A counterexample to the a-theorem

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Central charge c in 2d

$$T(z)T(0) \sim \frac{c}{2z^4} + \cdots$$

$$\langle T \rangle = -\frac{cR}{12}$$

- Captures the asymptotic growth of states.
- (Need to use modular transformation)
- Additive.

Zamolodchikov's c-theorem

Let

$$F(r) = z^4 \langle T_{zz}(z,\bar{z}) T_{zz}(0,0)\rangle,$$
$$G(r) = 4z^3\bar{z} \langle T_{zz}(z,\bar{z}) T_{z\bar{z}}(0,0)\rangle,$$
$$H(r) = 16z^2\bar{z}^2 \langle T_{z\bar{z}}(z,\bar{z}) T_{z\bar{z}}(0,0)\rangle,$$

and define

$$C(r) = 2F(r) - G(r) - \frac{3}{8}H(r).$$

- $C(r)$ is the central charge c for CFTs.
- $r\frac{\partial}{\partial r}C(r) = -\frac{3}{2}H \leq 0$.
- # DOF decreases along the RG flow

4d central charges: a and c

$$\langle T^\mu_\mu \rangle = \frac{c}{16\pi^2}\text{Weyl}^2 - \frac{a}{16\pi^2}\text{Euler}$$

- Only c appears in 2pt functions.
- Additive.
- No modular tr. ⟶ no immediate relation to #DOF.

	a	c
$\mathcal{N}=1$ chiral mult.	$1/48$	$1/24$
$\mathcal{N}=1$ vector mult.	$3/16$	$1/8$

Conjectural 'a-theorem'

[Cardy 1988]

In 2d, it was $c \propto \int_{S^2} \langle T \rangle$.

Why don't we choose $a \propto \int_{S^4} \langle T \rangle$ in 4d?

- He somehow chose S^4, which is conformally flat.
- Weyl^2 happened to dropped out.
- Anyway his proposal stood the test of time...

Test 1 [Cardy 1988]

Consider massless QCD with N_c colors and N_f flavors.

UV:	quarks & gluons	$a \sim N_c^2 + N_c N_f$
IR:	pions	$a \sim N_f^2$

- a-theorem violated if $\dfrac{N_f}{N_c} \gtrsim 15.1$
- loses asymptotic freedom if $\dfrac{N_f}{N_c} \gtrsim 5.5$
- (Chiral symmetry ceases to break at much lower N_f/N_c)
- [Ball-Damgaard 2001] checked other G and matter contents

Test 2 [Anselmi, Freedman et al. 1998]

- Supersymmetry relates a, c to 't Hooft anomalies of R symmetry:

$$a = \frac{3}{32}\left[3\,\mathrm{tr}\,R^3 - \mathrm{tr}\,R\right],$$
$$c = \frac{1}{32}\left[9\,\mathrm{tr}\,R^3 - 5\,\mathrm{tr}\,R\right]$$

- R symmetry must be non-anomalous,
- This condition alone sometimes fixes it
- e.g. SQCD in Seiberg's conformal window
- $a_{IR} < a_{UV}$, but other combinations $a + kc$ don't work

a-maximization

Let the trial a function be

$$a(R) = \frac{3}{32}\left[3\,\mathrm{tr}\,R^3 - \mathrm{tr}\,R\right],$$

a function of trial R symmetry,

$$R = R_0 + t_1 F_1 + \cdots.$$

The right R maximizes $a(R)$.

- Q should have charge **1** under R.
- Marginal terms in the superpotential W have charge **2** under R.
- UV: some terms in W irrelevant ⟶ less condition on R
- IR: some terms in W marginal ⟶ more condition on R
- Maximization in a smaller subset gives smaller number.

[Freedman, Gubser et al. 1999]

- Use AdS/CFT correspondence:

$$a \sim c \sim \Lambda^{-3/2}$$

- RG flow $\sim$ flow of scalars changing vacuum energy $\Lambda = V(\phi)$
- Null energy condition

$$T_{\mu\nu} n^\mu n^\nu \geq 0$$

guarantees monotony of $V(\phi)$ along the flow.

- Use AdS/CFT correspondence:

$$a \sim c \sim \frac{\pi^3 N^2}{4\,\mathrm{vol}\,X_5}$$

for N D3-branes on the cone over X_5.

- Bishop's theorem: $\mathrm{vol}\,X_5 \leq \mathrm{vol}\,S^5 \longrightarrow a(X_5) \geq a(S^5)$.
- Flow associated to 'Higgsing' which moves all D3-branes away from the tip.

a-theorem?

So far so good.

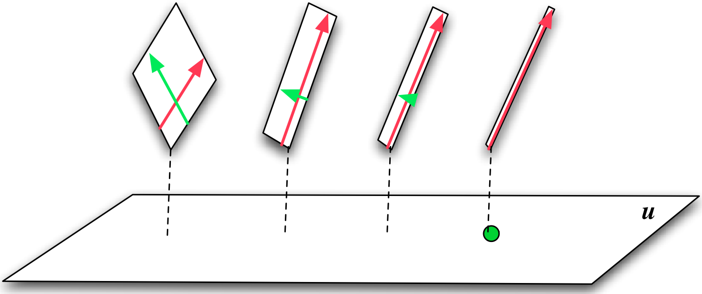
a-theorem?

So far so good.

But there is another class of SCFTs which is far less understood ...

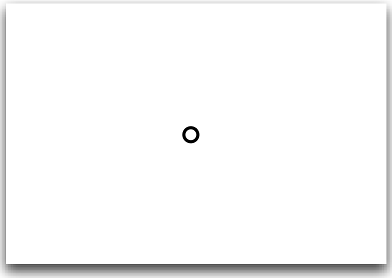
Contents

Seiberg-Witten theory



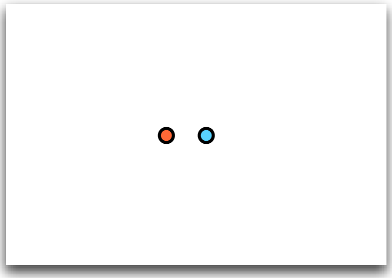
- SW curve parametrized by the vev $u = \langle \mathrm{tr}\, \phi^2 \rangle$
- Electron mass = $\int_A \lambda_{SW}$, Monopole mass = $\int_B \lambda_{SW}$

pure $\mathcal{N} = 2$ $SU(2)$: classical and quantum



- Enhanced $SU(2)$ symmetry at the origin $u = 0$ $\longrightarrow$

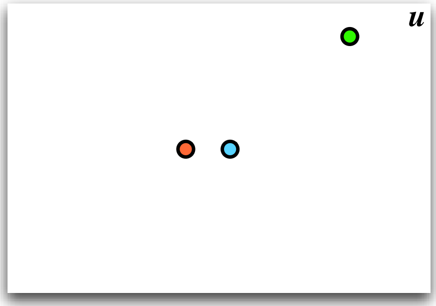
pure $\mathcal{N} = 2$ $SU(2)$: classical and quantum



- Enhanced $SU(2)$ symmetry at the origin $u = 0$ $\longrightarrow$
- Monopole point $u = \Lambda^2$
- Dyon point $u = -\Lambda^2$

$N_f = 1$

- $m \gg \Lambda \longrightarrow u \sim m^2/4, \qquad u \sim \pm 2(m\Lambda^3)^{1/2}$



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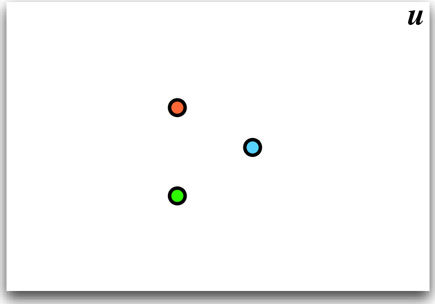
- $m \gg \Lambda \longrightarrow u \sim m^2/4, \qquad u \sim \pm 2(m\Lambda^3)^{1/2}$
- $m = 0 \longrightarrow u = 3\Lambda^2 \exp \frac{2\pi i k}{3}$



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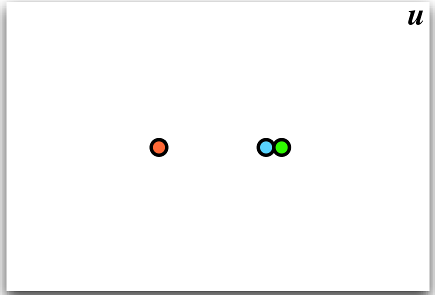
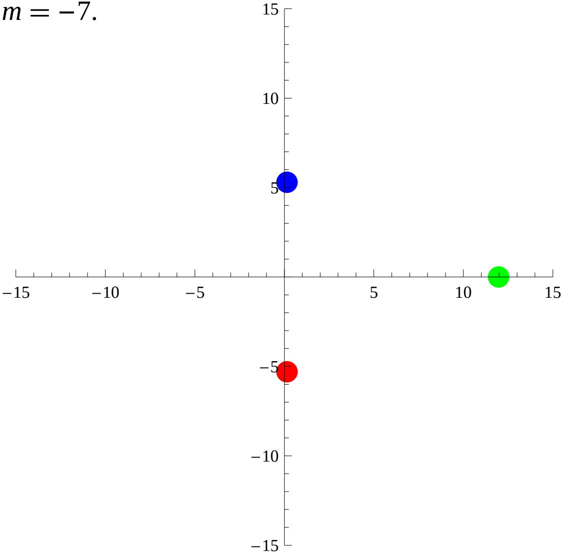
- $m = 0 \longrightarrow u = 3\Lambda^2 \exp \frac{2\pi i k}{3}$
- $m = 3\Lambda \longrightarrow u = 3\Lambda^2$ (double), $\qquad u = -15\Lambda^2/4$



Some gratuitous animation...

$m = -7.$



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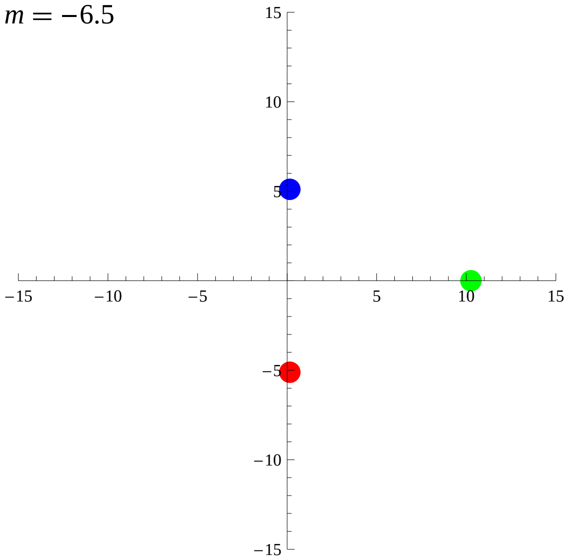

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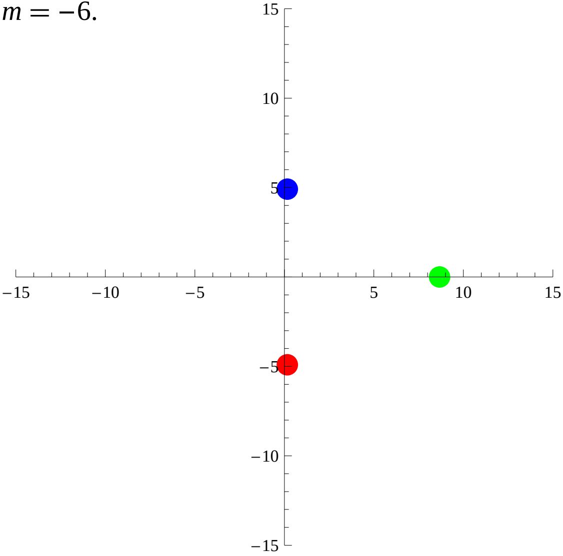
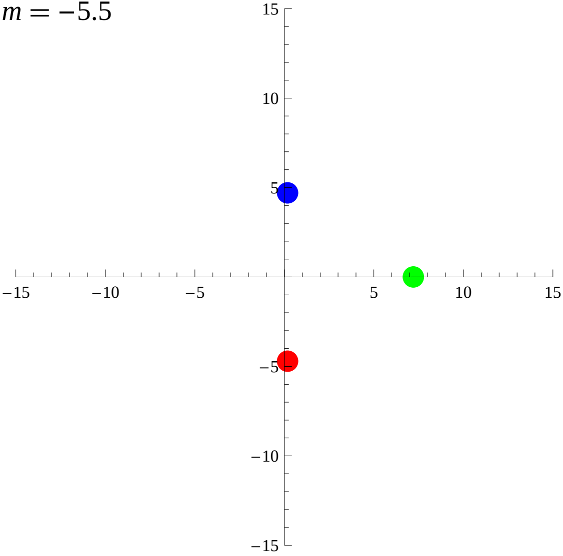
$m = -6.$



Some gratuitous animation...

$m = -5.5$



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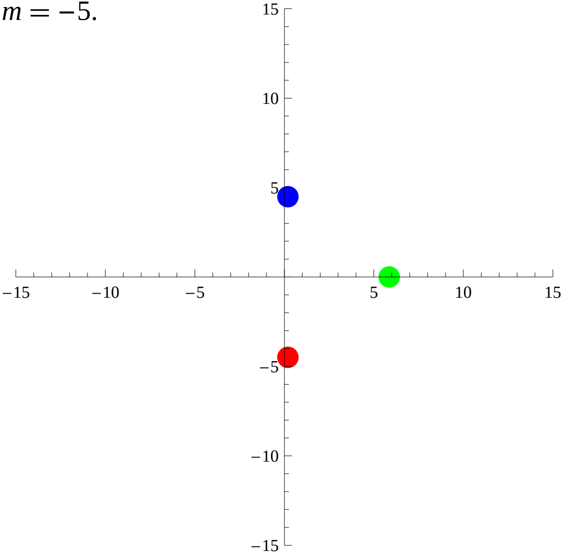
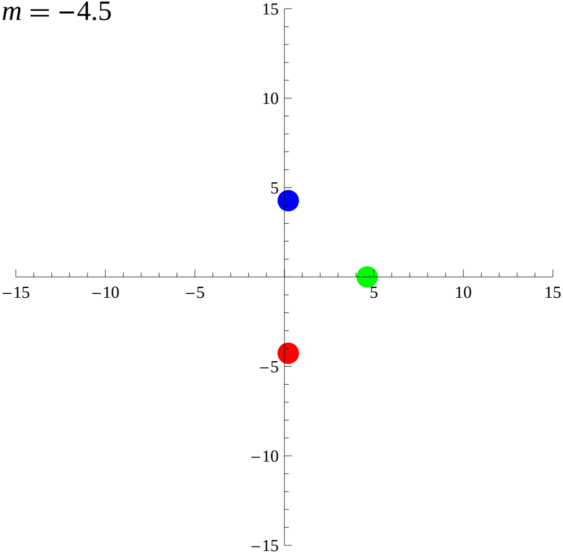


Some gratuitous animation...

$m = -4.5$



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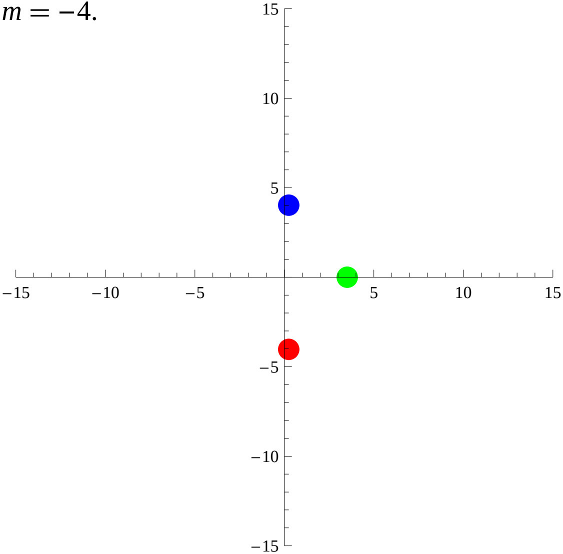
$m = -4.$



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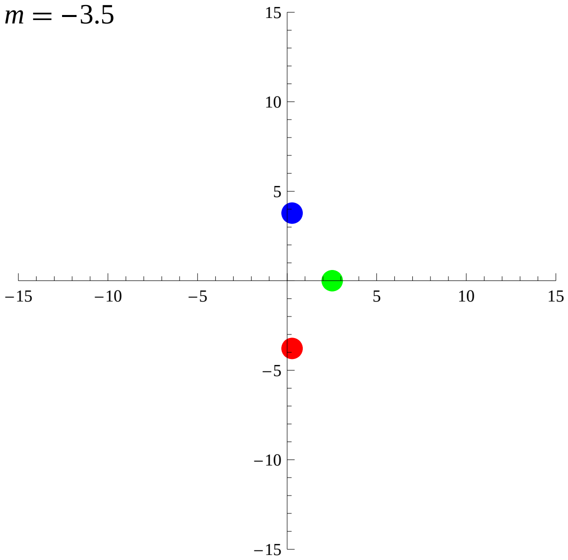
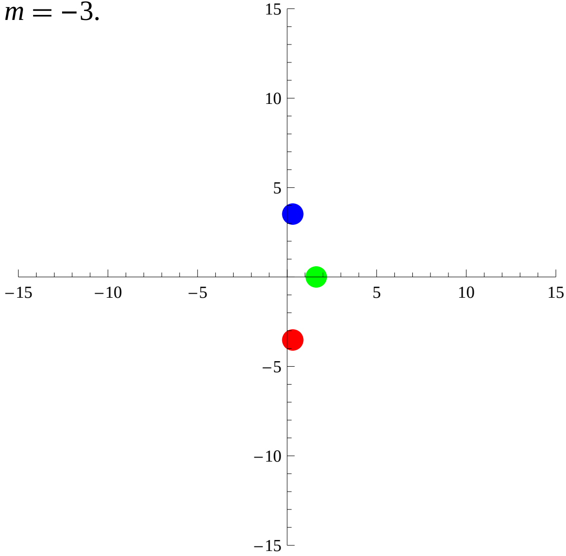


Some gratuitous animation...

$m = -3.$



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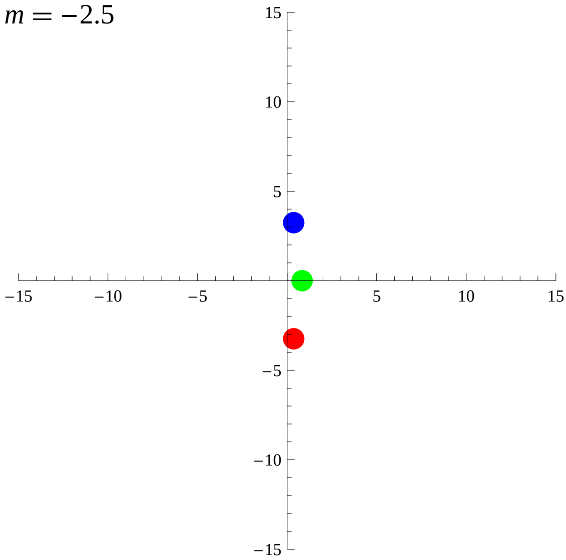
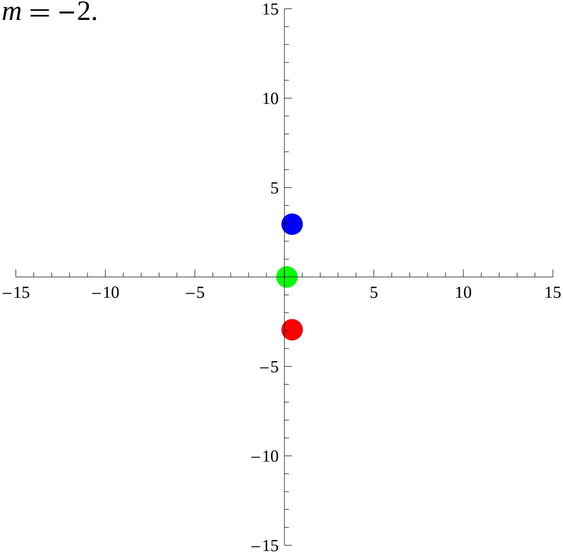


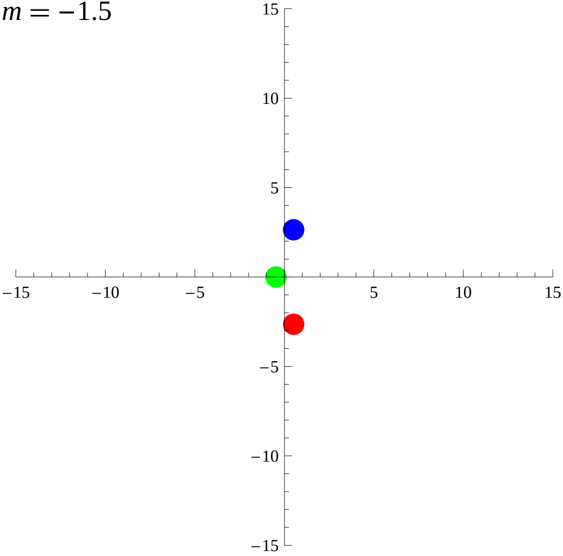
Some gratuitous animation...

$m = -2.$



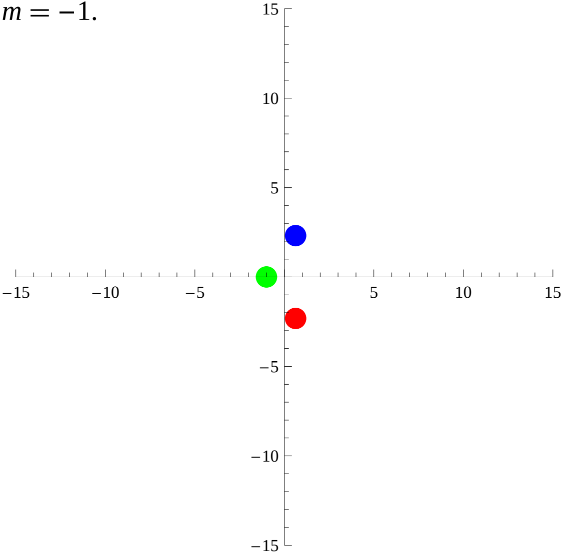
Some gratuitous animation...

$m = -1.5$



Some gratuitous animation...

$m = -1.$



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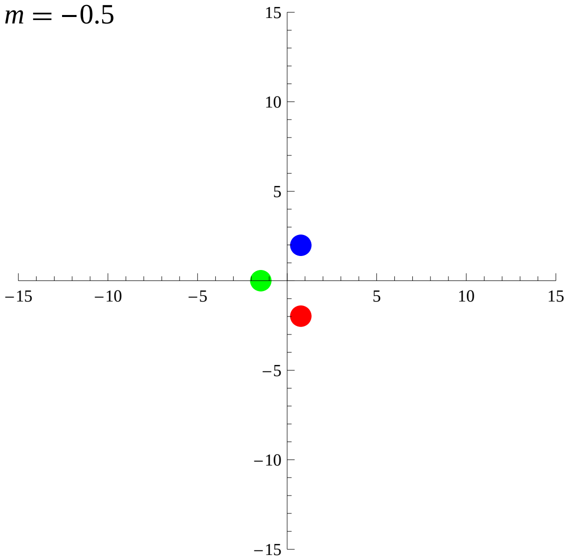
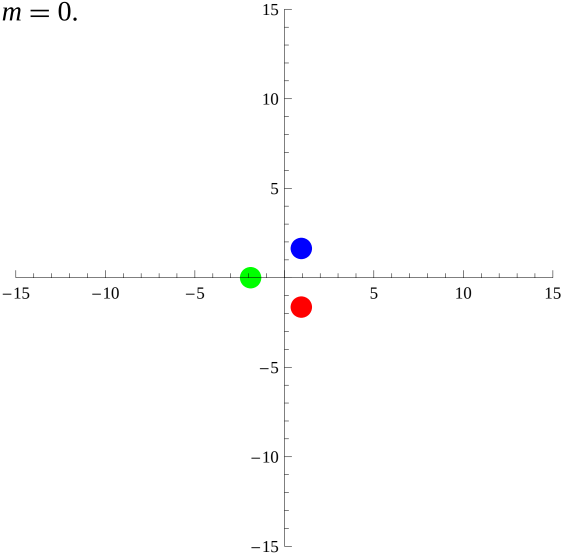


Some gratuitous animation...

$m = 0.$



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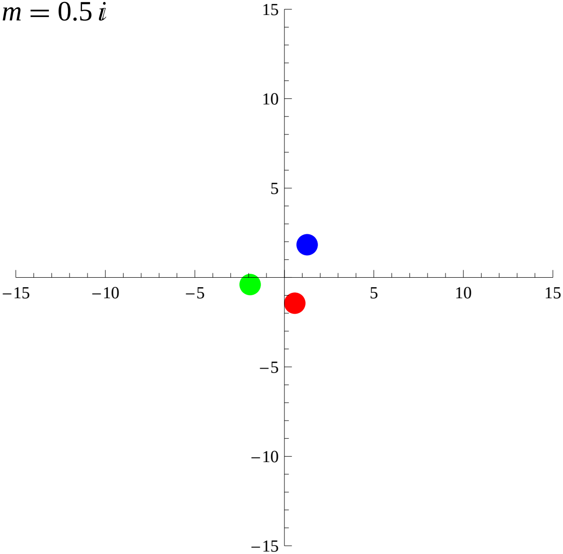
$m = 0.5\, i$



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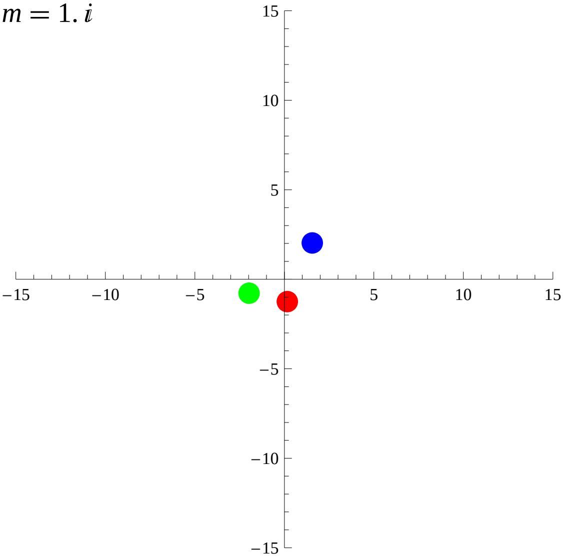

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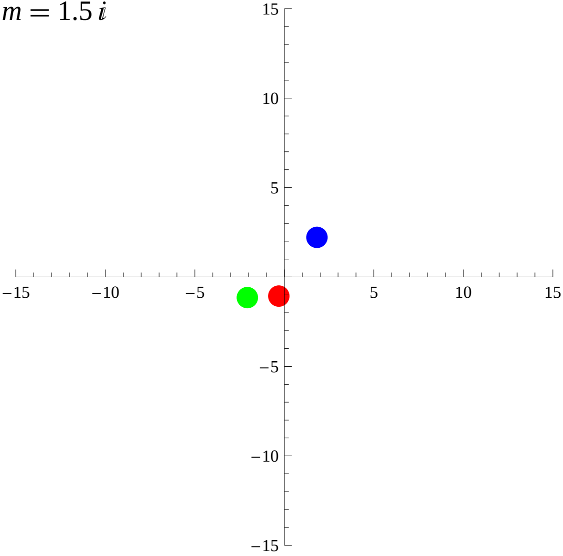
$m = 1.5\,i$



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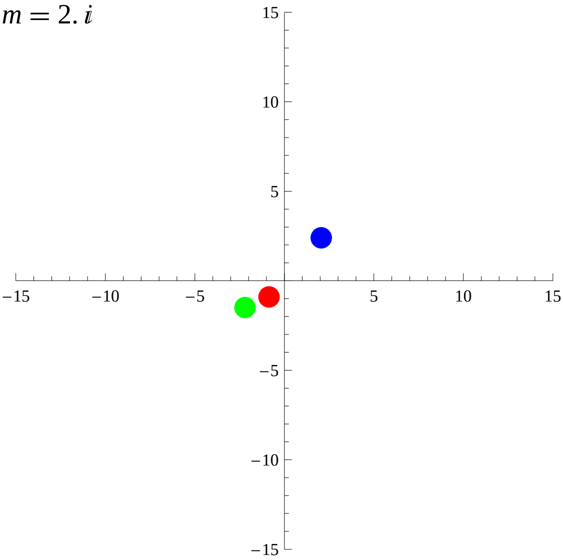
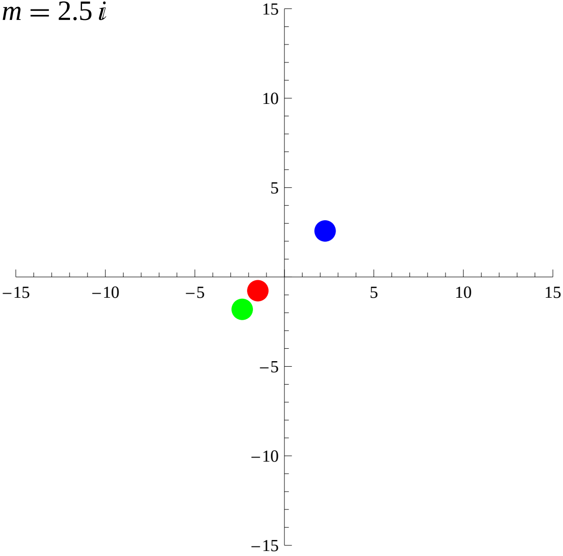


Some gratuitous animation...

$m = 2.5\,i$



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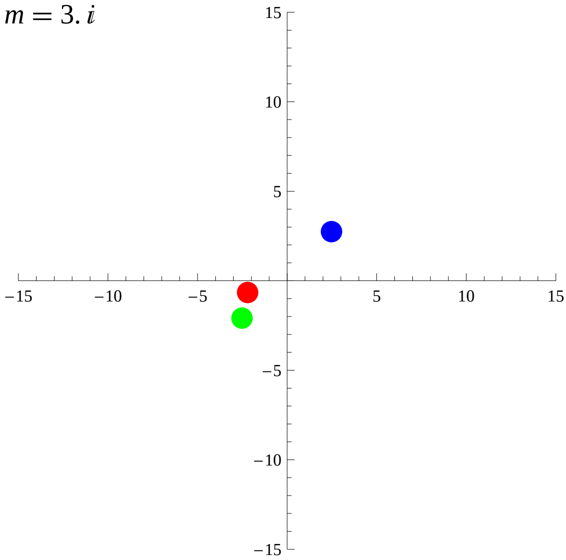
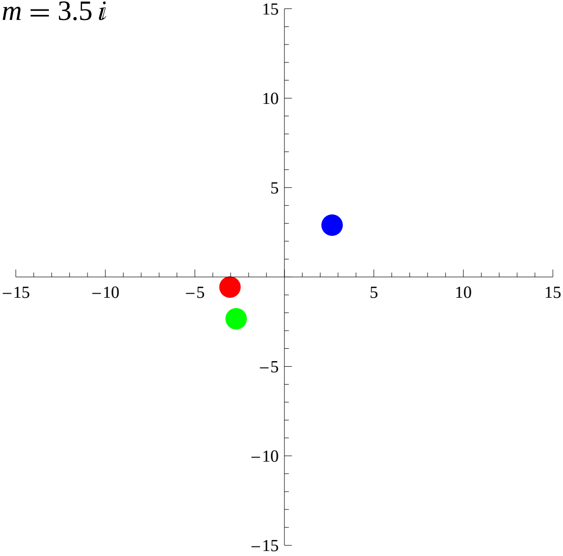


Some gratuitous animation...

$m = 3.5\,i$



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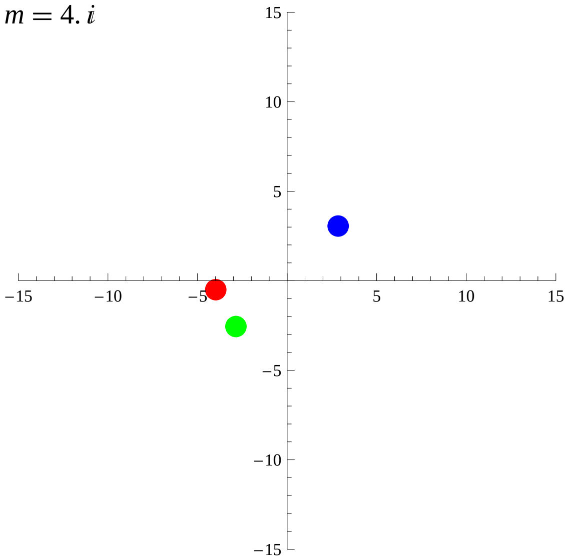

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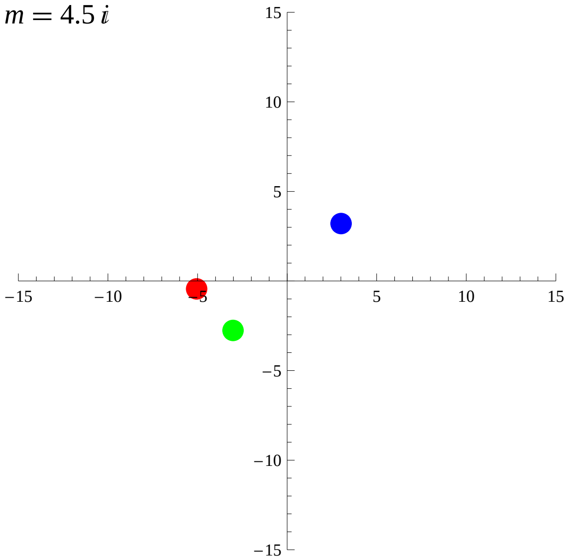
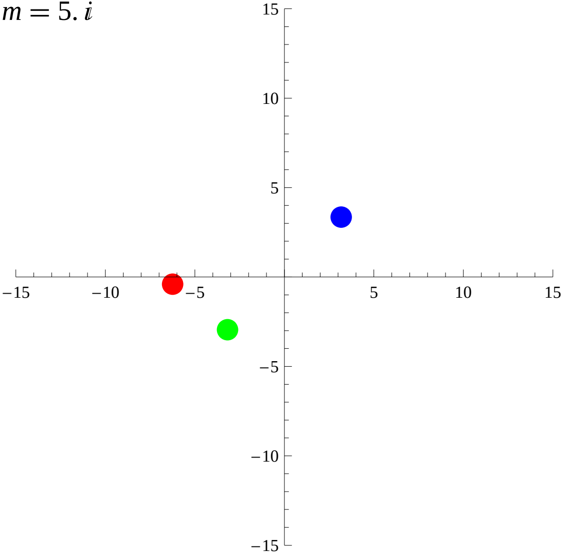


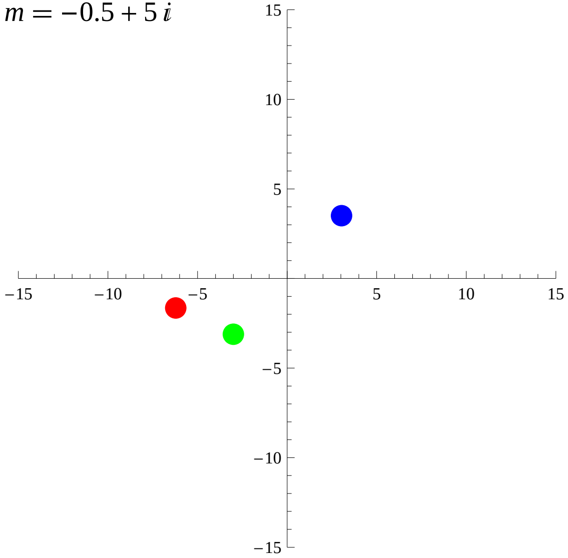
Some gratuitous animation...

$m = 5.\,i$



Some gratuitous animation...

$m = -0.5 + 5\,i$



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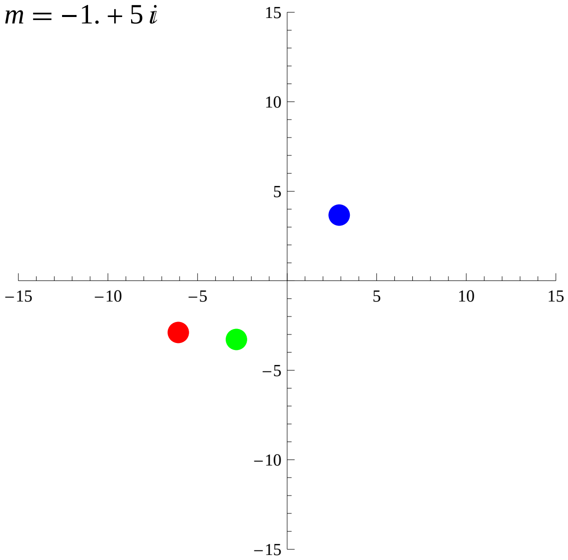

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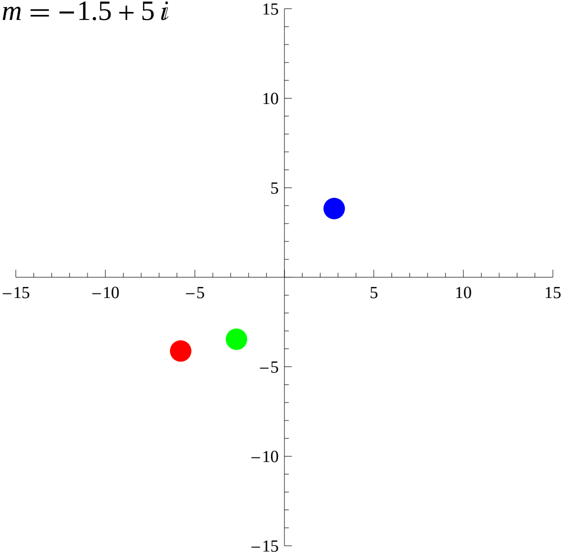
$m = -1.5 + 5\,i$



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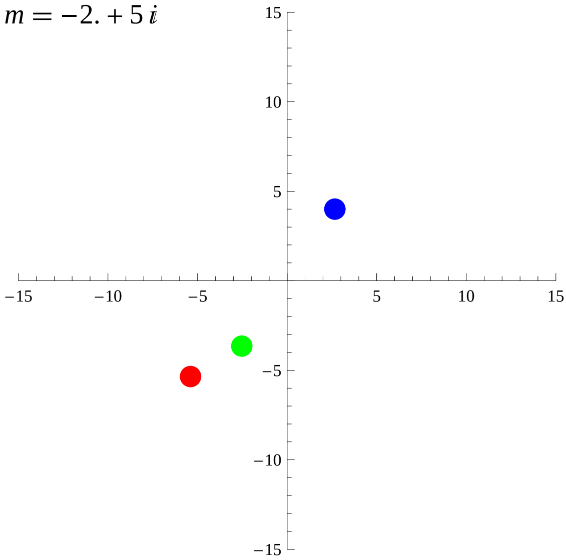
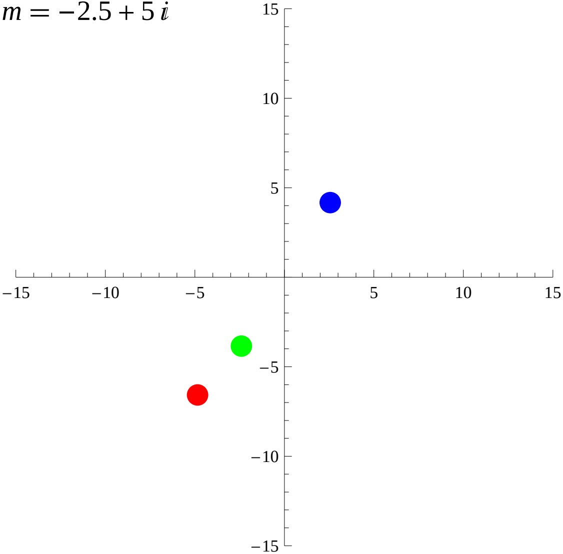


Some gratuitous animation...

$m = -2.5 + 5\,i$



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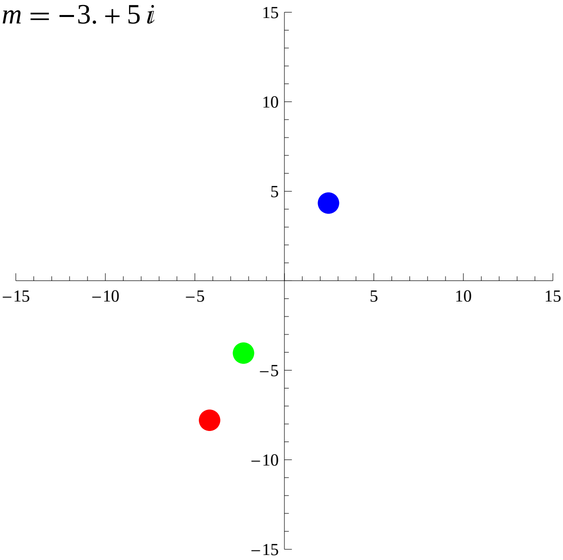

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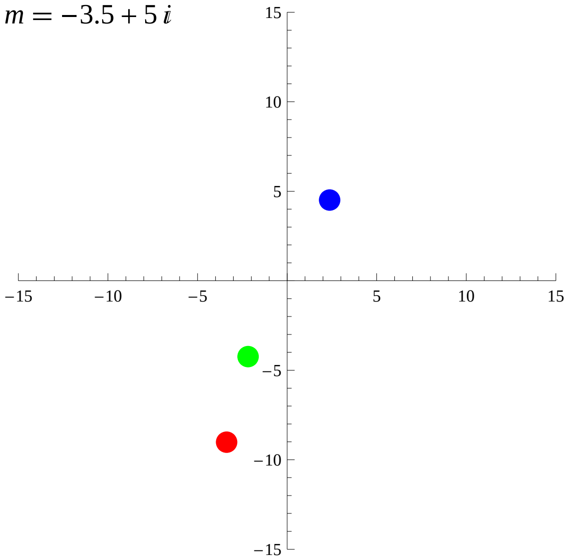
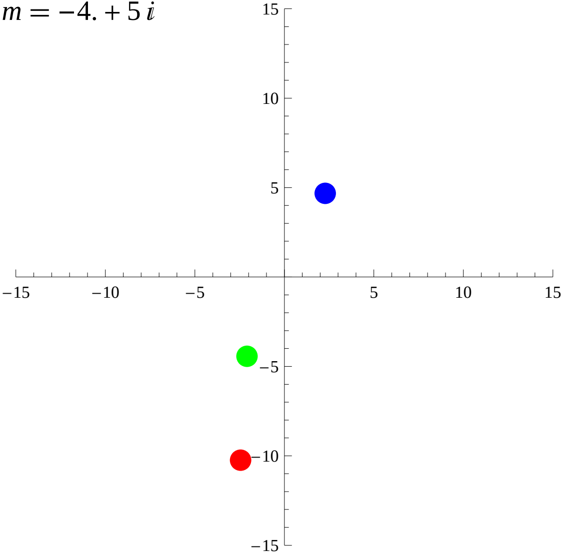


Some gratuitous animation...

$m = -4. + 5\,i$



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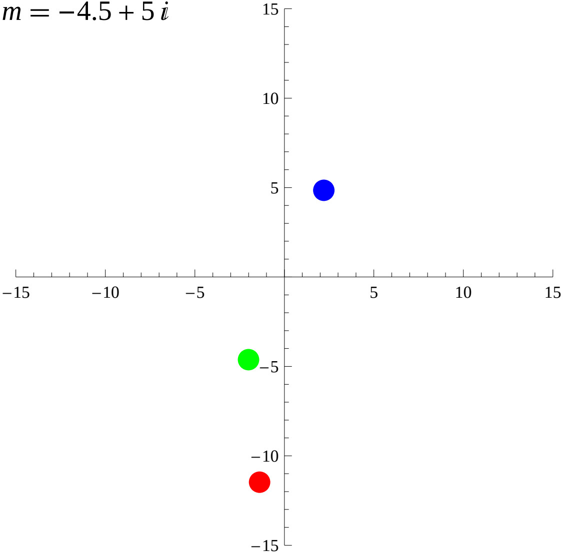

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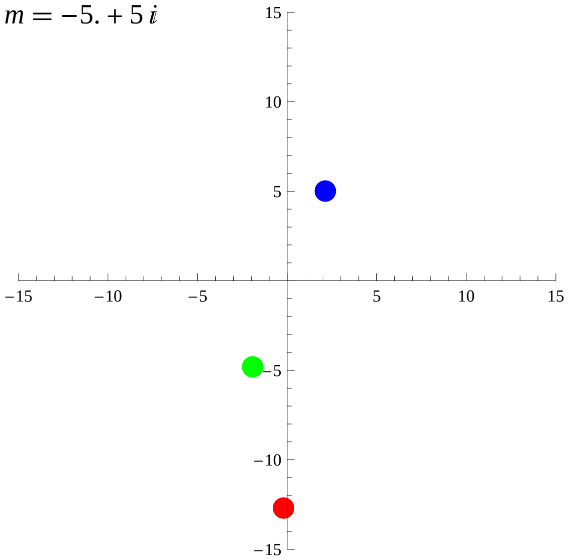

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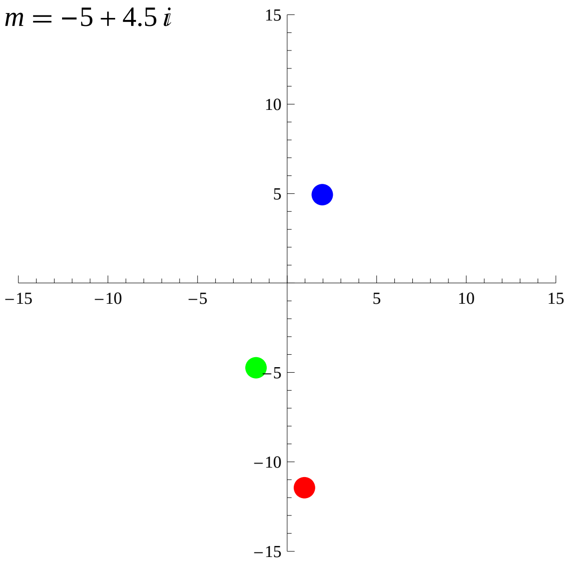
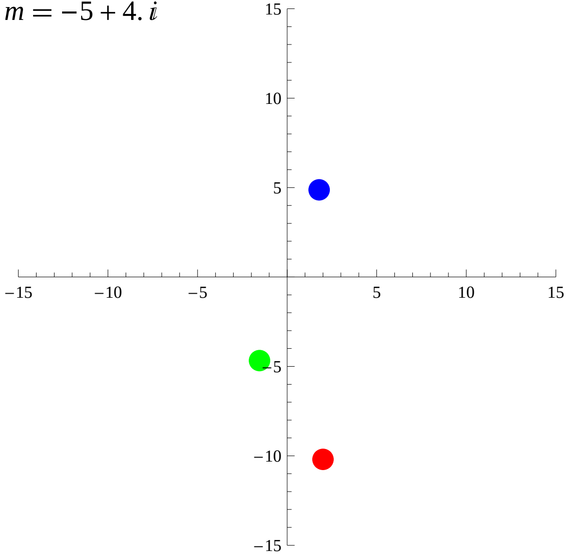


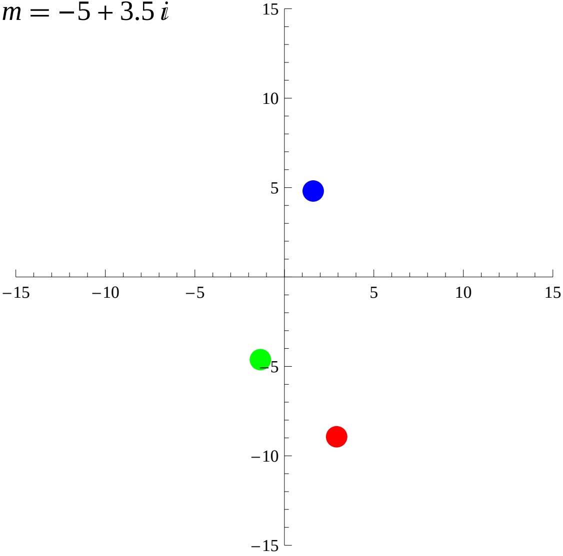
Some gratuitous animation...

$m = -5 + 4.\,i$



Some gratuitous animation...

$m = -5 + 3.5\,i$



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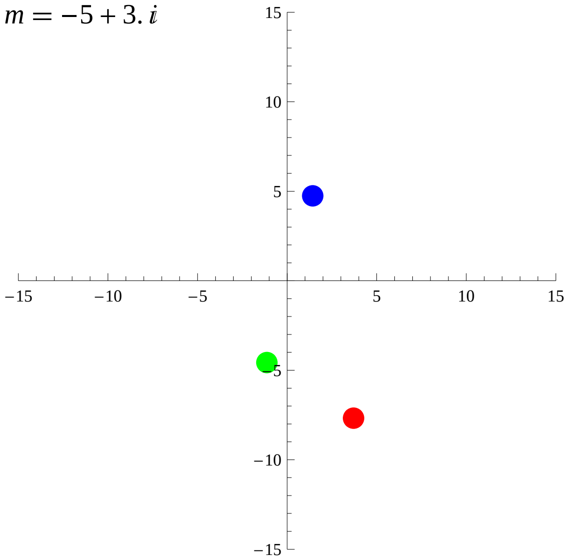
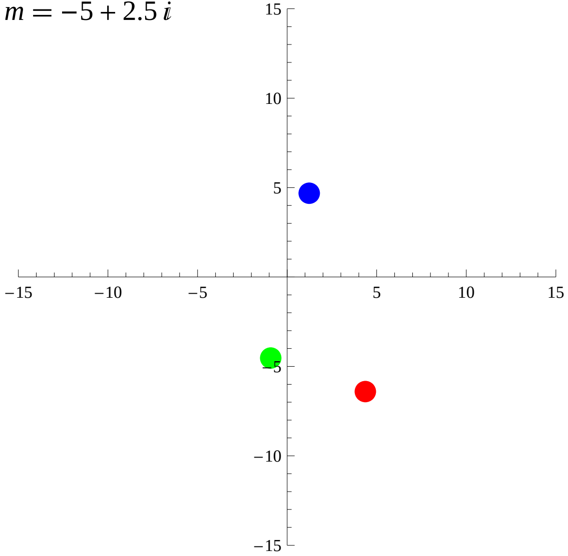


Some gratuitous animation...

$m = -5 + 2.5\,i$



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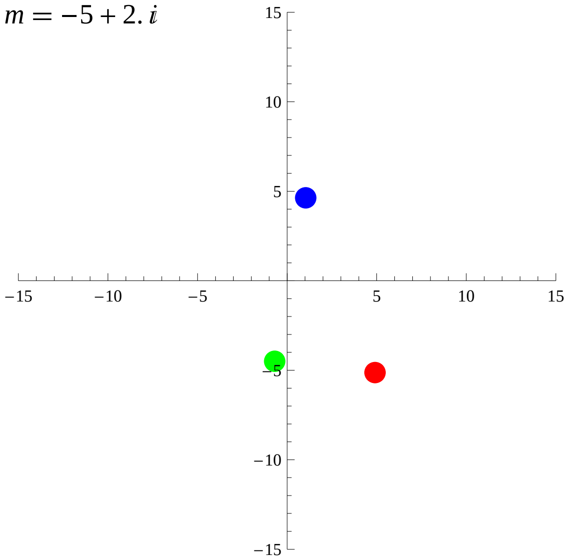

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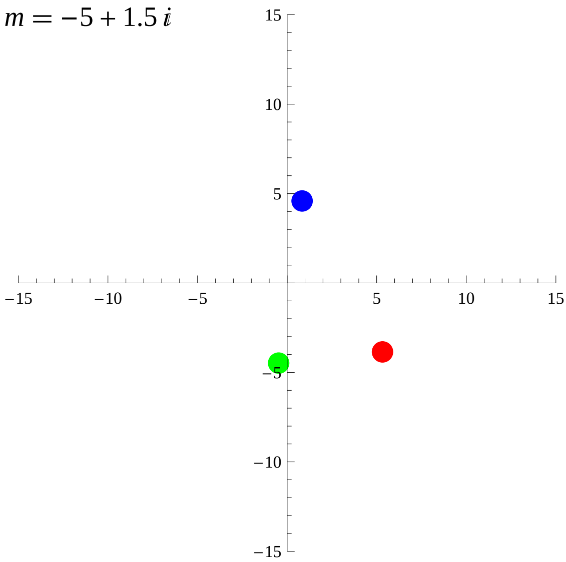
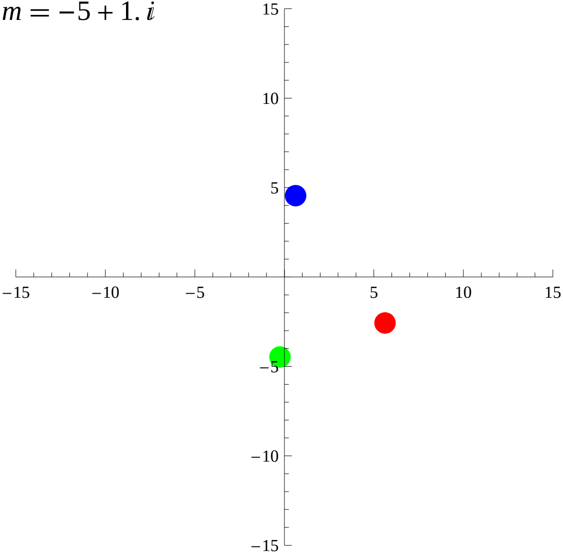


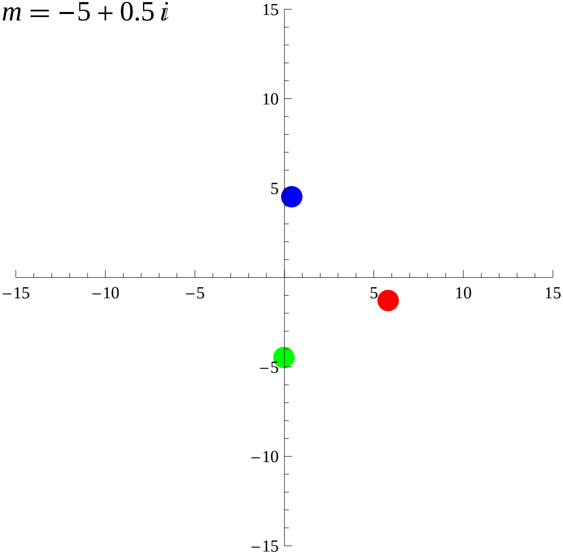
Some gratuitous animation...

$m = -5 + 1.\,i$



Some gratuitous animation...

$m = -5 + 0.5\,i$



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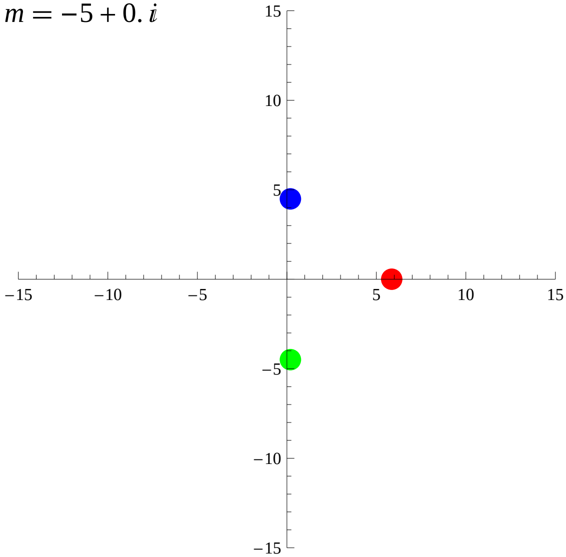

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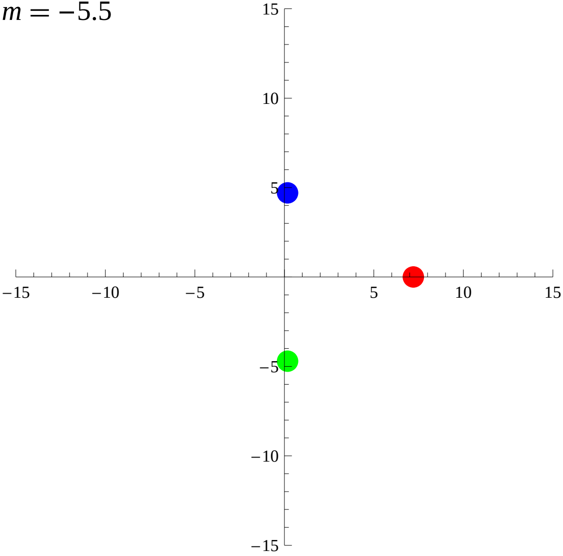
$m = -5.5$



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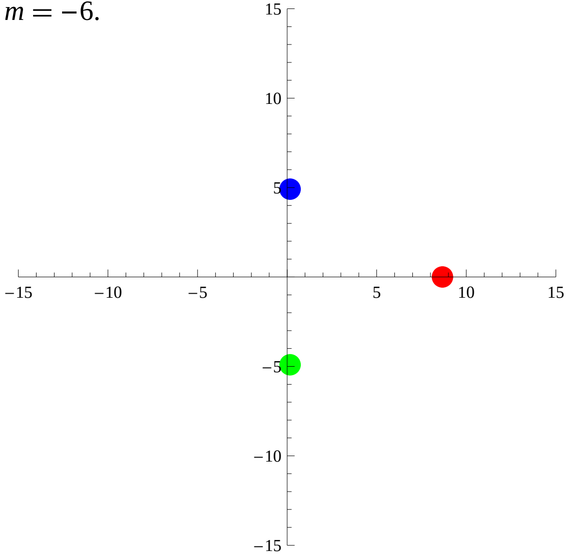

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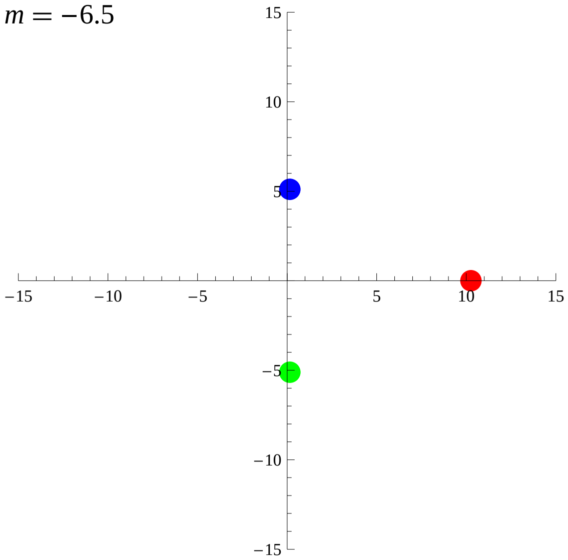
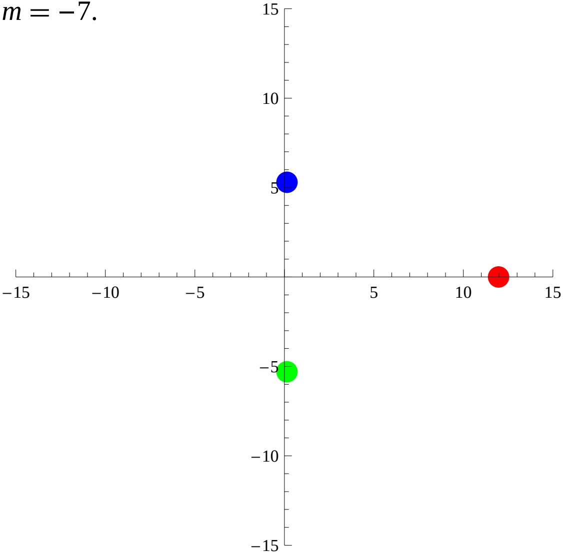


Some gratuitous animation...

$m = -7.$



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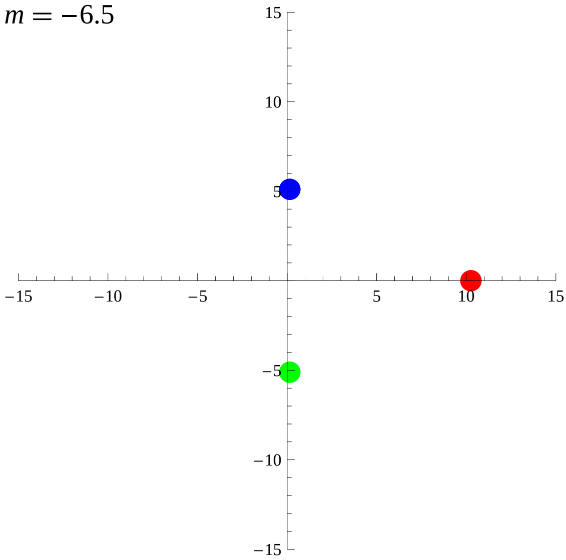
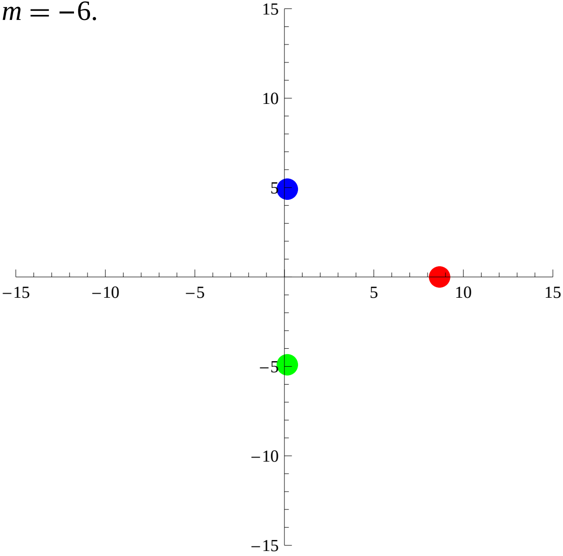


Some gratuitous animation...

$m = -6.$



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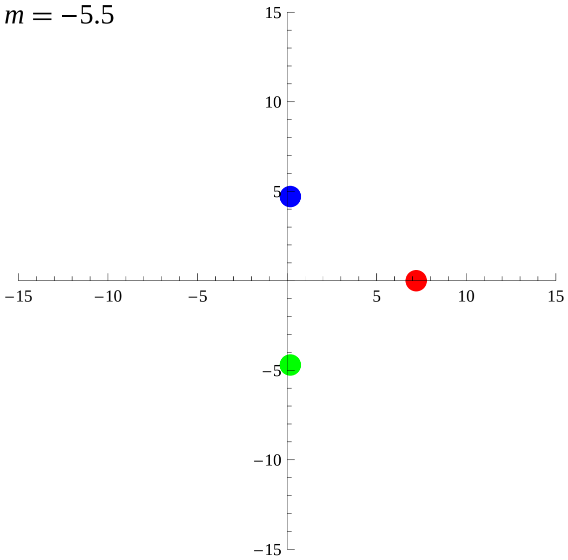
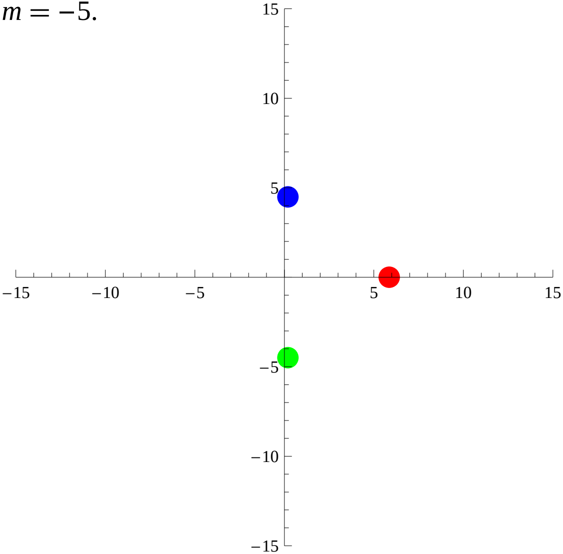
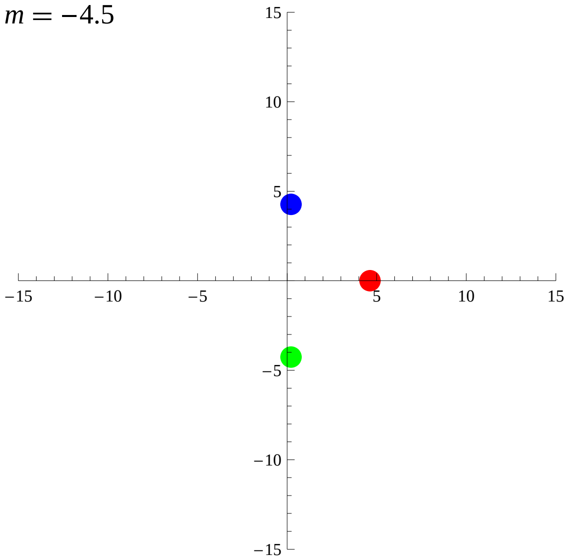


Some gratuitous animation...

$m = -5.$

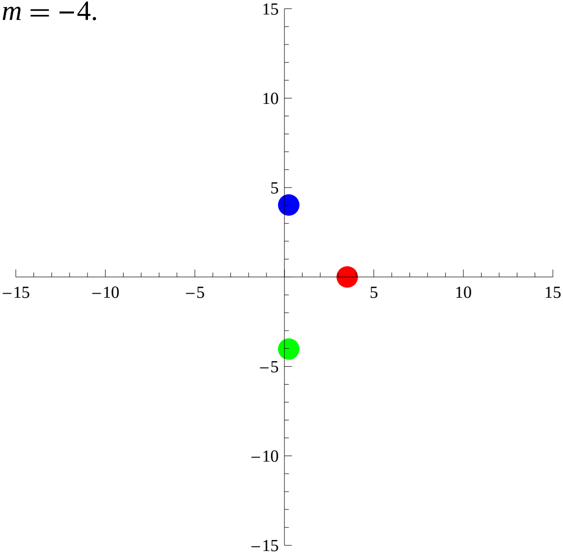


Some gratuitous animation...



Some gratuitous animation...

$m = -4.$



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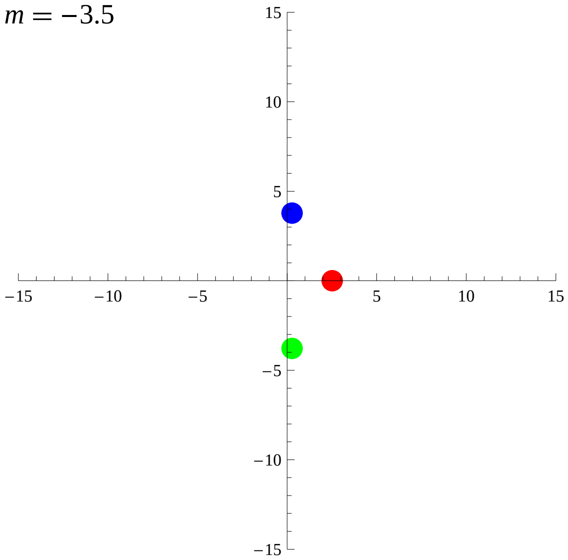
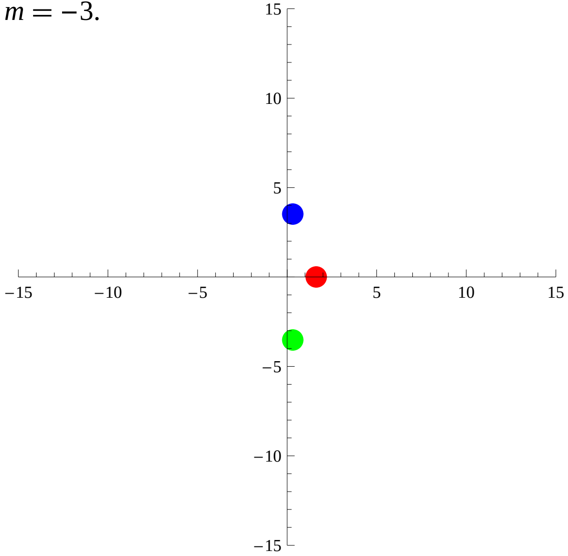
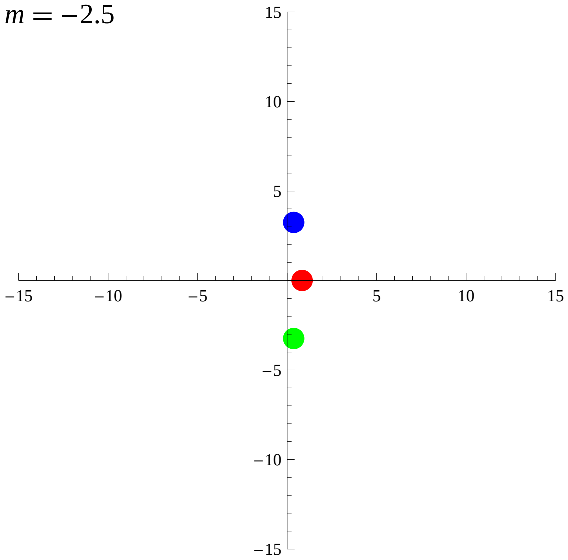


Some gratuitous animation...

$m = -3.$

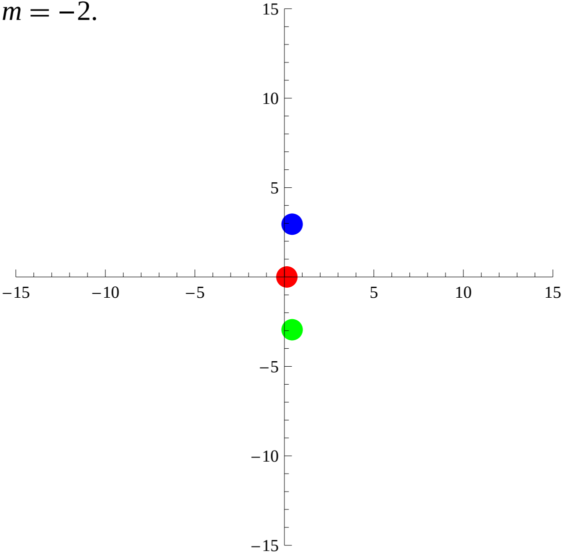


Some gratuitous animation...

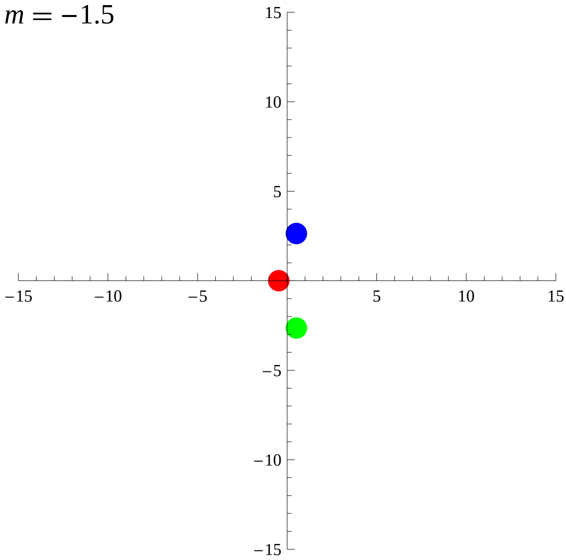


Some gratuitous animation...

$m = -2.$

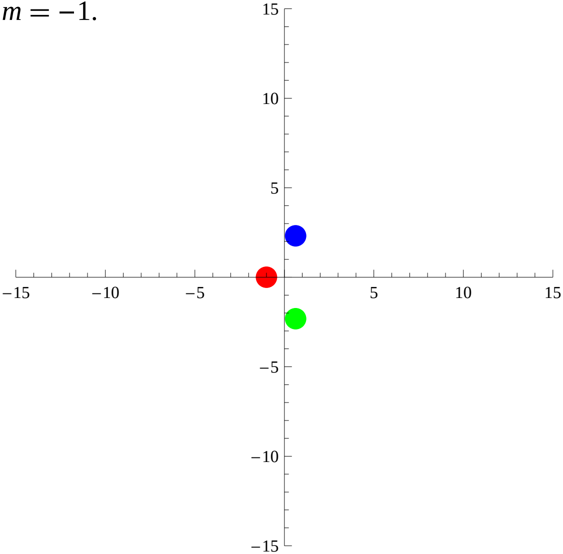


Some gratuitous animation...

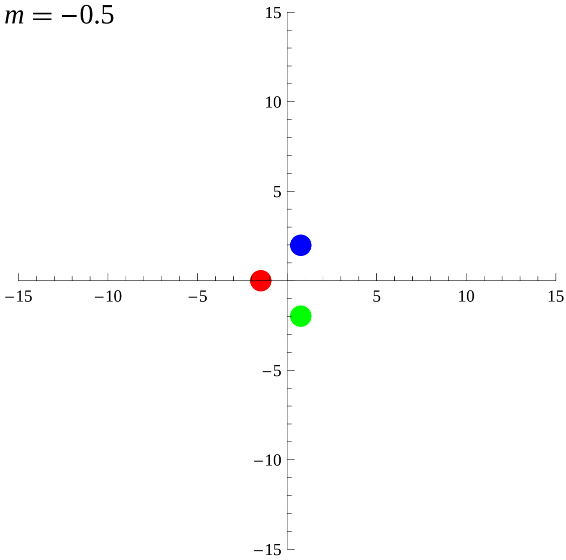


Some gratuitous animation...

$m = -1.$

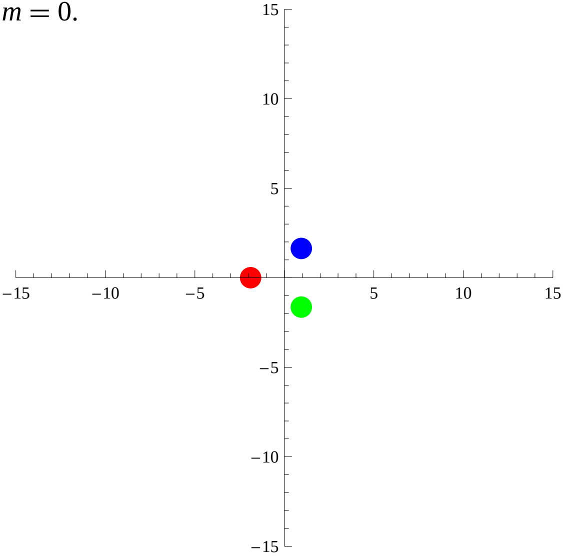


Some gratuitous animation...



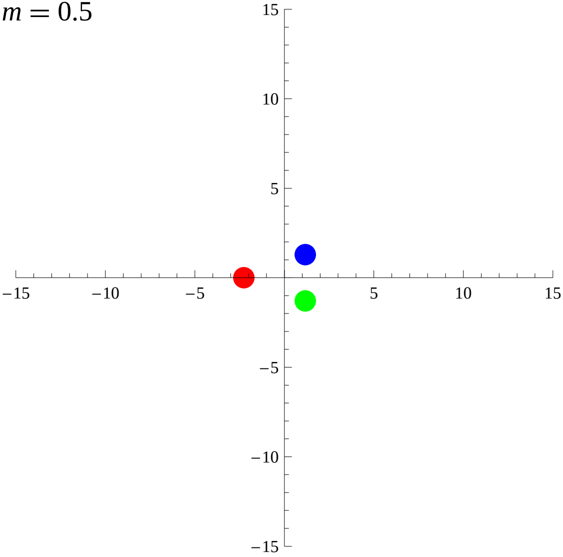
Some gratuitous animation...

$m = 0.$



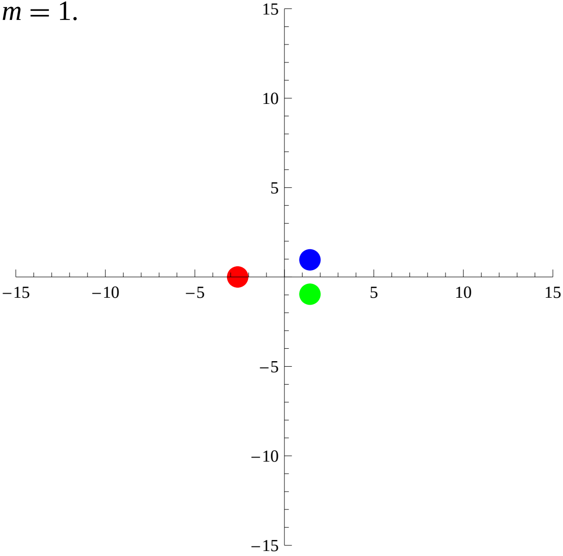
Some gratuitous animation...

$m = 0.5$



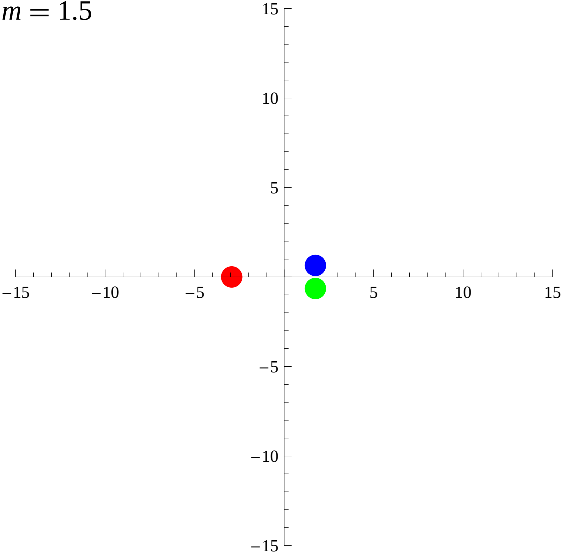
Some gratuitous animation...

$m = 1.$



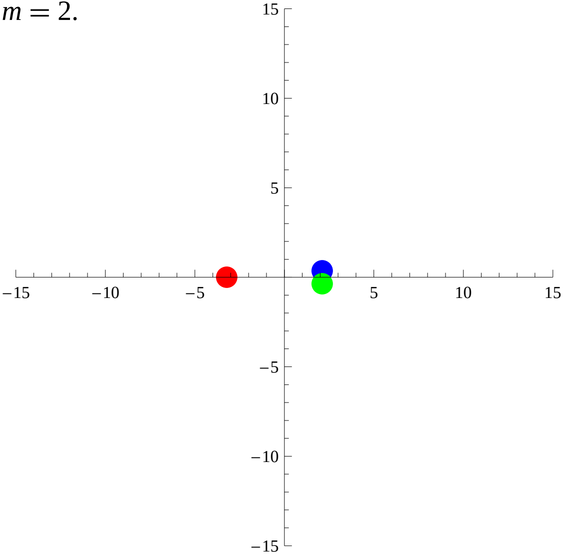
Some gratuitous animation...

$m = 1.5$



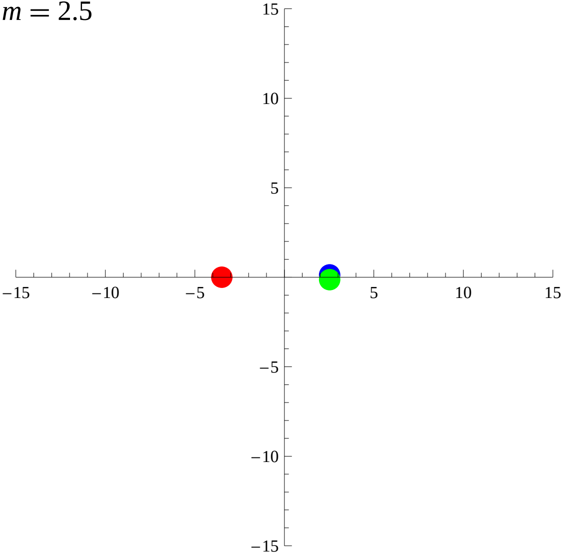
Some gratuitous animation...

$m = 2.$



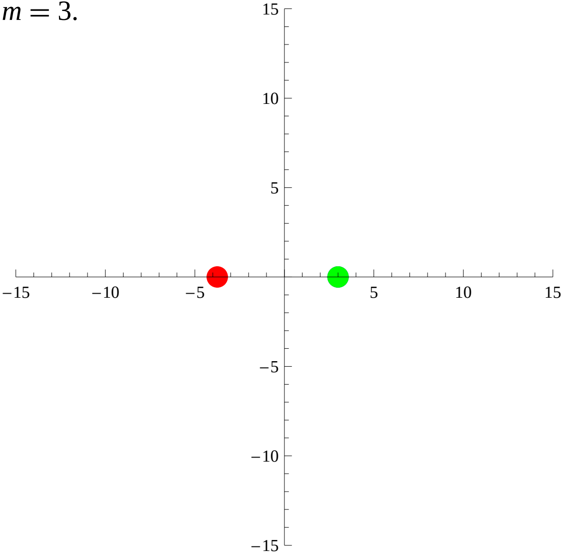
Some gratuitous animation...

$m = 2.5$



Some gratuitous animation...

$m = 3.$



Strongly coupled $\mathcal{N} = 2$ SCFT

- Electron & Monopole both massless at $u = 0$

  → likely to be a conformal theory

  [Argyres-Douglas, Argyres-Plesser-Seiberg-Witten,Eguchi-Hori-Ito-Yang]
- Suppose in a conformal theory there is an operator $F_{\mu\nu}$
- If

$$\partial_\mu F^{\mu\nu} = \partial_\mu \tilde{F}^{\mu\nu} = 0$$

  it is free, zero anomalous dimension.
- If not, it's guaranteed that both

$$\partial_\mu F^{\mu\nu} \neq 0, \qquad \partial_\mu \tilde{F}^{\mu\nu} \neq 0$$

  → There should be both electric & magnetic sources.

Argyres-Douglas point

- SW curve close to the AD point:

$$y^2 = \tilde{x}^3 + \tilde{m}\tilde{x} - \tilde{u}$$

- $D(\tilde{x}) : D(\tilde{y}) : D(\tilde{u}) : D(\tilde{m}) = 2 : 3 : 4 : 6$
- $D(\int_A \lambda_{SW}) = 1$, $\lambda_{SW} = \dfrac{\tilde{u}d\tilde{x}}{y}$

  → $D(\tilde{u}) = 6/5$, $D(\tilde{m}) = 4/5$.
- Recall $u = \mathrm{tr}\,\phi^2$, so $D_{UV}(u) = 2$. Strongly coupled !
- That's all what was known about $\mathcal{N} = 2$ SCFT before Nov 2007.

a and c of AD points

- a and c measure the response of the CFT to the external gravity

$$\langle T^{\mu}_{\mu} \rangle = a \times \text{Euler} + c \times \text{Weyl}^2$$

- the best way to couple $\mathcal{N} = 2$ supersymmetric theory to gravity = topological twisting.
- Are a and c encoded in the topological theory ?
  Yes ! in the so-called $A^{\chi} B^{\sigma}$ term which is known for 10 yrs by [Witten, Moore, Mariño, Losev, Nekrasov, Shatashvili]

Topological twisting

$$\delta\phi = \epsilon^{\alpha}\psi_{\alpha}$$

- On a curved bkg, constant ϵ_{α} not possible $\longrightarrow$ no global susy

Topological twisting

$$\delta\phi = \epsilon_i^\alpha \psi_\alpha^i$$

- On a curved bkg, constant ϵ_α^i not possible → no global susy
- They are $SU(2)_R$ doublet

Topological twisting

$$\delta\phi = \epsilon_i^\alpha \psi_\alpha^i$$

- On a curved bkg, constant ϵ_α^i not possible → no global susy
- They are $SU(2)_R$ doublet
- Introduce external $SU(2)_R$ gauge field ($a = 1, 2, 3$)

$$F_{\mu\nu,R}^a = R_{\mu\nu\rho\sigma}\Omega^{\rho\sigma,a}$$

i.e. self-dual part of metric connection.

- ϵ_i^α : $2 \times 2 = 1 + 3$
- One global susy preserved !

$A^\chi B^\sigma$ term

Just as nontrivial $\tau(u)F\tilde{F}$ is generated, on a curved manifold

$$S_{\text{curved}} = [\log A(u)]R\tilde{\tilde{R}} + [\log B(u)]R\tilde{R} + \cdots$$

are generated. Then we have

$$\langle O_1 O_2 \cdots \rangle = \int [du] e^{-S} A(u)^\chi B(u)^\sigma O_1 O_2 \cdots$$

with

$$\chi = \frac{1}{32\pi^2}\int d^4x\sqrt{g}R_{abcd}\tilde{\tilde{R}}_{abcd},$$

$$\sigma = \frac{1}{48\pi^2}\int d^4x\sqrt{g}R_{abcd}\tilde{R}_{abcd}.$$

$A^\chi B^\sigma$ and R anomaly

$$\langle O_1 O_2 \cdots \rangle = \int [du] A(u)^\chi B(u)^\sigma O_1 O_2 \cdots$$

means, on a curved manifold, $\langle O_1 O_2 \cdots \rangle$ nonzero only if

$$R(O_1) + R(O_2) + \cdots = -\chi R(A) - \sigma R(B) - R([du])$$

i.e. the vacuum has the R-anomaly

$$\Delta R = \chi R(A) + \sigma R(B) + \frac{\chi + \sigma}{2} r + \frac{\sigma}{4} h$$

where r, h = # of free vectors / hypers

R-anomaly in physical/twisted theories

$$a = \frac{3}{32}\left[3\,\mathrm{tr}\,R^3_{\mathcal{N}=1} - \mathrm{tr}\,R_{\mathcal{N}=1}\right], \quad c = \frac{1}{32}\left[9\,\mathrm{tr}\,R^3_{\mathcal{N}=1} - 5\,\mathrm{tr}\,R_{\mathcal{N}=1}\right]$$

can also be represented as

$$\partial_\mu R^\mu_{\mathcal{N}=1} = \frac{c-a}{24\pi^2} R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} + \frac{5a-3c}{9\pi^2} F^{\mathcal{N}=1}_{\mu\nu}\tilde{F}^{\mu\nu}_{\mathcal{N}=1}$$

Using $R_{\mathcal{N}=1} = R_{\mathcal{N}=2}/3 + 4I_3/3$ etc., we have

$$\partial_\mu R^\mu_{\mathcal{N}=2} = \frac{c-a}{8\pi^2} R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} + \frac{2a-c}{8\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

R-anomaly in physical/twisted theories

Twisting sets

$$F^a_{\mu\nu} = \text{anti-self-dual part of } R_{\mu\nu\rho\sigma}$$

so

$$\partial_\mu R^\mu = \frac{c-a}{8\pi^2} R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma} + \frac{2a-c}{8\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a$$

becomes

$$\partial_\mu R^\mu = \frac{2a-c}{16\pi^2} R_{\mu\nu\rho\sigma}\tilde{\tilde{R}}_{\mu\nu\rho\sigma} + \frac{c}{16\pi^2} R_{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma}.$$

Therefore

$$\Delta R = 2(2a-c)\chi + 3c\,\sigma$$

$A^\chi B^\sigma$ and a, c

Comparing

$$\Delta R = 2(2a - c)\chi + 3c\,\sigma$$

and

$$\Delta R = \chi R(A) + \sigma R(B) + \frac{\chi + \sigma}{2} r + \frac{\sigma}{4} h$$

we have

$$a = \frac{1}{4} R(A) + \frac{1}{6} R(B) + \frac{5}{24} r, \qquad c = \frac{1}{3} R(B) + \frac{1}{6} r.$$

A and B have been calculated

[Witten,Moore,Mariño,Nekrasov,Losev,Shatashivili]

⟶ taking their R-charges, we get a and c.

Determination of $A^\chi B^\sigma$

- again, it's gravitational analog of $\tau(u)F\tilde{F}$:

$$[\log A(u)]R\tilde{\tilde{R}} + [\log B(u)]R\tilde{R}.$$

- $\tau(u)$ was determined from
  - Holomorphy
  - Semiclassical behavior
  - Behavior around the singular point in the moduli
- The same works for $A^\chi B^\sigma$.

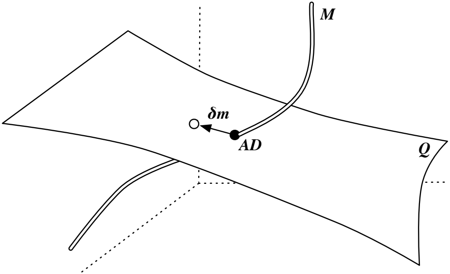
Determination of $A^\chi B^\sigma$

$$A(u)^2 = \det \frac{\partial u_i}{\partial a_I} \qquad B(u)^8 = \Delta$$

- $u_i = \mathrm{tr}\,\phi^i$: gauge-invariant coordinates
- a^I: special coordinates i.e. masses of BPS particles
- Δ: the discriminant of the SW curve

Argyres-Douglas points

- Consider $U(N_c)$ with N_f quarks
- There's a subspace where $U(N_c - 1)$ with N_f massless quarks is realized semiclassically: $\phi = \mathbf{diag}(m, 0, 0, \ldots, 0)$
- A monopole locus intersects at the strongly-coupled region



Argyres-Douglas points

$U(n+1)$ with $N_f = 2n$ flavors:

$$y^2 = P(x)^2 - \Lambda^2 x^{2n}$$

where

$$P(x) = \langle \det(x-\phi) \rangle = x^{n+1} + u_1 x^n + \cdots + u_{n+1}$$

and

$$\lambda_{SW} = x d \log \frac{1 - y/P}{1 + y/P}.$$

Take $u_2 = u_3 = \cdots = 0$:

$$y^2 = x^{2n}[(x+u_1)^2 - \Lambda^2].$$

Then let $u_1 = \Lambda$:

$$y^2 = x^{2n+1}(x + 2\Lambda).$$

Argyres-Douglas points

Curve: $y^2 = x^{2n+1}(x+2\Lambda)$ with $\lambda_{SW} \sim x d(y/P) \sim x^{1-n} dy.$

$$\longrightarrow \qquad D(x) = \frac{2}{3}, \qquad D(u_j) = j D(x).$$

Therefore
$$\begin{cases} A^2 = \det \dfrac{\partial u_i}{\partial a^I} \sim x^{n^2/2}, \\ B^8 = \Delta = \prod (e_i - e_j)^2 \sim x^{2n(2n+1)}. \end{cases}$$

Use our formula

$$a = \frac{1}{4} R(A) + \frac{1}{6} R(B) + \frac{5}{24} r, \qquad c = \frac{1}{3} R(B) + \frac{1}{6} r.$$

and get

$$a = \frac{14n^2 + 19n}{72}, \qquad c = \frac{4n^2 + 5n}{18}.$$

A counterexample to the a-theorem

Flow associated to Higgsing:

$$\mathbf{SU}(N+1) \text{ with } 2N \text{ flavors:} \quad a \sim \tfrac{7}{36} N^2$$

$$\mathbf{SU}(N) \text{ with } 2N \text{ flavors:} \quad a \sim \tfrac{7}{24} N^2$$

- Violation occurs first at $N = 4$. No violation for $N = 2, 3$.
- One way to understand the factor $2/3$: our formula implies

$$4(2a - c) = \sum_j [2D(u_j) - 1].$$

originally conjectured by Aharony and Argyres.

$$D_{UV}(u_j) = jD_{UV}(x) = \frac{2}{3}j, \qquad D_{IR}(u_j) = jD_{IR}(x) = j.$$

Compatibility to the previous results

a-maximization-based proofs

Our R symmetry is completely accidental,
't Hooft anomaly matching unusable

AdS/CFT-based proofs

Our theories can not have weakly-curved AdS duals,
because $a/c \to 7/8$ in the large N limit.

Possible objections...

- Is our method correct?

  → Yes I believe so. More on the next slide.

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- Is Argyres-Douglas points really superconformal?
  → If so it's more fascinating !

Is our method correct?

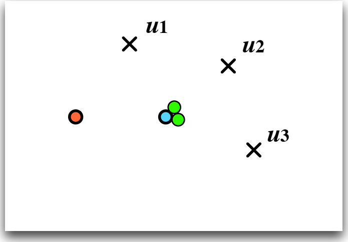
Gives the same a and c for AD points of

- pure **SU(3)**
- **SU(2)** with one massive flavor

as it should be,
because they're known to be the same theory in the infrared.

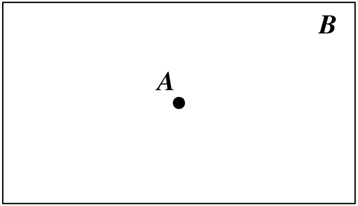
Is our method correct?

$USp(2N)$ + N_f flavors + 1 antisymmetric
= N D3-branes probing O7 + N_f D7's



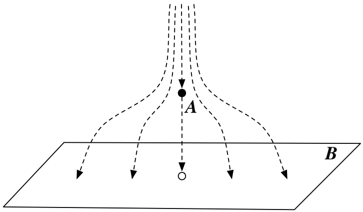
- Can be analyzed both purely field theoretically and holographically. [Aharony-YT] [Shapere-YT]
- Gives exactly the same result, including $1/N$ corrections.

Is the direction of the flow correct?



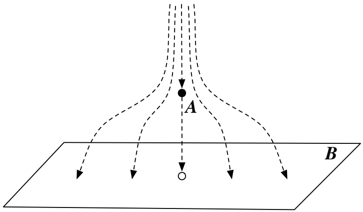
- Consider $U(1)$ theory with a charged quark.
- What Seiberg-Witten theory tells us:
- At A the quark becomes massless
- At generic points in B it's just pure $U(1)$, varying coupling
- all at the extreme infrared.

Is the direction of the flow correct?



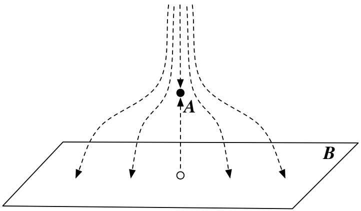
- RG scale is an additional direction.
- At A the quark is massless, and at zero coupling.
- Mass term brings it to pure $U(1)$ at zero coupling.

Is the direction of the flow correct?



- We believe the situation is the same with our case
- At A one has the AD point of $\mathbf{SU}(N+1)$ with $2N$ quarks
- On B one has $\mathbf{SU}(N)$ with $2N$ quarks.

Is the direction of the flow correct?



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Summary

- Brief history of 'a-theorem'.
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- Strongly coupled,
- R symmetry completely accidental,
- No weakly-curved AdS dual.

- I would be happier if I can calculate η/s of this thing.

R.I.P.

a-theorem

1988 — 2008

We hardly knew ye...