4d SYM on a graph and 3d mirror symmetry

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based on [1007.0992]
with **Francesco Benini** and **Dan Xie**.

May 2011

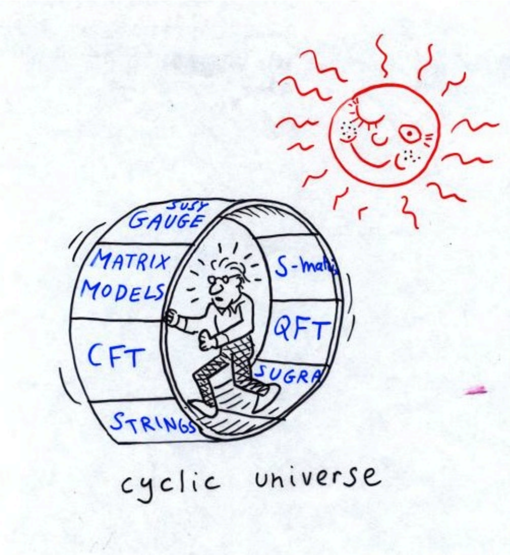
I'm going to talk about **3d mirror symmetry**,

which was introduced by **Nati Seiberg** and **Ken Intriligator** in 1996.

It's **15 years** ago, at the height of the 2nd revolution.

Ooguri-san also wrote papers on 3d mirror.

Alas I came late to the party. I started my graduate study in 2002.

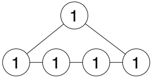


(taken from Robbert Dijkgraaf's talk, Strings 2002)

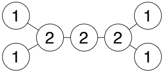
	Coulomb	Higgs
5d $\mathcal{N} = \mathbf{1}$	very special	**hyperkähler**
	$\downarrow$	$\downarrow$
4d $\mathcal{N} = \mathbf{2}$	special Kähler	**hyperkähler**
	$\downarrow$	$\downarrow$
3d $\mathcal{N} = 4$	**hyperkähler**	**hyperkähler**

Basic examples are: [Intriligator-Seiberg]

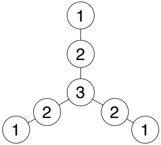
U(1) with n quarks $\leftrightarrow$



SU(2) with n quarks $\leftrightarrow$



"The E_6 theory" $\leftrightarrow$



which exchange

$$\mathbb{C}^2/\Gamma_{A,D,E}$$

and

1-instanton moduli space of A, D, E

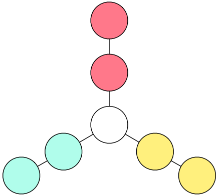
A and D had a nice brane interpretation.

[Hanany-Witten], [de Boer-Hori-Ooguri-Oz-Yin]

E looked rather, uh, exceptional,

and non-Lagrangian on the 'original' side.

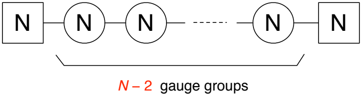
[Gaiotto] found that the E_6 theory is a member T_3
of an infinite family T_N, having $\mathbf{SU}(N)^3$ flavor symmetry in general.



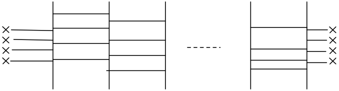
$\mathbf{SU}(3)^3 \subset E_6$

$T_{N\geq 3}$ are all non-Lagrangian.

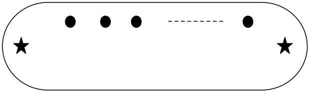
Recall the construction of T_N. Start from 4d $\mathcal{N} = 2$ conformal quiver



Realize in IIA

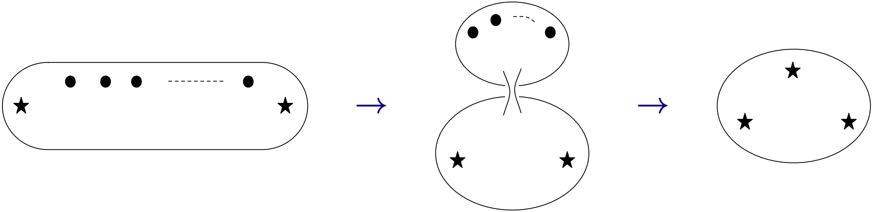


Lift to M



Note $\star$ carries $\mathbf{SU}(N)$ flavor symmetry, and $\bullet$ carries $\mathbf{U}(1)$.

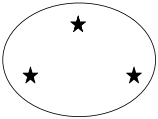
Change the marginal couplings to the strongly-coupled region



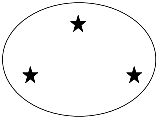
The resulting theory is T_N with (at least) $\mathbf{SU}(N)^3$ symmetry.

You wrapped non-Lagrangian 6d theory on a space without S^1.
Of course you get a non-Lagrangian theory.

4d T_N theory is

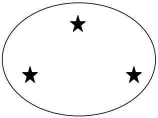
N M5s on 

which is non-Lagrangian. 3d T_N theory is the 4d T_N on S^1, i.e.

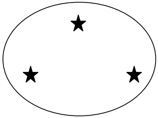
N M5s on  $\times S^1$

which is non-Lagrangian, in this frame.

3d T_N theory is the 4d T_N on S^1, i.e.

N M5s on  $\times S^1$

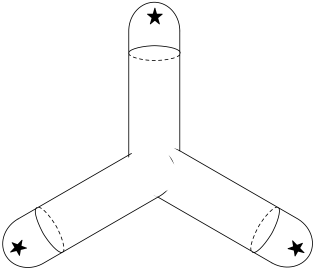
which is non-Lagrangian, in this frame.
But, first compactify along S^1. Then 3d T_N is

5d SYM on 

This **should have a Lagrangian**...It has a Lagrangian in 5d already!
and this is the **mirror description**.

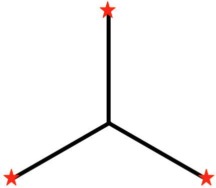
The mirror of 3d T_N is

5d SYM on



which is

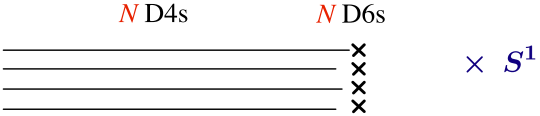
4d SYM on



What happens at $\star$?

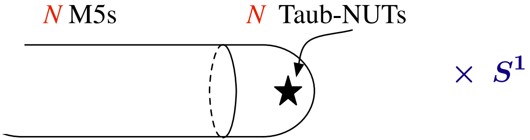
To get

we started from IIA,



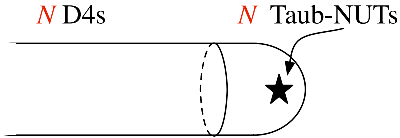
To get

we started from IIA, lifted it to M,



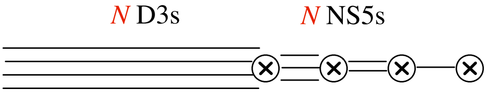
To get

we started from IIA, lifted it to M, reduced along S^1,



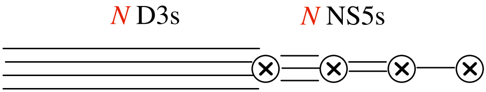
To get

we started from IIA, lifted it to M, reduced along S^1, took the T-dual

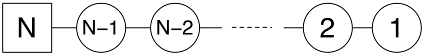


The boundary condition ⋆

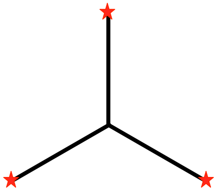
is



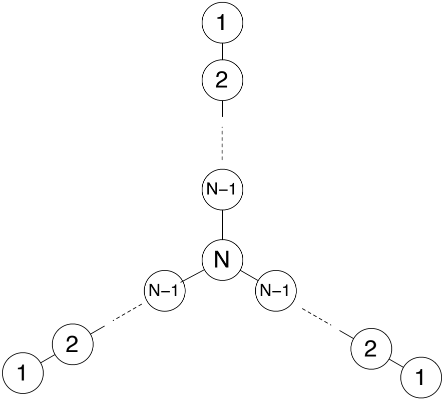
i.e. at the boundary of 4d SYM $\mathbf{U}(N)$, we have 3d quiver



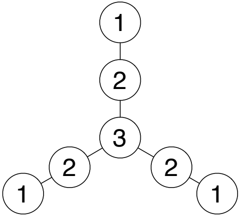
The mirror of 3d T_N, which is 4d SYM on



then becomes



For T_3 i.e. the E_6 theory, the mirror is then



reproducing [Intriligator-Seiberg].

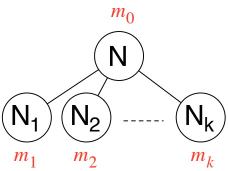
Each $\mathbf{U(1)}$ factor gives a 'magnetic' conserved current

$$j = \star F \quad \rightarrow \quad d \star j = dF = 0$$

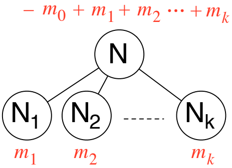
How do we see the enhancement from $\mathbf{U(1)^5}$ to E_6?

[Gaiotto-Witten] showed two things.

1. When [2N]—(N), the monopole operator with charge 1 is of **dimension 2** and **vector** $\rightarrow$ a conserved current.
2. If $2N = N_1 + \cdots + N_k$ and there is a dimension-2 vector magnetic monopole of the form

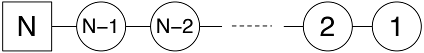


then

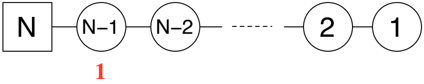


is also a dimension-2 vector magnetic monopole.

For example, Every node of



is $N_f = 2N_c$, and therefore has conserved monopole currents of the form



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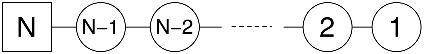

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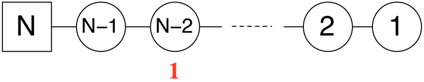

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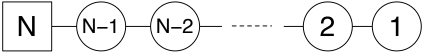

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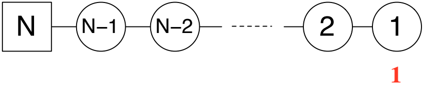

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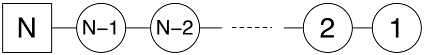

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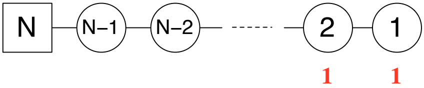

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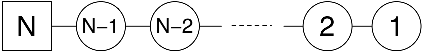

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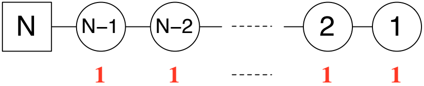

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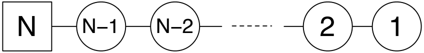

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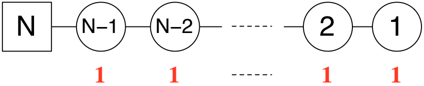
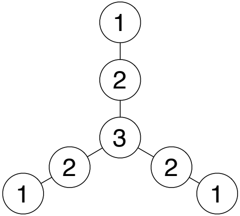


Together with $j = *F$,
they generate $\mathbf{SU}(N)$ symmetry acting on the **Coulomb branch**.

Leftmost $\mathbf{SU}(N)$ acts on the **Higgs branch**.

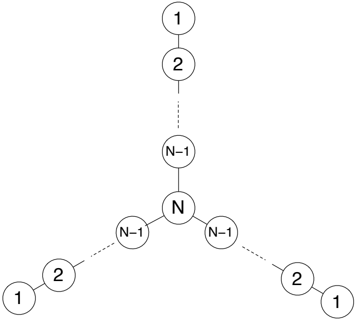
For T_3 theory



every node satisfies $N_f = 2N_c$.

Conserved currents from monopoles fill E_6 root system.

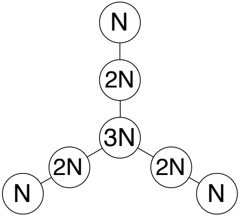
For $T_{N>3}$ theory,



every node **except the center** satisfies $N_f = 2N_c$.

Conserved currents from monopoles only give $\mathbf{SU}(N)^3$.

Every node of



satisfies $N_f = 2N_c$.

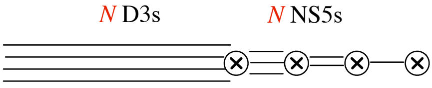
Conserved currents from monopoles give E_6.

Quantum-corrected Coulomb branch is supposed to be

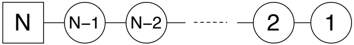
k-instanton moduli of E_6

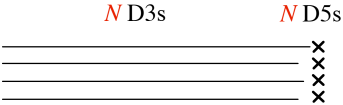
This might, one day, give a substitute for ADHM construction for E_n.

[Gaiotto-Witten] discussed the S-duality of the B.C.



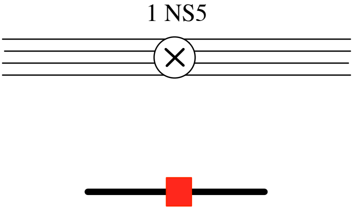
coupling to the quiver



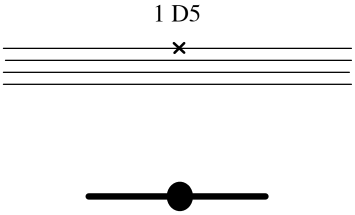


'Dirichlet' BC

Another example discussed was



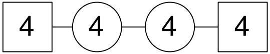
split into $\mathbf{U}(N) \times \mathbf{U}(N)$ quiver



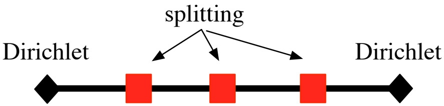
add a quark

note that they are BC of $\mathbf{U}(N)^2$ theory: 

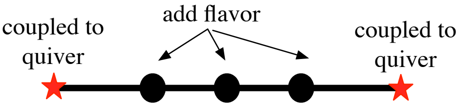
This streamlines the 3d mirror construction greatly. E.g.



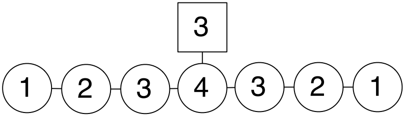
is the low energy limit of

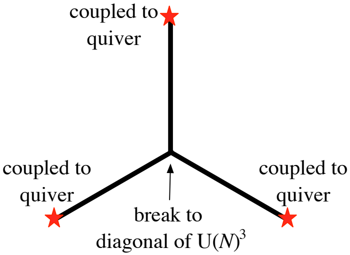


which is the S-dual of

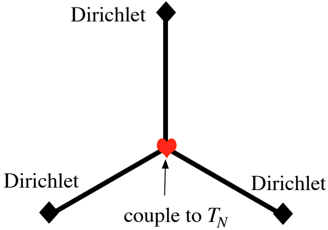


whose low-energy limit is



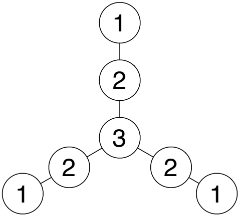


← S-dual →



low-energy limit ↓

↓ low-energy limit



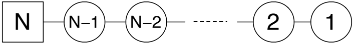
← mirror →

non-Lagrangian T_N

← S-dual →

‘Dirichlet’ BC

coupling to the quiver

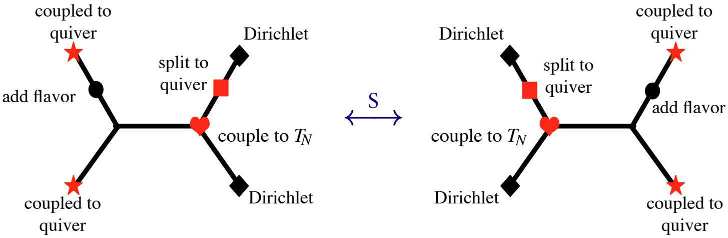


← S-dual →

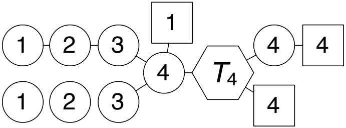
break $\mathbf{U}(N)^3$ to diagonal $\mathbf{U}(N)$

coupling to non-Lagrangian T_N

Then we can do stupid things like



so we see



is self-mirror.

Summarizing,

- Construction of 3d mirrors can be streamlined using **4d theory on graphs**.
- 3d versions of Gaiotto's non-Lagrangian theories all have **Lagrangian mirrors**.
- All these can be checked at the level of **partition functions**, using the matrix model of Kapustin-Willet-Yaakov.
  [Benvenuti-Pasquetti] (last week)
  [Nishioka-Yamazaki-YT] (to appear soon...)

Announcement:

Four String Generations at IPMU

May 2030

Everyone is invited!