# Classification of $\mathcal{N} = 6$ theories of ABJM type

Yuji Tachikawa (IAS)

in collaboration with
Martin Schnabl (IAS $\longrightarrow$ FZÚ, Prague)

arXiv:0807.1102

McGill Workshop,
Montréal, September, 2008

# Contents

# Motivation

**$\mathcal{N} = 8$ Lagrangian [Bagger-Lambert,Gustavsson]**

- Relevant to the study of M2-branes
- Classification via 3-algebra [Papadopoulos,Gauntlett-Gutowski] [Nagy]

# Motivation

## $\mathcal{N} = 8$ Lagrangian [Bagger-Lambert,Gustavsson]

- Relevant to the study of M2-branes
- Classification via 3-algebra [Papadopoulos,Gauntlett-Gutowski] [Nagy]

## $\mathcal{N} = 4$ Lagrangian [Gaiotto-Witten]

- Arose from the study of boundary cond. of $\mathcal{N} = 4$ $d = 4$.
- Lagrangians clearly related to BLG...

# Motivation

### $\mathcal{N} = 8$ Lagrangian [Bagger-Lambert,Gustavsson]

- Relevant to the study of M2-branes
- Classification via 3-algebra [Papadopoulos,Gauntlett-Gutowski] [Nagy]

### $\mathcal{N} = 4$ Lagrangian [Gaiotto-Witten]

- Arose from the study of boundary cond. of $\mathcal{N} = 4$ $d = 4$.
- Lagrangians clearly related to BLG...

### $\mathcal{N} = 6$ Lagrangian [Aharony-Bergman-Jafferis-Maldacena]

- Membrane on orbifolds.
- Enhancement mechanism quite understandable.
- Lagrangians clearly related to Gaiotto-Witten...

# Motivation

**Aim**

- classification of $\mathcal{N} = \mathbf{8}$ lagrangians
- classification of $\mathcal{N} = \mathbf{6}$ lagrangians
- Better understanding of relations among [BLG,GW,ABJM]

[Hosomichi-Lee-Lee-Lee-Park] appeared one week before ours.

- Theirs use their $\mathcal{N} = \mathbf{4}$ formalism.
- Ours use more pedestrian $\mathcal{N} = \mathbf{2}$ formalism.
- Today's talk combine aspects of both.

# Personal Motivation

Completely new class of Lagrangians unfolding in front of my eyes.
[BLG,Gaiotto-Witten,ABJM]

Simply Amazing.
Aren't Lagrangians with arbitrary $(d, \mathcal{N})$ totally explored in 1980s ?

# Personal Motivation

Completely new class of Lagrangians unfolding in front of my eyes. [BLG,Gaiotto-Witten,ABJM]

Simply Amazing.
Aren't Lagrangians with arbitrary $(d, \mathcal{N})$ totally explored in 1980s ?

I just love Lie algebras.

# Contents

# Contents

# SUSY enhancement

## $d = 4$ $\mathcal{N} = 2$ in $\mathcal{N} = 1$ notation

- $V$, $\Phi_p$: adjoint of $G$
- $Q_a$, $\tilde{Q}^a$: chiral multiplets in $R$ and $R^*$
- $W = T^{pa}{}_b Q_a \tilde{Q}^b \Phi_p$ with a specific coef.

- has $\mathbf{SU(2)}_R \curvearrowright Q, \tilde{Q}^\dagger$
- Not Manifest in $\mathcal{N} = 1$ formalism

# SUSY enhancement

## When $Q$, $\tilde{Q}$ are adjoints

- $W = f^{pqr} Q_p \tilde{Q}_q \Phi_r$ has $\mathbf{SU(3)}_F$
- Manifest in $\mathcal{N} = 1$ formalism

- do not commute with $\mathbf{SU(2)}_R$
- combine to form $\mathbf{SU(4)}_R \curvearrowright Q, \tilde{Q}, \Phi$.

# SUSY enhancement

## When $Q$, $\tilde{Q}$ are adjoints

- $W = f^{pqr} Q_p \tilde{Q}_q \Phi_r$ has $\mathbf{SU(3)}_F$
- Manifest in $\mathcal{N} = 1$ formalism

- do not commute with $\mathbf{SU(2)}_R$
- combine to form $\mathbf{SU(4)}_R \curvearrowright Q, \tilde{Q}, \Phi$.

## In 3d

- classically conformal → quartic superpotential
- [Schwarz,2004] tried to use four adjoints
- no suitable $f^{pqrs}$, contrary to 4d case with $f^{pqr}$.

*We now know how to circumvent this...*

# $\mathcal{N}$-extended susy in 3d

- the same number of supercharges with $\mathcal{N}/2$-extended susy in 4d
- $\mathbf{SO}(\mathcal{N})_R$ symmetry

| $\mathcal{N}$ | $\phi$ | $\psi$ | $Q$ | |
|---|---|---|---|---|
| **3** | **2** | **2** | **3** | hyper |
| **4** | **2** | $\mathbf{2}'$ | **4** | hyper |
| | $\mathbf{2}'$ | **2** | **4** | twisted-hyper |
| **5** | **4** | **4** | **5** | |
| **6** | **4** | **4** | **6** | |
| **7** | **8** | **8** | **7** | |
| **8** | $\mathbf{8}_S$ | $\mathbf{8}_C$ | $\mathbf{8}_V$ | ultra |
| | $\mathbf{8}_C$ | $\mathbf{8}_S$ | $\mathbf{8}_V$ | twisted-ultra |

- These multiplets don't have vectors in it ! Only possible in 3d
- need not be adjoint !

# Super Chern-Simons in 3d

- $\mathcal{N}=2$ formalism in 3d $\simeq \mathcal{N}=1$ formalism in 4d.
- $\mathcal{N}=2$ super CS for arbitrary $G$, chiral matter $Q$, superpotential $W$

$$S = kS_{\mathcal{N}=2CS} + \int d^4\theta Q^\dagger e^V Q + \int d^2\theta W + c.c.$$

- $\mathcal{N}=3$ when chiral matters are $A$ in $R$, $B$ in $R^*$, $\Phi$ in adjoint

$$S = kS_{\mathcal{N}=2CS} + k\int d^2 \operatorname{tr}\Phi^2 + c.c.$$
$$+ \int d^4\theta(A^\dagger e^V A + B^\dagger e^{-V} B) + \int d^2\theta(A\Phi B) + c.c.$$

- $\mathrm{SU}(2)_R \curvearrowright (A, B^\dagger)$
- no kinetic term for $\Phi$ ⟶ can be integrated out

# $\mathcal{N} = 3$ super Chern-Simons in 3d

- $\mathcal{N} = 3$ when chiral matters are $A$ in $R$, $B$ in $R^*$

$$S = kS_{\mathcal{N}=2CS} + \int d^4\theta (A^\dagger e^V A + B^\dagger e^{-V} B) + \int d^2\theta f_{a\bar{b}c\bar{d}} A^a B^{\bar{b}} A^c B^{\bar{d}} + c.c.$$

- $f_{a\bar{b}c\bar{d}} = (k^{-1})_{pq} T^p_{a\bar{b}} T^q_{c\bar{d}}$
- this can have enhanced symmetry,
- which does not commute with $\mathbf{SU(2)}_R$ ⟶ enhanced SUSY !

# Example: $\mathcal{N} = 4$

- Take $G = U(N)_k \times U(N)_{-k}$, $A$ and $B^\dagger$ in the bifundamental
- $W = \mathrm{tr}\,\Phi_1^2 - \mathrm{tr}\,\Phi_2^2 + \mathrm{tr}\,AB\Phi_1 + \mathrm{tr}\,BA\Phi_2$

  $\longrightarrow$ $W = \mathrm{tr}(AB)^2 - \mathrm{tr}(BA)^2 = 0$ !
- has $\mathrm{U}(1)_A \times \mathrm{U}(1)_B$ acting on $A$ and $B$ separately
- does not commute with $\mathrm{SU}(2)_R \curvearrowright A, B^\dagger$.

| | $A$ | $\psi_A$ | $B$ | $\psi_B$ |
|---|---|---|---|---|
| $J_3 \in \mathrm{SU}(2)_R$ | $+1$ | $-1$ | $-1$ | $+1$ |
| $\mathrm{U}(1)_A$ | $+1$ | $+1$ | $0$ | $0$ |
| $\mathrm{U}(1)_B$ | $0$ | $0$ | $+1$ | $+1$ |

- $\mathrm{SO}(4)_R \curvearrowright A, B^\dagger, \mathcal{N} = 4$ !
- [Gaiotto-Witten], although they used $\mathcal{N} = 1$ formalism ...

# Example: $\mathcal{N} = 6$

- Take $G = \mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$, $A_i$ and $B^{i\dagger}$, $i = 1, 2$
- $W = \mathrm{tr}\,\Phi_1^2 - \mathrm{tr}\,\Phi_2^2 + \mathrm{tr}\,A_i B^i \Phi_1 + \mathrm{tr}\,B^i A_i \Phi_2$

  $\longrightarrow$ $W = \mathrm{tr}(A_i B^i)^2 - \mathrm{tr}(B^i A_i)^2 = \epsilon^{ij}\epsilon_{ab}\,\mathrm{tr}\,A_i B^a A_j B^b$
- has $\mathrm{SU}(2)_A \times \mathrm{SU}(2)_B$ acting on $A_{1,2}$ and $B^{1,2}$ separately
- does not commute with $\mathrm{SU}(2)_R \curvearrowright A_{1,2}, B^{1,2\dagger}$.
- $\mathrm{SU}(4)_R \curvearrowright A_{1,2}, B^{1,2\dagger}$, $\mathcal{N} = 6$ !
- [Aharony-Bergman-Jafferis-Maldacena]

# Example: $\mathcal{N} = 8$

- Take $G = \mathbf{SU(2)}_k \times \mathbf{SU(2)}_{-k}$, $C_{1,2,3,4} = (A_{1,2}, B^{1,2})$
- $W = \mathrm{tr}\,\Phi_1^2 - \mathrm{tr}\,\Phi_2^2 + \mathrm{tr}\,A_i B^i \Phi_1 + \mathrm{tr}\,B^i A_i \Phi_2$

  $\longrightarrow$ $W = \mathrm{tr}(A_i B^i)^2 - \mathrm{tr}(B^i A_i)^2 = \epsilon^{ij}\epsilon_{ab}\,\mathrm{tr}\,A_i B^a A_j B^b$
- has $\mathbf{SU(4)}$ acting on $C_{1,2,3,4}$

- does not commute with $\mathbf{SU(2)}_R \curvearrowright C, C^\dagger$.
- $\mathbf{SO(8)}_R \curvearrowright C, C^\dagger$, $\mathcal{N} = 8$ !
- [Bagger-Lambert, Gustavsson]

# Half-hypermultiplets

- A hyper of gauge group $G$ : $A$ in $R$ and $B$ in $R^*$
- A half-hyper of gauge group $G$: $Q$ in $R$ which is pseudo-real
- $\mathbf{SU(2)}_R$ acts on $(Q, Q^\dagger)$
- e.g. a doublet of **SU(2)**.
- two half-hyper $Q, \tilde{Q}$ in **2** of **SU(2)** forms a full-hyper
- Why rarely discussed ?
  → odd number of half-hypers often afflicted with
  Witten's global anomaly
- No need to worry in 3d.

# $\mathcal{N} = 4$

$Q$ : half-hyper in $R$.

$$W = k^{pq}\Phi_p\Phi_q + t^p_{ab}\Phi_p Q^a Q^b \longrightarrow W = f_{abcd}Q^aQ^bQ^cQ^d$$

where

$$f_{abcd} = (k^{-1})_{pq} t^p_{ab} t^p_{cd}$$

$R$ pseudo-real $\longrightarrow$ $t^p_{ab} = t^p_{ba}$. Suppose furthermore $f_{a(bcd)} = 0$.

$$\longrightarrow W = 0. \longrightarrow \mathrm{U}(1)_F \curvearrowright Q.$$

| | $Q$ | $\psi_Q$ |
|---|---|---|
| $J_3 \in \mathrm{SU}(2)_R$ | $+1$ | $-1$ |
| $\mathrm{U}(1)_F$ | $+1$ | $+1$ |

Enhancement to $\mathrm{SO}(4)_R = \mathrm{SU}(2) \times \mathrm{SU}(2)$. [Gaiotto-Witten]

# $\mathcal{N}=5$

$Q_{1,2}$ : two half-hypers in $R$. $\mathrm{SO}(2)_F \curvearrowright Q_{1,2}$.

$$W = k^{pq}\Phi_p\Phi_q + t^p_{ab}\Phi_p Q^a_i Q^b_i \longrightarrow W = f_{abcd}Q^a_i Q^b_i Q^c_j Q^d_j$$

where

$$f_{abcd} = (k^{-1})_{pq} t^p_{ab} t^p_{cd}$$

$R$ pseudo-real $\longrightarrow$ $t^p_{ab} = t^p_{ba}$. Suppose furthermore $f_{a(bcd)} = 0$.

$$\longrightarrow W = f_{abcd}\epsilon^{ij}\epsilon^{kl}Q^a_i Q^b_j Q^c_k Q^d_l.$$

$$\longrightarrow \mathrm{SU}(2)_F \curvearrowright Q_{1,2}.$$

Enhancement to $\mathrm{USp}(4)_R \curvearrowright (Q_{1,2}, Q^\dagger_{1,2})$ [Hosomichi-Lee[3]-Park]

# $\mathcal{N} = 6$

$A_{1,2}$ in $R$, $B^{1,2}$ in $R^*$, $\mathbf{SU(2)}_F \curvearrowright A_{1,2}, B^{1,2}$.

$$W = k^{pq}\Phi_p\Phi_q + t^p_{a\bar{b}}\Phi_p A^a_i B^{\bar{b}i} \longrightarrow W = f_{abcd} A^a_i B^{\bar{b}i} A^c_j B^{\bar{d}j}$$

where

$$f_{abcd} = (k^{-1})_{pq} t^p_{a\bar{b}} t^p_{c\bar{d}}$$

Suppose furthermore $f_{a\bar{b}c\bar{d}} = -f_{c\bar{b}a\bar{d}}$.

$$\longrightarrow W = f_{abcd}\epsilon^{ik}\epsilon_{jl} A^a_i B^{\bar{b}j} A^c_k B^{\bar{d}l}.$$

$$\longrightarrow \mathbf{SU(2)}_A \curvearrowright A_{1,2}, \qquad \mathbf{SU(2)}_B \curvearrowright B^{1,2}.$$

Enhancement to $\mathbf{SU(4)}_R \curvearrowright (A_{1,2}, B^\dagger_{1,2})$ [ABJM]

# $\mathcal{N} = 8$

$A_{1,2}$ in $R$, $B^{1,2}$ in $R^* = R$, strictly real.

$$W = k^{pq}\Phi_p\Phi_q + t^p_{ab}\Phi_p A^a_i B^{bi} \longrightarrow W = f_{abcd}A^a_i B^{bi} A^c_j B^{dj}$$

where

$$f_{abcd} = (k^{-1})_{pq} t^p_{ab} t^p_{cd}$$

$R$ strictly real: $t^p_{ab} = -t^p_{ba}$. Suppose furthermore $f_{abcd} = f_{[abcd]}$.

$$\longrightarrow W = f_{abcd}\epsilon^{ijkl} C^a_i C^b_j C^c_k C^d_l.$$

$$\longrightarrow \mathbf{SU(4)}_F \curvearrowright C_{1,2,3,4} = (A_{1,2}, B^{1,2})$$

Enhancement to $\mathbf{SO(8)}_R \curvearrowright (C_{1,2,3,4}, {C_{1,2,3,4}}^\dagger)$.

# Fundamental Identities

## $\mathcal{N} = 8$ [Gustavsson]

$$f^{efg}{}_b f^{cba}{}_d + f^{fea}{}_b f^{cbg}{}_d + f^{gaf}{}_b f^{ceb}{}_d + f^{age}{}_b f^{cfb}{}_d = 0$$

equivalent to

$$f_{abcd} = (k^{-1})_{pq} t^p_{ab} t^q_{cd}, \qquad f_{abcd} = -f_{cbad}.$$

## $\mathcal{N} = 6$ [Bagger-Lambert]

$$f^{efg}{}_b f^{cba}{}_d + f^{fea}{}_b f^{cbg}{}_d + f^{*gaf}{}_b f^{ceb}{}_d + f^{*age}{}_b f^{cfb}{}_d = 0$$

equivalent to

$$f_{a\bar{b}c\bar{d}} = (k^{-1})_{pq} t^p_{a\bar{b}} t^q_{c\bar{d}}, \qquad f_{a\bar{b}c\bar{d}} = -f_{c\bar{b}a\bar{d}}.$$

N.B. our $f_{a\bar{b}c\bar{d}}$ = their $f_{ac\bar{d}\bar{b}}$

# Summary

$$f_{abcd} = (k^{-1})_{pq} t^p_{ab} t^p_{cd}$$

- $\mathcal{N} = 4$ if half-hyper in $\boldsymbol{R}$ pseudo-real, and
$$f_{a(bcd)} = 0$$
- $\mathcal{N} = 5$ if two half-hypers in $\boldsymbol{R}$ pseudo-real and
$$f_{a(bcd)} = 0$$
- $\mathcal{N} = 6$ if two hypers in $\boldsymbol{R} \oplus \boldsymbol{R}^*$ and
$$f_{a\bar{b}c\bar{d}} = -f_{c\bar{b}a\bar{d}}$$
- $\mathcal{N} = 8$ if two hypers in $\boldsymbol{R} \oplus \boldsymbol{R}^*$, $\boldsymbol{R}$ strictly real, and
$$f_{abcd} = -f_{cbad}$$

# Contents

# $\mathcal{N} = 4, 5$

The conditions

$$f_{abcd} = (k^{-1})_{pq} t^p_{ab} t^p_{cd}, \qquad t^p_{ab} = t^p_{ba}$$

and

$$f_{a(bcd)} = 0$$

are equivalent to the Jacobi identity of a superalgebra [Gaiotto-Witten]

$$[B^p, B^q] = f^{pq}{}_r B^r,$$
$$[B^p, F_a] = t^p_{ab} J^{bc} F_c,$$
$$\{F_a, F_b\} = (k^{-1})_{pq} t^p_{ab} B^q.$$

$t^p_{ab}$ : pseudoreal

$J^{bc}$ : the fundamental antisymmetric tensor.

N.B. Info on $k$ also encoded in the superalgebra !

# Lie superalgebra

Classification done by [Kac,Scheunert-Nahm-Rittenberg].
Assume the bosonic part = semisimple + **U(1)**s

| | boson | fermion |
|---|---|---|
| $\mathbf{PSU}(N\|N)$ | $\mathbf{SU}(N)\times\mathbf{SU}(N)$ | $\mathbf{N}\times\bar{\mathbf{N}}\oplus\bar{\mathbf{N}}\times\mathbf{N}$ |
| $\mathbf{SU}(N\|M)$ | $\mathbf{SU}(N)\times\mathbf{SU}(M)\times\mathbf{U}(1)$ | $\mathbf{N}\times\bar{\mathbf{M}}\oplus\bar{\mathbf{M}}\times\mathbf{N}$ |
| $\mathbf{OSp}(N\|2M)$ | $\mathbf{SO}(N)\times\mathbf{USp}(2M)$ | $\mathbf{N}\times\mathbf{2M}$ |
| $D(2,1;\alpha)$ | $\mathbf{SO}(4)\times\mathbf{USp}(2)$ | $\mathbf{2}\times\mathbf{2}'\times\mathbf{2}$ |
| $G(3)$ | $G_2\times\mathbf{USp}(2)$ | $\mathbf{7}\times\mathbf{2}$ |
| $F(4)$ | $\mathbf{SO}(7)\times\mathbf{USp}(2)$ | $\mathbf{8}\times\mathbf{2}$ |
| $P(N)$ | $\mathbf{SU}(N+1)$ | **sym.** $\oplus$ **antisym.** |
| $Q(N)$ | $\mathbf{SU}(N+1)$ | **adj.** |

$P(N)$, $Q(N)$ are really weird objects, e.g. no invariant supertrace.
Their fermionic parts are not pseudoreal, etc.

# $\mathcal{N} = 6, 8$

$\mathcal{N} = 6$ is when the pseudoreal representation decomposes as $R \oplus R^*$

| gauge group | matter |
|---|---|
| $\mathrm{SU}(N)\times\mathrm{SU}(N)$ | $\mathbf{N} \times \bar{\mathbf{N}} \oplus \bar{\mathbf{N}} \times \mathbf{N}$ |
| $\mathrm{SU}(N)\times\mathrm{SU}(M) \times \mathrm{U}(1)$ | $\mathbf{N} \times \bar{\mathbf{M}} \oplus \bar{\mathbf{M}} \times \mathbf{N}$ |
| $\mathrm{SO}(2)\times\mathrm{USp}(2M)$ | $\mathbf{2M}_{+1} \oplus \mathbf{2M}_{-1}$ |

N.B. Vector representation $\mathbf{N}$ of $\mathrm{SO}(N)$ is reducible only for $N = 2$ !

$\mathcal{N} = 8$ is when the pseudoreal representation decomposes as $R \oplus R^*$, and furthermore $R$ is strictly real.

| gauge group | matter |
|---|---|
| $\mathrm{SU}(2)\times\mathrm{SU}(2)$ | $\mathbf{2} \times \mathbf{2} \oplus \mathbf{2} \times \mathbf{2}$ |

Done. [Hosomichi-Lee-Lee-Lee-Park]

# $\mathcal{N} = 6, 8$ contd.

$\mathcal{N} = 6$ fund. identity

$$f_{a\bar{b}c\bar{d}} = -f_{c\bar{b}a\bar{d}}$$

is a special case of $\mathcal{N} = 4, 5$ fund. identity

$$f_{A(BCD)} = 0 \qquad \text{where} \qquad t^p_{AB} = \begin{pmatrix} 0 & t^p_{a\bar{b}} \\ -t^p_{\bar{b}a} & 0 \end{pmatrix}$$

as discussed in [Hosomichi-Lee$^3$-Park].

Martin and I were rather stupid,
and didn't understand this until we read their paper...

Instead we had (almost) classified directly representations which satisfy the $\mathcal{N} = 6$ fund. identity when their paper came out.

# $\mathcal{N} = 6, 8$ contd.

We later found our method was quite similar to
what Kac and Nahm did 30 years ago.

Yet we submitted our paper on arXiv, with a guilty conscience ...

I explain below how the uniqueness of $\mathcal{N} = \mathbf{8}$ can be proved,
without relying on the magic table provided by Nahm & Kac.

$\mathcal{N} = \mathbf{6}$ is a bit more complicated, but of similar flavor.

# $\mathcal{N} = 8$

$$f(a,b,c,d) = \sum_{l=1}^{L} \frac{2\pi}{k_l} g^{(l)}_{pq} \langle\, b | T^p_{(l)} | a \,\rangle \langle\, d | T^p_{(l)} | c \,\rangle$$

$$= \sum_{l=1}^{L} \frac{2\pi}{k_l} \Big[ \sum_i \langle\, b | H^i_{(l)} | a \,\rangle \langle\, d | H^i_{(l)} | c \,\rangle +$$

$$\sum_{\alpha\in\Delta} \frac{|\alpha|^2}{2} \langle\, b | E^{\alpha}_{(l)} | a \,\rangle \langle\, d | E^{-\alpha}_{(l)} | c \,\rangle \Big].$$

We require

$$f(a,b,c,d) = -f(c,b,a,d)$$

when $a$, $b$, $c$, $d$ are all real.

# $\mathcal{N} = 8$

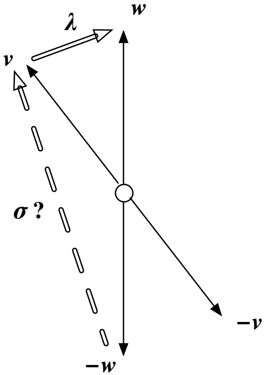
Now, take $a = |w\rangle + |w\rangle^*$, $b = |v\rangle + |v\rangle^*$
where $w$: highest weight state, $|v\rangle = T^{-\lambda}|w\rangle$.

$$0 = -f(a,a,b,b) = f(a,b,a,b)$$
$$= \sum_{\ell=1}^{L} k_\ell \sum_{\rho\in\Delta_\ell} \left|(\langle w| + \langle w|^*)T_\ell^\rho(|v\rangle + |v\rangle^*)\right|^2.$$

Contradicts unless there's a root $\sigma$ s.t.

$$|v\rangle = T^\sigma |w\rangle^*.$$

Only two steps $T^{-\lambda}T^{-\sigma}$
from the highest weight $|w\rangle$ to the lowest $|-w\rangle$.



# $\mathcal{N} = 8$

Only two steps $T^{-\lambda}T^{-\sigma}$ from the h.w. $|w\rangle$ to the l.w. $|-w\rangle$.

This is a rather strong condition. $w = (\lambda + \sigma)/2$.
Implies the length squared (LS) of $w$ is $\leq 1$.
Such representations are rare.

| | | |
|---|---|---|
| 1. | fundamental of $\mathbf{SU}(N)$, | LS=1-1/N. |
| 2. | vector of $\mathbf{SO}(N)$, | LS=1. |
| 3. | spinor of $\mathbf{SO(7)}$, $\mathbf{SO(8)}$, $\mathbf{SO(9)}$, | LS=3/4, 1, 1. |
| 4. | vector of $\mathbf{USp}(2N)$, | LS=1/2. |
| 5. | antisymmetric traceless of $\mathbf{USp}(2N)$, | LS=1 |
| 6. | $\mathbf{26}$ of $F_4$. | LS=1. |

and combination thereof.
Only $\mathbf{SU(2)} \times \mathbf{SU(2)}$ with bifundamental is allowed.

# Contents

# Summary

## Reviewed

- SUSY enhancement à la ABJM from $\mathcal{N} = \mathbf{3}$ to $\mathcal{N} = \mathbf{4, 5, 6, 8}$
- Classification
  - via Lie superalgebra
  - direct analysis ...

## To be done

- Dynamics of newly found $\mathcal{N} = \mathbf{5, 6}$ theories ?
- $\mathcal{N} = \mathbf{4, 5}$ theory in terms of 3-algebras ?
- Non-ABJM-type enhancement ? unlikely, but not ruled out ...