the anomaly-free condition for R_{SC}.

- λ_a behaves like coupling constant for $F^a_{\mu\nu}$.

◇ **Recall**

AdS	CFT
A_μ	J^μ
ϕ	$\mathcal{X}$

Higgs mechanism $\leftrightarrow$ **anomalous symmetry**

◇ **It's because**

$$\langle \exp(\int A_\mu J^\mu + \phi \mathcal{X}) \rangle \text{ : invariant under } \quad \delta A_\mu = \partial_\mu \epsilon, \quad \delta\phi = \epsilon$$

$\Rightarrow$

$$\langle \exp(\int \epsilon(\partial_\mu J^\mu - X)) \rangle = 0.$$

- Let us denote by K_a^X the isometry for

$$\delta\phi_a = \epsilon, \qquad \phi_a \leftrightarrow \text{tr}\, F^a \wedge F^a.$$

- It can be shown

AdS		CFT
K_a^X		$\text{tr}\, F^a \wedge F^a$
$P_a^{r=1,2}$	$\leftrightarrow$	$\text{tr}\, \epsilon^{\alpha\beta}\lambda_\alpha^a \lambda_\beta^a$
$P_a^{r=3}$		$\text{tr}\, F_{\mu\nu}^a F_{\mu\nu}^a$

- $P_a^{r=3}$ enters in the P-function as $P_a^{r=3} m_I^a h^I$

The AdS dual of the coupling constant

exactly acts as the Lagrange multiplier !

5. Conclusion

DONE

✓ a-maximization is P-minimization or the attractor eq.

✓ Other correspondences. Please see my paper !

TO DO

◇ Dual of Higgsing, Dual of Unitarity Bound Hit, ...

◇ Include $\hat{c}_I A^I \wedge \mathrm{tr}\, R \wedge R$

◇ Calculate c_{IJK} for IIB on $\mathrm{AdS} \times X_5$ (in progress)