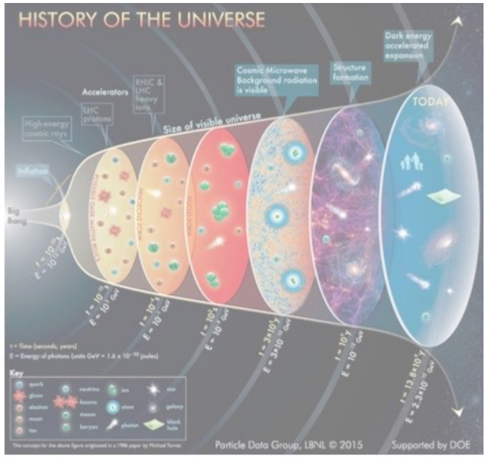

Figure credit;
Particle Data Group
at Lawrence Berkeley National Lab.

素粒子特論

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@お茶の水女子大学, 2023年夏学期

前回までのあらすじ

・[第一部] スライドで全体的な話

・[第二部] 主に黒板を使って各論

0. 標準模型

A. 大統一理論 ← 今日ココから

一番難しいかも.
たくさん質問して下さい

A 大統一理論 Grand Unified Theory (GUT)

予備知識

- non-Abelian ✓
- 群論 少し ✓
- Standard Model ✓

A-1 Standard Model

A-2 GUT

A-3 SU(5) rep.

A-4 coupling unification

A-5 comments

A-1 Standard Model

$$\mathcal{L} = \sum_{\text{matters}} \bar{\psi} i\gamma^\mu \underline{D_\mu} \psi - \frac{1}{4} \sum_{\substack{\text{gauge}\\\text{bosons}}} F^{(a)}_{\mu\nu} F^{(a)\mu\nu} + \mathcal{L}_{\substack{\text{Higgs}\\\text{Yukawa}}}$$

$$\left(\partial_\mu - igA^{(a)}_\mu \underline{T^{(a)}} \right) \psi$$

generators (matrices)

▶ gauge bosons

$SU(3)$	$G_\mu^{1\sim 8}$	gluons
$SU(2)$	$W_\mu^{1\sim 3}$	
$U(1)$	B_μ	

$$\left. \begin{matrix} W_\mu^{1\sim 3} \\ B_\mu \end{matrix} \right] \xrightarrow[\text{mechanism}]{\text{Higgs}} \left[\begin{matrix} W_\mu^{\pm} \quad Z_\mu \\ A_\mu \leftarrow \text{photon} \end{matrix} \right.$$

matters

$\circledast$

$$\text{quarks}\begin{cases} Q_i & (\overset{SU(3)}{3}\,,\ \overset{SU(2)}{2})_{\overset{U(1)}{1/6}} \\ u_i^c & (\bar{3}\,,\ 1)_{-2/3} \\ d_i^c & (\bar{3}\,,\ 1)_{1/3} \end{cases}$$

$$\text{leptons}\begin{cases} L_i & (1\,,\ 2)_{-1/2} \\ e_i^c & (1\,,\ 1)_{1} \end{cases}$$

黒板に残す

$$\overbrace{\begin{pmatrix} u_L^{(1)} & u_L^{(2)} & u_L^{(3)} \\ d_L & d_L & d_L \end{pmatrix}}^{SU(3)} \Big\}_{SU(2)}$$

$$\left(u_R^{c\,(1)} \quad u_R^{c\,(2)} \quad u_R^{c\,(3)}\right)$$

$$\left(d_R^c \quad * \quad *\right)$$

$$\begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}$$

$$\left(e_R^c\right)$$

注
全て left-handed
に揃えてある

$i = 1 \sim 3$
generations
families

例 ①

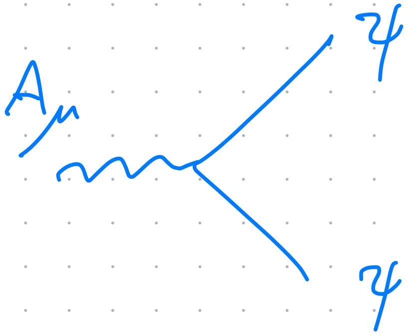
U(1)

$$\psi \xrightarrow{\text{ゲージ変換}} e^{i\overset{\text{変換パラメータ}}{\alpha(x)}}\psi$$

$$\mathcal{L} \supset i\bar{\psi}\gamma^\mu D_\mu \psi$$

$$= i\bar{\psi}\gamma^\mu(\partial_\mu - ieA_\mu\psi)$$

$$\supset e\bar{\psi}\gamma^\mu\psi A_\mu$$



例 ②

SU(2)

$$L_1 = \begin{pmatrix}\nu_e \\ e\end{pmatrix} \xrightarrow[\text{変換}]{SU(2)\ \text{ゲージ}} \underbrace{\exp\left[-i\sum_{a=1}^{3}\overset{\text{変換パラメータ}}{\lambda^a(x)}\left(\frac{1}{2}\underset{\text{Pauli}}{\sigma^a}\right)\right]}_{2\times2\ \text{matrix}}\begin{pmatrix}\nu_e \\ e\end{pmatrix}$$

$$\mathcal{L} \supset i \bar{L}_1 \gamma^\mu D_\mu L_1$$

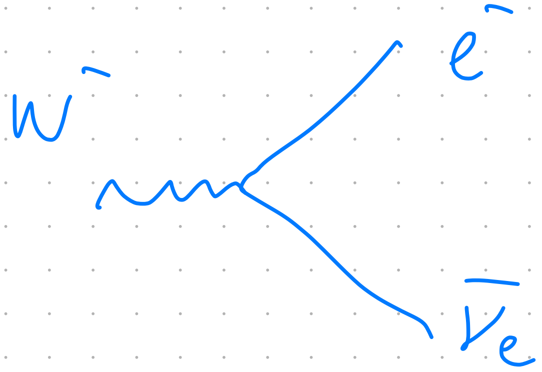
$$= i(\bar{\nu}_e \ \ \bar{e})\gamma^\mu\left[\partial_\mu - i g_2 \sum_{a=1}^{3} W_\mu^a \left(\frac{1}{2}\sigma^a\right)\right]\begin{pmatrix}\nu_e\\ e\end{pmatrix}$$

$$\sigma^1 = \begin{pmatrix} & 1\\ 1 & \end{pmatrix}$$

$$\sigma^2 = \begin{pmatrix} & -i\\ i & \end{pmatrix}$$

$$\supset \frac{1}{\sqrt{2}} g_2 \left(W_\mu^- \bar{e}\gamma^\mu \nu_e + W_\mu^+ \bar{\nu}_e \gamma^\mu e\right)$$

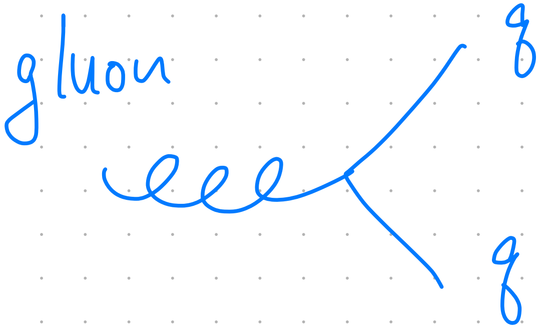
$$\left(W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right)\right)$$



例 ③ SU(3)

$$\mathcal{L} \supset i\bar{q}\gamma^\mu D_\mu q$$

$$\supset i\left(\bar{q}_G\ \bar{q}_B\ \bar{q}_R\right)\gamma^\mu\left[\partial_\mu - i g_3 \sum_{a=1}^{8} G_\mu^a \underset{\substack{\uparrow \\ 3\times 3 \\ \text{Gell-Mann Matrices}}}{T^a}\right]\begin{pmatrix} q_G \\ q_B \\ q_R \end{pmatrix}$$



3つのゲージ群

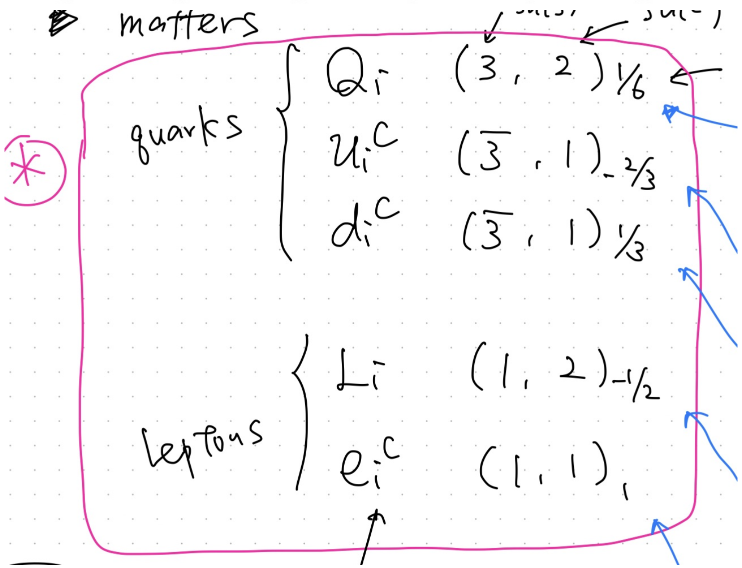
クォーク、レプトンの量子数がバラバラ

→ 統一できないか？

A-2　GUT

条件
① $SU(3) \times SU(2) \times U(1)$ を部分群にもつ
② コレを表現に含む

this ⊛



- matters

quarks	Q_i	$(3, 2)_{1/6}$
	u_i^c	$(\bar{3}, 1)_{-2/3}$
	d_i^c	$(\bar{3}, 1)_{1/3}$
leptons	L_i	$(1, 2)_{-1/2}$
	e_i^c	$(1, 1)_1$

▶ ①より

rank $\geqq 4$ （少なくとも4つの可換な生成子

$$\underbrace{T_3 \quad T_8}_{SU_3} \quad \underset{SU_2}{\sigma_3} \quad \underset{U_1}{Y}$$

を含む）

simplest :

rank $= 4$ → (コンパクトな単純Lie群では) 5つしかない

$SU(5)$

$SO(8)$

$SO(9)$

$Sp(8)$

F_4

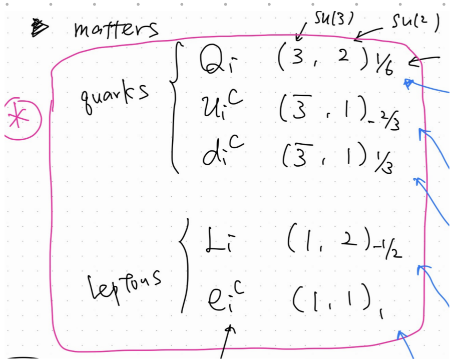
複素表現を持たない
→ ②を満たせない。

SU(5) を考えよう！

A-3 SU(5) の表現

fundamental rep of SU(5) = 5 rep から始めよう

▶ ㊟ より 足して5になる組み合わせは



▶ matters

SU(3) SU(2)

quarks	Q_i	$(3, 2)_{1/6}$
	u_i^c	$(\bar{3}, 1)_{-2/3}$
	d_i^c	$(\bar{3}, 1)_{1/3}$
leptons	L_i	$(1, 2)_{-1/2}$
	e_i^c	$(1, 1)_1$

$u^c + L$ or $d^c + L$ のみ

$(3 + 2)$ $(3 + 2)$

▶ $u^c + L$

~~$(\bar{3}, 1)_{2/3} \oplus (1, 2)_{-1/2}$~~

~~$Y\begin{pmatrix} u^c \\ L \end{pmatrix} = \begin{pmatrix} -2/3 & & & & \\ & -2/3 & & & \\ & & -2/3 & & \\ & & & -1/2 & \\ & & & & -1/2 \end{pmatrix}\begin{pmatrix} u^c \\ L \end{pmatrix}$~~

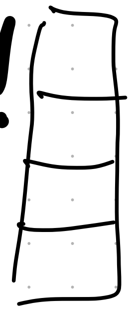
U(1) charge →

tracelers ではない

SU(5)表現に含まれない

▶ $d^c + L$

$(\bar{3},1)_{1/3} \oplus (1,2)_{-1/2} = \bar{5}$ rep in $SU(5)$!!



$$\begin{matrix}3\{\\2\{\end{matrix}\begin{pmatrix}d^c\\L\end{pmatrix}\xrightarrow[\text{trf}]{\text{gauge}}\exp\left[-i\sum_{a=1}^{24}\lambda^a(x)\underbrace{\left(T^a_{SU(5)}\right)^*}\right]\begin{pmatrix}d^c\\L\end{pmatrix}$$

$$\overset{SU(3)}{\left(\begin{array}{c|c}T^{1\sim8}_{SU(3)}&\\\hline&0\end{array}\right)}+\overset{SU(2)}{\left(\begin{array}{c|c}0&\\\hline&\frac{1}{2}\sigma^{1\sim3}_{SU(2)}\end{array}\right)}$$

$$+\boxed{\sqrt{\frac{3}{5}}}\overset{U(1)}{\left(\begin{array}{ccc|cc}1/3&&&&\\&1/3&&&\\&&1/3&&\\\hline&&&-1/2&\\&&&&-1/2\end{array}\right)}+\overset{\text{remaining }12}{\left(\begin{array}{c|c}&▨\\\hline ▨&\end{array}\right)}$$

normalization $\boxed{\mathrm{tr}\left(T^a T^b\right) = \frac{1}{2}\delta^{ab}}$

▶ 残りの matter も、、、

$$\boxed{\underset{Q}{(3,2)_{1/6}} \oplus \underset{u^c}{(\bar{3},1)_{2/3}} \oplus \underset{e^c}{(1,1)_1}} = 10 \text{ rep. of } su(5)$$

!!

▶ つまり

$$\boxed{\begin{aligned} \circledast &= Q + u^c + d^c + L + e^c \\ &= \bar{5} + 10 \qquad \text{in } su(5) \end{aligned}}$$

量子数がピッタリ。

ではゲージボソンは？

$$\begin{array}{c}\text{adjoint} \\ 24 \\ \text{in } SU(5)\end{array} = \underset{G_\mu^{1\sim 8}}{(8,1)} \oplus \underset{W_\mu^{1\sim 3}}{(1,3)} \oplus \underset{B_\mu}{(1,1)_0}$$

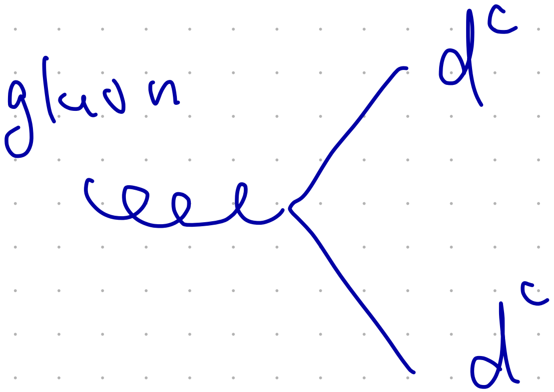
$$\oplus \underbrace{(3,2)_{-5/6} \oplus (\bar{3},2)_{5/6}}_{X,\ Y \text{ bosons}}$$

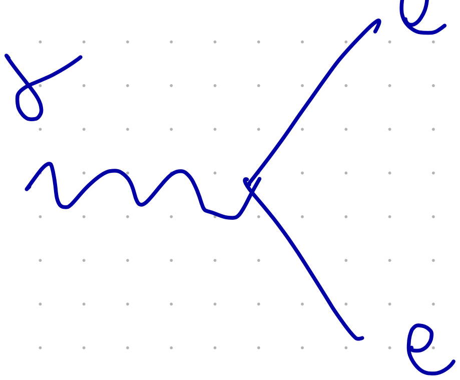
→ SU(5)の破れに伴って重くなる

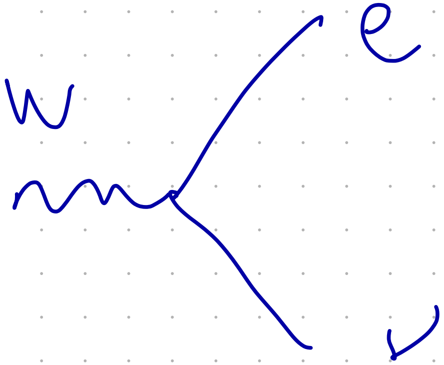
例えば

$$D_\mu \begin{pmatrix} d^c \\ L \end{pmatrix} = \left[\partial_\mu - i g_5 A_\mu^a T^a_{SU(5)}\right] \begin{pmatrix} d^c \\ L \end{pmatrix}$$

この中に

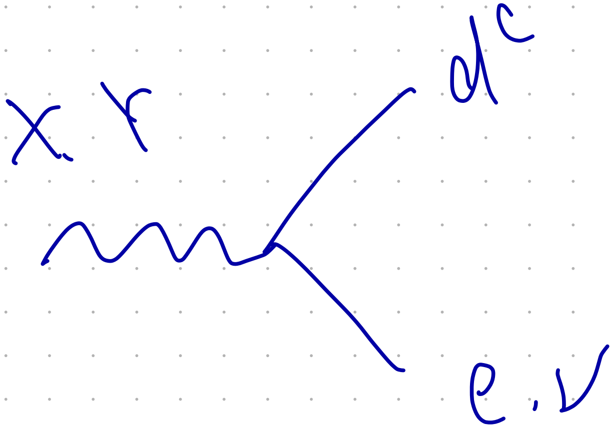






など全部含まれる

(さらに



クォーク ⟷ レプトン

⟶ 陽子崩壊)

5/16(火)　(4日目)

あらすじ

- スライド全体像（1日目⊕2日目）

§0　SM復習（2日目）

§A　GUT　（3日目〜）

A-1　SM復習

A-2　GUT

A-3　SU(5)の表現　↑

A-4　↓今日ココから

A-4 gauge coupling unification

$$\left(F_{\mu\nu}F^{\mu\nu} \longrightarrow \frac{1}{g^2}F_{\mu\nu}F^{\mu\nu} \right.$$

$$\left. \text{by } A_\mu \longrightarrow \frac{1}{g}A_\mu \right)$$

と再定義

$$\mathcal{L} \supset \frac{1}{g_{GUT}^2}\sum_{a=1}^{24} F^a_{\mu\nu}F^{a\mu\nu}$$

$$= \frac{1}{g_3^2}G^a_{\mu\nu}G^{a\mu\nu} + \frac{1}{g_2^2}W^a_{\mu\nu}W^{a\mu\nu} + \frac{1}{g_1^2}B_{\mu\nu}B^{\mu\nu}$$

(+ X Y bosons)

$$\boxed{\Rightarrow \quad g_1 = g_2 = g_3 = g_{GUT} \quad @ \text{ unification scale } (= \text{GUT が破れたスケール})}$$

となるはず

▶ running gauge coupling

場の量子論で計算

R.G. eq
繰り込み群

$$\frac{d}{d\ln\mu}\alpha_i^{-1}(\mu) = -\frac{1}{2\pi} b_i \quad (@ \text{1-loop})$$

$$\alpha_i = \frac{1}{4\pi} g_i^2 \quad (i = 1 \sim 3)$$

SUSY

	INPUT $\alpha_i(\mu = m_Z)$	b_i^{SM}	b_i^{MSSM}
SU(3)	$\simeq 0.1129$	-7	-3
SU(2)	$\frac{\alpha}{\sin^2\theta_w}$	$-19/6$	$+1$
U(1)	$\left(\frac{5}{3}\right)\alpha_Y = \frac{5}{3}\frac{\alpha}{\cos^2\theta_w}$	$+41/10$	$+33/5$

A-3 参照

$\alpha(m_Z) \simeq \frac{1}{128}$

$\sin^2\theta_w \simeq 0.231$

1. Physical Constants

Table 1.1: Revised 2021 by D. Robinson (LBNL). Reviewed by P. Mohr (NIST). Mainly from "CODATA Recommended Values of the Fundamental Physical Constants: 2018," E. Tiesinga, D.B. Newell, P.J. Mohr, and B.N. Taylor, NIST SP961 (May 2019) [1]. The electron charge magnitude e, and the Planck, Boltzmann, and Avogadro constants h, k, and N_A, now join c as having defined values; the free-space permittivity and permeability constants ϵ_0 and μ_0 are no longer exact. These changes affect practically everything else in the Table. Figures in parentheses after the values are the 1-standard-deviation uncertainties in the last digits; the fractional uncertainties in parts per 10^9 (ppb) are in the last column. The full 2018 CODATA Committee on Data for Science and Technology set of constants are found at `https://physics.nist.gov/constants`. The last set of constants (beginning with the Fermi coupling constant) comes from the Particle Data Group. See also "The International System of Units (SI)," 9th ed. (2019) of the International Bureau of Weights and Measures (BIPM), `https://www.bipm.org/utils/common/pdf/si-brochure/SI-Brochure-9-EN.pdf`.

Quantity	Symbol, equation	Value	Uncertainty (ppb)
speed of light in vacuum	c	$299\ 792\ 458$ m s^{-1}	exact
Planck constant	h	$6.626\ 070\ 15\times10^{-34}$ J s (or J/Hz) [§]	exact
Planck constant, reduced	$\hbar \equiv h/2\pi$	$1.054\ 571\ 817\ldots \times 10^{-34}$ J s	exact*
		$= 6.582\ 119\ 569\ldots \times 10^{-22}$ MeV s	exact*
electron charge magnitude	e	$1.602\ 176\ 634\times10^{-19}$ C	exact
conversion constant	$\hbar c$	$197.326\ 980\ 4\ldots$ MeV fm	exact*
conversion constant	$(\hbar c)^2$	$0.389\ 379\ 372\ 1\ldots$ GeV^2 mbarn	exact*
electron mass	m_e	$0.510\ 998\ 950\ 00(15)$ MeV/c^2 $= 9.109\ 383\ 7015(28)\times10^{-31}$ kg	0.30
proton mass	m_p	$938.272\ 088\ 16(29)$ MeV/c^2 $= 1.672\ 621\ 923\ 69(51)\times10^{-27}$ kg	0.31
		$= 1.007\ 276\ 466\ 621(53)$ u $= 1836.152\ 673\ 43(11)\ m_e$	0.053, 0.060
neutron mass	m_n	$939.565\ 420\ 52(54)$ MeV/c^2 $= 1.008\ 664\ 915\ 95(49)$ u	0.57, 0.48
deuteron mass	m_d	$1875.612\ 942\ 57(57)$ MeV/c^2	0.30
unified atomic mass unit[**]	u = (mass ^{12}C atom)/12	$931.494\ 102\ 42(28)$ MeV/c^2 $= 1.660\ 539\ 066\ 60(50)\times10^{-27}$ kg	0.30
permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854\ 187\ 8128(13)\ \times10^{-12}$ F m^{-1}	0.15
permeability of free space	$\mu_0/(4\pi \times 10^{-7})$	$1.000\ 000\ 000\ 55(15)$ N A^{-2}	0.15
fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	$7.297\ 352\ 5693(11)\times10^{-3} = 1/137.035\ 999\ 084(21)$[†] [‡‡]	0.15
classical electron radius	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	$2.817\ 940\ 3262(13)\times10^{-15}$ m	0.45
(e^- Compton wavelength)/2π	$\lambda\!\!\!^{-}_e = \hbar/m_e c = r_e\alpha^{-1}$	$3.861\ 592\ 6796(12)\times10^{-13}$ m	0.30
Bohr radius ($m_{\text{nucleus}} = \infty$)	$a_\infty = 4\pi\epsilon_0\hbar^2/m_e e^2 = r_e\alpha^{-2}$	$0.529\ 177\ 210\ 903(80)\times10^{-10}$ m	0.15
wavelength of 1 eV/c particle	$hc/(1\ \text{eV})$	$1.239\ 841\ 984\ldots \times 10^{-6}$ m	exact*
Rydberg energy	$hcR_\infty = m_e e^4/2(4\pi\epsilon_0)^2\hbar^2 = m_e c^2\alpha^2/2$	$13.605\ 693\ 122\ 994(26)$ eV	1.9×10^{-3}
Thomson cross section	$\sigma_T = 8\pi r_e^2/3$	$0.665\ 245\ 873\ 21(60)$ barn	0.91
Bohr magneton	$\mu_B = e\hbar/2m_e$	$5.788\ 381\ 8060(17)\times10^{-11}$ MeV T^{-1}	0.30
nuclear magneton	$\mu_N = e\hbar/2m_p$	$3.152\ 451\ 258\ 44(96)\times10^{-14}$ MeV T^{-1}	0.31
electron cyclotron freq./field	$\omega^e_{\text{cycl}}/B = e/m_e$	$1.758\ 820\ 010\ 76(53)\times10^{11}$ rad s^{-1} T^{-1}	0.30
proton cyclotron freq./field	$\omega^p_{\text{cycl}}/B = e/m_p$	$9.578\ 833\ 1560(29)\times10^{7}$ rad s^{-1} T^{-1}	0.31
gravitational constant[‡]	G_N	$6.674\ 30(15)\times10^{-11}$ m^3 kg^{-1} s^{-2}	2.2×10^4
		$= 6.708\ 83(15)\times10^{-39}\ \hbar c\ (\text{GeV}/c^2)^{-2}$	2.2×10^4
standard gravitational accel.	g_N	$9.806\ 65$ m s^{-2}	exact
Avogadro constant	N_A	$6.022\ 140\ 76\times10^{23}$ mol^{-1}	exact
Boltzmann constant	k	$1.380\ 649\times10^{-23}$ J K^{-1}	exact
		$= 8.617\ 333\ 262\ldots \times 10^{-5}$ eV K^{-1}	exact*
molar volume, ideal gas at STP	$N_A k$ (273.15 K)/(101 325 Pa)	$22.413\ 969\ 54\ldots \times 10^{-3}$ m^3 mol^{-1}	exact*
Wien displacement law constant	$b = \lambda_{\text{max}}T$	$2.897\ 771\ 955\ldots \times 10^{-3}$ m K	exact*
Stefan-Boltzmann constant	$\sigma = \pi^2 k^4/60\hbar^3 c^2$	$5.670\ 374\ 419\ldots \times 10^{-8}$ W m^{-2} K^{-4}	exact*
Fermi coupling constant[‡‡]	$G_F/(\hbar c)^3$	$1.166\ 378\ 8(6)\times10^{-5}$ GeV^{-2}	510
weak-mixing angle	$\sin^2\widehat{\theta}(M_Z)\ (\overline{\text{MS}})$	$0.231\ 21(4)$[††]	1.7×10^5
$W^\pm$ boson mass	m_W	$80.377(12)$ GeV/c^2[¶]	1.5×10^5
Z^0 boson mass	m_Z	$91.1876(21)$ GeV/c^2	2.3×10^4
strong coupling constant	$\alpha_s(m_Z)$	$0.1179(9)$	7.6×10^6

$\pi = 3.141\ 592\ 653\ 589\ 793\ 238\ \ldots$

$e = 2.718\ 281\ 828\ 459\ 045\ 235\ldots$

$\gamma = 0.577\ 215\ 664\ 901\ 532\ 860\ldots$

1 in $\equiv$ 0.0254 m

1 G $\equiv 10^{-4}$ T

1 eV $= 1.602\ 176\ 634 \times 10^{-19}$ J (exact)

kT at 300 K $= [38.681\ 727\ 0718\ldots]^{-1}$eV (exact*)

1 Å $\equiv$ 0.1 nm

1 dyne $\equiv 10^{-5}$ N

(1 kg)$c^2 = 5.609\ 588\ 603\ldots \times 10^{35}$eV(exact*)

0 °C $\equiv$ 273.15K

1 barn $\equiv 10^{-28}$ m^2

1 erg $\equiv 10^{-7}$ J

1 C $= 2.997\ 924\ 58 \times 10^9$ esu

1 atmosphere $\equiv$ 760 Torr $\equiv$ 101 325Pa

[§] CODATA recommends that the unit be J/Hz to stress that in $h = E/\nu$ the frequency ν is in cycles/sec (Hz), not radians/sec.

[*] These are calculated from exact values and are exact to the number of places given (*i.e.* no rounding).

[**] The molar mass of ^{12}C is 11.999 999 9958(36) g.

[†] At $Q^2 = 0$. At $Q^2 \approx m_W^2$ the value is $\sim 1/128$.

[‡] Absolute laboratory measurements of G_N have been made only on scales of about 1 cm to 1 m.

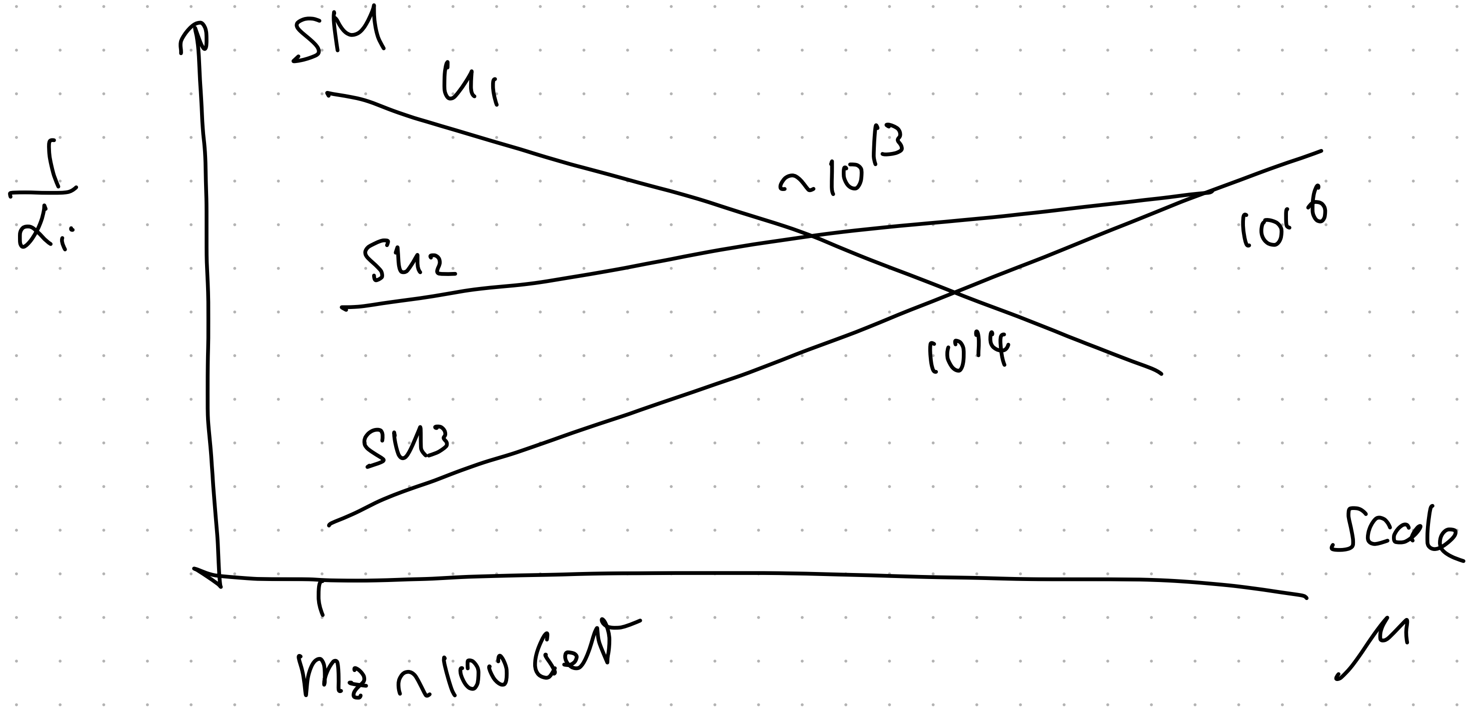
[‡‡] See the discussion in Ch. 10, "Electroweak model and constraints on new physics."

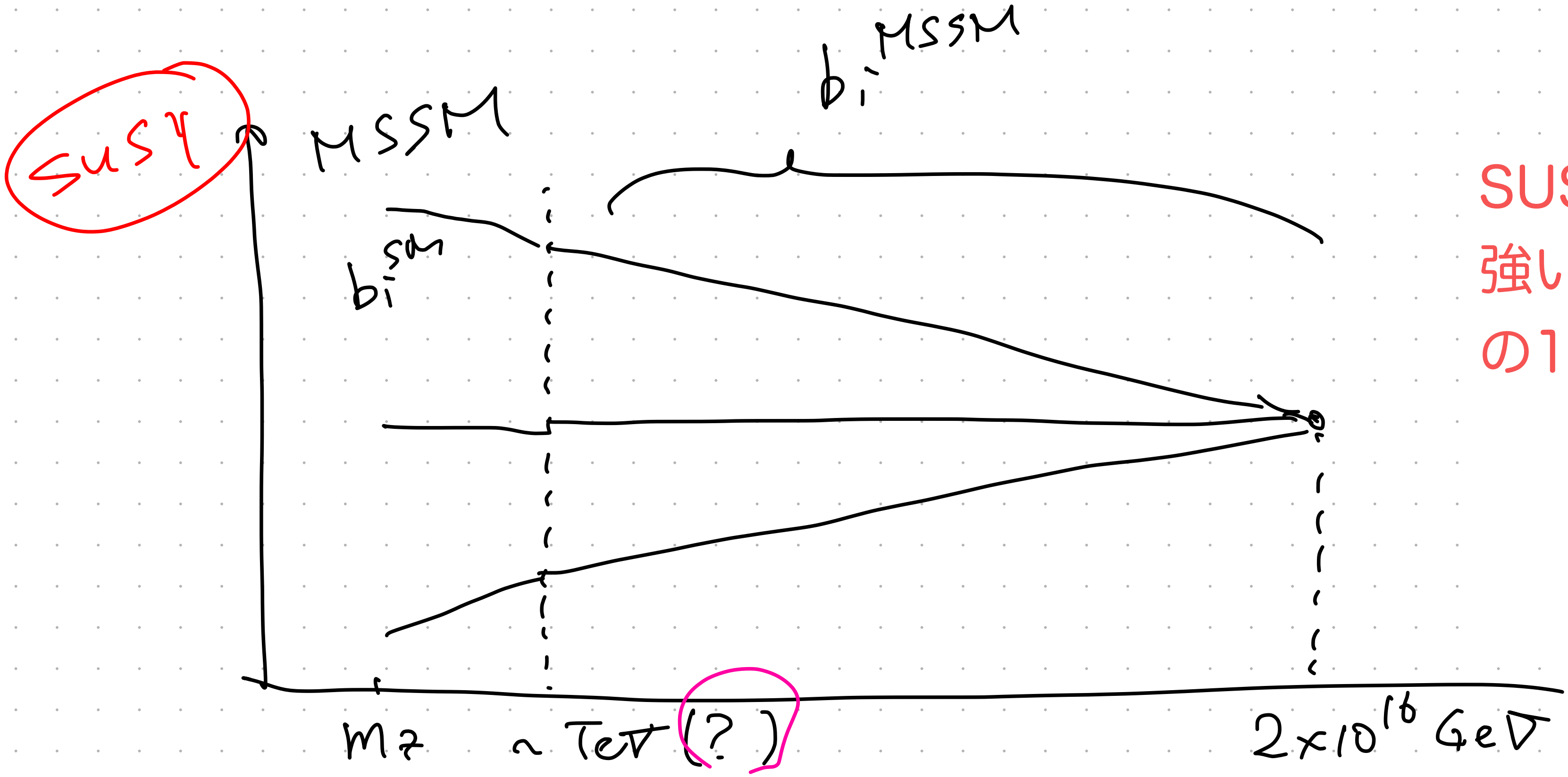
[††] The corresponding $\sin^2\theta$ for the effective angle is 0.23153(4).

[¶] See the "Mass and width of the W boson" review.

References

[1] E. Tiesinga *et al.*, Rev. Mod. Phys. **93**, 025010 (2021).





SUSY GUT の
強い motivation
の1つ。

※ Another check

If $\exists$ GUT @ $\forall \mu$ ← any scale

$$\frac{\alpha_2^{-1} - \alpha_1^{-1}}{\alpha_3^{-1} - \alpha_2^{-1}}(\mu) = \frac{b_1 - b_2}{b_2 - b_3} = \begin{cases} \text{MSSM} \quad \frac{7}{5} = 1.4 \\ \\ \text{SM} \quad \frac{218}{115} = 1.895\ldots \end{cases}$$

$$\frac{\alpha_2^{-1} - \alpha_1^{-1}}{\alpha_3^{-1} - \alpha_2^{-1}}(m_Z) \cong 1.4$$

$(1.39\ldots)$?

ref for SUSY GUT

textbook S. Raby Supersymmetric Grand Unified Theories (Springer)

arXiv hep-ph/9911272 Mohapatra

A-5 Other Comments

SU(5) singlet = SM singlet

▶ **SO(10)**

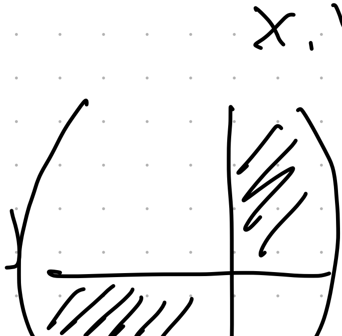
$$\underbrace{Q+u^c+d^c+L+e^c}_{\bar{5}+10 \text{ in } SU(5)} + \underbrace{N_R}_{} = 16$$

of SO(10)

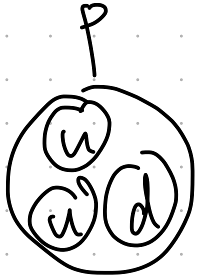
$N_R \rightarrow$ (seesaw §B
leptogenesis §E)

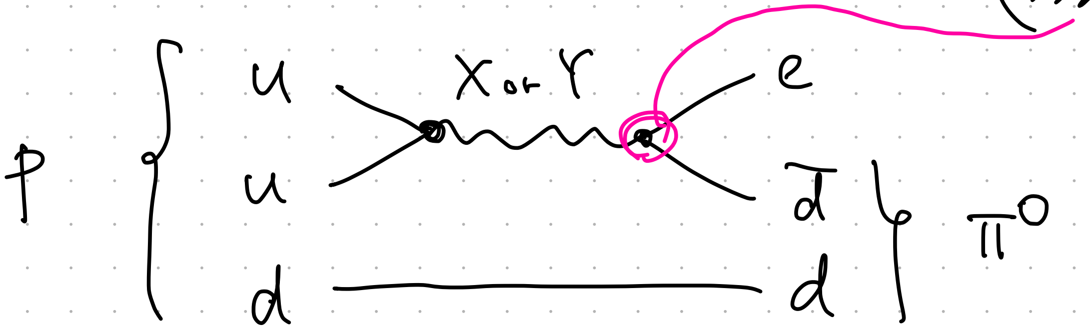
▶ **proton decay**

$$(\bar{d^c}\ \ L)\cancel{D}_\mu \begin{pmatrix} d^c \\ L \end{pmatrix} \supset (d^c\ \ L)$$



$$\begin{pmatrix} d^c \\ L \end{pmatrix}$$





$$\boxed{p \to e^+\pi^0}$$

$$\Gamma \propto \left(\frac{g^2}{m_X^2} \right)^2 m_p^5$$

▶ Doublet-Triplet Splitting problem

SM Higgs $\in$ 5 of SU(5)

2 of SU(2)

H

$$5_H = \begin{pmatrix} H_c \\ H \end{pmatrix}$$

colored Higgs

$\downarrow$ SM

$m_H \simeq 100$ GeV $\ll$ $m_{H_c} \gtrsim 10^{16}$ GeV

(proton decay)

2-A

$\longrightarrow$ fine-tuning problem