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# Tachyon Condensation in Cubic Open Bosonic String Field Theory

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**Abstract**

Progress on understanding tachyon condensation for the bosonic open string using Witten's cubic string field theory is reviewed. Numerical evidence is given for Sen's conjectures regarding the energy gap between the unstable and stable vacua, and for the decoupling of open string modes in the stable vacuum. The tachyon effective potential is studied in Feynman-Siegel gauge, and singularities appearing in this potential are discussed.

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# 1 Introduction

In this lecture I summarize results which have been achieved using Witten's cubic string field theory to study tachyon condensation in the bosonic open string. The basic philosophy of the work I review in this lecture is to take the cubic open string field theory, write everything explicitly in terms of the bosonic open string Fock space, and directly calculate as much as possible about the structure of Lorentz-invariant vacua for the theory. Most of the results described in this lecture appeared originally in the papers [1, 2, 3].

There are many aspects of the string field theory approach to tachyon condensation which I do not touch on in this lecture. The lectures of Zwiebach at this meeting describe many interesting features and formal properties of the cubic bosonic open string field theory. Berkovits describes an approach to open superstring field theory, and Harvey discusses constructions of solitons in noncommutative field theory which are relevant to the construction of lower-dimensional D-branes from tachyon condensation in string field theory.

I have attempted to make this writeup fairly self-contained, and to follow as closely as possible the material presented in the original Komaba lecture. I have added some additional comments in places where further relevant developments have occurred since the workshop. I have made no attempt to be complete with references to all relevant papers here, but have included pointers to most of the papers whose work is directly mentioned here. For further references and background, the reader should consult the original papers referenced here.

In Section 2 I review the Fock space description of level-truncated open bosonic string field theory, and review briefly Sen's conjectures regarding tachyon condensation in the open bosonic string. In Section 3 I describe the level truncation approach to string field theory. In Section 4 I discuss the effective potential for the tachyon. In Section 5 I discuss some interesting features related to the U(1) gauge field on the brane. In Section 6 I make some observations regarding physics in the stable vacuum, and Section 7 contains conclusions.

# 2 String Field Theory

## 2.1 Perturbative bosonic string theory

The bosonic string Fock space associated with states of a single string is generated by acting with mode operators $\alpha_n^\mu, b_n, c_n$ on the Fock space vacuum $|\hat{0}\rangle$, where the mode operators and the vacuum state satisfy the relations

$$
\begin{aligned}
\alpha_n^\mu|\hat{0}\rangle &= 0, \qquad n \geq 0 \\
b_n|\hat{0}\rangle &= 0, \qquad n \geq 0 \\
c_n|\hat{0}\rangle &= 0, \qquad n > 0 \\
[\alpha_n^\mu, \alpha_m^\nu] &= n\eta^{\mu\nu}\delta_{n+m} \\
\{b_n, c_m\} &= \delta_{n+m}
\end{aligned}
$$

We define the ghost number one vacuum

$$|\hat{0}\rangle = c_1|0\rangle.$$

in terms of the SL(2,R) invariant vacuum $|0\rangle$.

Space-time fields are associated with ghost number one states in the Fock space, beginning with

$$\begin{array}{rcl} |\hat{0}\rangle & \rightarrow & \phi \\ \alpha^{\mu}_{-1}|\hat{0}\rangle & \rightarrow & A_{\mu} \\ & \vdots & \end{array}$$

The field $\phi$ has $\frac{1}{2}M^2 = -1$, corresponding to a space-time tachyon. The existence of this tachyon indicates that the canonical vacuum for the theory is unstable. To study the true vacuum, it is necessary to use a nonperturbative formalism. String field theory is such a formalism. In the string field theory approach, a single string field

$$\Phi = \left(\phi + A_{\mu}\alpha^{\mu}_{-1} + \frac{1}{\sqrt{2}}B_{\mu\nu}\alpha^{\mu}_{-1}\alpha^{\nu}_{-1} + \beta b_{-1}c_{-1} + \cdots\right)|\hat{0}\rangle \tag{1}$$

encodes all physical space-time fields. Throughout most of this talk we will use Feynman-Siegel gauge, in which we restrict attention to states satisfying $b_0|S\rangle = 0$. All string field theory interactions are encoded in Witten's cubic action [4]

$$S = -\frac{1}{2}\int \Phi \star Q\Phi - \frac{g}{3}\int \Phi \star \Phi \star \Phi \tag{2}$$

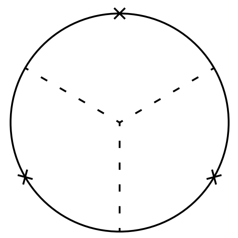
where $Q$ is the BRST operator and $\star$ is the string field theory star product. The quadratic terms

$$\int |\Phi\rangle \star Q|\Phi\rangle = \langle\Phi|c_0\left(p^2 + \frac{1}{2}M^2\right)|\Phi\rangle \tag{3}$$

lead to a quadratic action whose leading terms are

$$\begin{aligned} S_2 &= \frac{1}{2\alpha'}\int \Phi \star Q\Phi \\ &= \frac{1}{2}\int d^{26}x\ \Big[\left(-\phi^2 + B_{\mu\nu}B^{\mu\nu} - \beta^2 + \cdots\right) \\ &\qquad +\partial_{\mu}\phi\,\partial^{\mu}\phi + \partial_{\mu}A_{\nu}\,\partial^{\mu}A^{\nu} \\ &\qquad +\partial_{\mu}B_{\nu\lambda}\,\partial^{\mu}B^{\nu\lambda} - \partial_{\mu}\beta\,\partial^{\mu}\beta + \cdots\Big] \end{aligned}$$

The cubic Witten interaction vertex



can be described by CFT correlation functions

$$\langle (h_1 \circ \mathcal{O}_1(0))\,(h_2 \circ \mathcal{O}_2(0))\,(h_3 \circ \mathcal{O}_3(0))\rangle$$

where $h_i$ are conformal maps taking the origin to 3 equally spaced points on the boundary of the disk. Alternatively, in Fock space language the cubic interactions can be described by a vertex operator

$$\int \Phi \star \Phi \star \Phi = \langle V|(\Phi \otimes \Phi \otimes \Phi) \tag{4}$$

An explicit Fock space representation of this vertex operator was worked out in [5, 6, 7], and is given by

$$\begin{aligned} \langle V| &= \delta(p_{(1)} + p_{(2)} + p_{(3)})(\langle 0|c_0^{(1)} \otimes \langle 0|c_0^{(2)} \otimes \langle 0|c_0^{(3)}) \\ &\quad \times \exp\left(\frac{1}{2}\alpha_n^{(r)\mu} N_{nm}^{rs} \eta_{\mu\nu} \alpha_m^{(s)\nu} + c_n^{(r)} X_{nm}^{rs} b_m^{(s)}\right). \end{aligned}$$

where $N_{nm}^{rs}, X_{nm}^{rs}$ are calculable constants. Explicit tables of these constants are given in [1].

Because we are primarily interested here in the Lorentz-invariant vacuum structure of the theory, we can restrict attention to scalar fields. Because of the twist symmetry [8] of the SFT action, we can further restrict attention to scalar fields at even levels (counting the tachyon $\phi$ as level zero). We can write the even level scalar string field as

$$\Phi = \sum_{i=1}^{\infty} \psi^i |s_i\rangle \tag{5}$$

where $|s_i\rangle$ are all even level scalar states of ghost number one in the string Fock space.

For example, at levels 0, 2 we have

$$\begin{array}{ll} |s_1\rangle = |\hat{0}\rangle & \phi = \psi^1 \\ |s_2\rangle = \alpha_{-1} \cdot \alpha_{-1}|\hat{0}\rangle & B_\mu{}^\mu = 26\sqrt{2}\psi^2 \\ |s_3\rangle = b_{-1}c_{-1}|\hat{0}\rangle & \beta = \psi^3 \end{array}$$

The complete potential for the 0-momentum scalar fields can be written as

$$V = \sum_{i,j} d_{ij}\psi^i\psi^j + g\kappa \sum_{i,j,k} t_{ijk}\psi^i\psi^j\psi^k \tag{6}$$

where $g$ is the string coupling and $d_{ij}, t_{ijk}$ are coefficients symmetric in their indices. The constant

$$\kappa = \frac{3^{7/2}}{2^7}$$

has been chosen so that

$$t_{111} = 1.$$

## 2.2 Tachyonic instability

The leading terms in the tachyon potential arise from neglecting all other fields:

$$V_0(\phi) = -\frac{1}{2}\phi^2 + g\kappa\phi^3$$

This potential has an instability at $\phi = 0$, and a minimum at $g\kappa\phi = 1/3$. Integrating out other fields changes the potential. There are several natural questions to ask about the resulting tachyon potential:

*a*) Is there a stable vacuum?

*b*) What is the energy of the stable vacuum, if it exists?

*c*) What is the physics of the stable vacuum?

*d*) What happens if the tachyon rolls in the direction of negative $\phi$?

In [9], Sen proposed specific answers to the first three of these questions. He suggested that the open bosonic strings should be thought of as ending on an unstable space-filling Dirichlet 25-brane. The condensation of the tachyon should correspond to the annihilation of the D25-brane. From this physical picture, Sen arrived at the following conclusions in answer to the above questions:

*a*) There should be a stable vacuum.

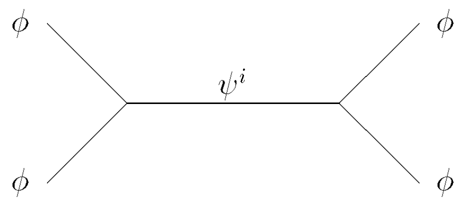
*b*) The energy difference between the stable and unstable vacua should be given by the D25-brane tension $-T_{25} = -(2\pi^2 g^2)^{-1}$ times the volume of space-time.

*c*) The stable vacuum should be identified with the closed string vacuum, so that open strings should decouple and only closed string excitations should be present.

In the remainder of this talk we address these questions, and give concrete evidence from specific calculations in level truncated string field theory for each of Sen's conclusions.

# 3 Level Truncation

Since the string field (1) contains an infinite number of space-time fields, in order to do actual calculations it is necessary to truncate the theory by dropping all but a finite number of fields. One systematic way of doing this is to drop all fields above a certain level. We define the theory at level truncation $(L, I)$ to include all fields up to level $L$, and all interactions of fields whose total level is less than or equal to $I$. We need to have at least $I \geq 2L$ to include the quadratic terms in the string field theory action (2). This form of level truncation in string field theory was first introduced by Kostelecky and Samuel [10]. The philosophy behind this approach is that saturating intermediate states with low level fields often gives an accurate systematic approximation scheme. A simple example of this is the tachyon 4-point function, which arises from diagrams of the form




We will discuss this 4-point function in more detail shortly.

Using *Mathematica*, we have computed all interactions in the action (6) up to level (10, 20). At this level we have 252 fields, and 138,202 cubic vertices. This computation was done using a very simple program, which took approximately 24 hours on a desktop PC. With a more serious programming effort and faster hardware, it might be possible to extend this calculation up to level (16, 32). Because both the number of fields and the computational complexity of each interaction grows exponentially in the level number, however, it is unlikely that it will be possible to go far beyond level (16, 32) in the forseeable future without some improvement in our analytic understanding of the problem.

We will not go into any of the details of the calculation here, but a few points are worth mentioning which simplify the calculation significantly. One feature of the cubic SFT action is that most of the calculation of the coefficients in (6) factorizes into separate parts for the matter and ghost sectors

$$\mathcal{H} = \mathcal{H}_{\text{mat}} \otimes \mathcal{H}_{\text{gh}} \tag{7}$$

These sectors only communicate through the appearance of the total level in the gauge-fixed BRST operator through $\frac{1}{2}M^2 = L_m + L_g - 1$.

Another possible simplification in the calculation follows from an observation made by Sen in [9], which is that the scalar Hilbert space decomposes into two subspaces

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2$$

where $\mathcal{H}_1$ contains only states formed from matter Virasoro generators and ghost modes. There are no cubic vertices in the action of type 112; the only vertices connecting the two subspaces are of the form 122. This indicates that the states in the subspace $\mathcal{H}_2$ cannot affect the classical effective potential for the tachyon or the classical stable vacuum configuration. As a result, we could perform all calculations relevant to the determination of the classical vacuum only in the subspace $\mathcal{H}_1$.

The following table compares the numbers of states in $\mathcal{H}$ and $\mathcal{H}_1$

| $L$ | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $h_L$ | 1 | 2 | 7 | 21 | 60 | 161 | 415 | 1021 | 2432 | 5620 | 12639 |
| $h_L^1$ | 1 | 2 | 6 | 17 | 43 | 102 | 231 | 496 | 1027 | 2060 | 4010 |

From this table we see that there is a significant difference between the number of fields only for large $L$. We found it easier to simply work in $\mathcal{H}$, so that all calculations could be done using the Fock space representation of the vertex operator. This also allows us to use the condition that the vacuum should lie in $\mathcal{H}_1$ as a check on our calculations, as we will discuss later. It might, however, be possible to implement an algorithm based on the Virasoro algebra, as discussed in [11], to reduce the amount of computational work needed to determine the vacuum as a state an $\mathcal{H}_1$.

# 4 Effective Tachyon Potential

## 4.1 Perturbative expansion of the effective tachyon potential

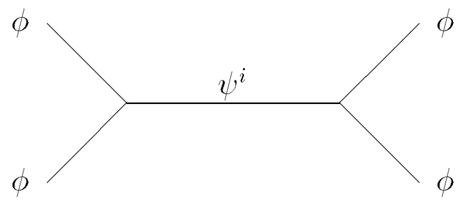
The effective potential for the tachyon field $\phi$ can be written as

$$V(\phi) = \sum_{n=2}^{\infty} c_n (g\kappa)^{n-2}\phi^n = -\frac{1}{2}\phi^2 + g\kappa\phi^3 + \cdots \tag{8}$$

where the coefficients $c_{n>3}$ can be calculated by integrating out all fields at level $L > 0$. The quartic coefficient $c_4$ was exactly computed by Kostelecky and Samuel [12] using an exact calculation of the general off-shell tachyon 4-point function done earlier by Samuel in Feynman-Siegel gauge using the cubic SFT [15]. The exact value for $c_4$ is given by

$$c_4 \approx -1.75 \pm 0.02 \tag{9}$$

where the uncertainty arises from numerical approximation to an integral. The term $c_4$ arises from SFT diagrams of the form

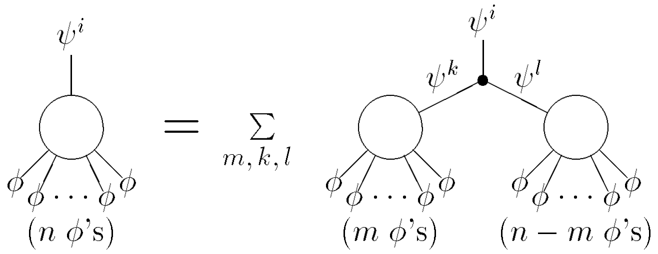



It was shown in [10] that integrating out the level 2 fields $B, \beta$ gives 72% of the exact value [12]. Including fields up to level 20 gives 96% of the exact value [2]. The following table indicates the contribution to $c_4$ when fields up to each level $L \leq 20$ are included.

| $L$ | # of fields | $c_4^{(L)}$ | $c_4^{(L)}/c_4^{\text{exact}}$ |
|---|---|---|---|
| 2 | 2 | $-1.259$ | 0.72 $\pm$0.01 |
| 4 | 7 | $-1.472$ | 0.84 $\pm$0.01 |
| 6 | 20 | $-1.556$ | 0.89 $\pm$0.01 |
| 8 | 55 | $-1.600$ | 0.92 $\pm$0.01 |
| 10 | 139 | $-1.628$ | 0.93 $\pm$0.01 |
| 12 | 331 | $-1.646$ | 0.94 $\pm$0.01 |
| 14 | 747 | $-1.660$ | 0.95 $\pm$0.01 |
| 16 | 1618 | $-1.670$ | 0.96 $\pm$0.01 |
| 18 | 3375 | $-1.677$ | 0.96 $\pm$0.01 |
| 20 | 6818 | $-1.684$ | 0.96 $\pm$0.01 |

Clearly the successive approximations to $c_4$ are converging, but the rate of convergence is not exceptionally fast.

To compute the coefficients $c_{n>4}$, the number of diagrams which must be computed grows exponentially in $n$. We can turn this calculation into a polynomial problem, however, using a simple set of recursion relations which can be diagrammatically represented through




Using these recursion relations is equivalent to solving the $N-1$ quadratic equations for the massive fields order by order in a formal power series expansion.

For example, let us consider a single extra field $\psi$ coupled through the potential

$$V=-\frac{1}{2}\phi^2+g\kappa\phi^3+\frac{1}{2}\psi^2+g\kappa\left(\frac{\gamma}{6}\psi^3+\beta\phi^2\psi\right). \tag{10}$$

An explicit solution for $\psi$ is given by

$$\psi=\frac{1}{g\kappa\gamma}\left(-1+\sqrt{1-2g^2\kappa^2\beta\gamma\phi^2}\right) \tag{11}$$

where the choice of root is determined by the condition that $\psi=0$ when $\phi=0$. This gives a potential

$$V=\sum_{n=2}^{\infty}c_n(g\kappa)^{n-2}\phi^n, \qquad c_{2n>4}=-\frac{(2n-5)!!}{n!}\beta^n\gamma^{n-2}$$

Note that the appearance of a singularity in the effective potential at $2g^2\kappa^2\beta\gamma\phi^2=1$ is indicated by the asymptotic form of the coefficients $c_n\sim(2\beta\gamma)^n$.

We have numerically computed the coefficients $c_n$ for $n\leq 60$, $(L,I)\leq(10,20)$. We find that these coefficients converge, although the rate of convergence decreases as $n$ increases. For $n>7$, the contribution from level 4 fields is greater than that from level 2; For $n>14$, the contribution from level 6 is greater than that from level 4; For $n>25$, the contribution from level 8 is greater than that from level 6, etc. For large values of $n$, fields at arbitrarily high levels must be included to get a good approximation to $c_n$.

In each level truncation, we can match the growth of the coefficients $c_n$ to an asymptotic function of the form

$$|c_n|\sim\alpha n^\eta\frac{1}{(g\kappa r_c)^n}\,.$$

The value $r_c$ indicates the radius of convergence for $V$ in the given level truncation. These values are given in the following table for each of the level truncations we have considered.

| level | (2, 6) | (4, 12) | (6, 18) | (8, 20) | (10, 20) |
|---|---|---|---|---|---|
| $gr_c$ | $\approx 0.345$ | $\approx 0.283$ | $\approx 0.265$ | $\approx 0.256$ | $\approx 0.252$ |

From this table and the signs of the $c_n$'s, it is clear that the values of $r_c$ seem to be converging fairly rapidly to an apparent singularity near

$$\phi \approx -0.25/g\,. \tag{12}$$

### 4.2 The nonperturbative stable vacuum

To find the stable vacuum, we need to solve $N$ coupled quadratic equations

$$\psi^i = \frac{-b + \mathrm{sign}(d_{ii})\sqrt{b^2 - 4ac}}{2a} \tag{13}$$

where the coefficients in the quadratic are given by

$$\begin{aligned} a &= 3t_{iii} \\ b &= \sum_{j \neq i} 6t_{iij}\psi^j + 2d_{ii} \\ c &= \sum_{j,k \neq i} 3t_{ijk}\psi^j\psi^k + \sum_{j \neq i} 2d_{ij}\psi^j \end{aligned}$$

for fields without ghosts. For fields with ghosts there are similar expressions. The branch choice taken for the solution of the quadratic is fixed by the condition that $\psi^i = 0$ for all fields in the perturbative vacuum.

In the level (0, 0) truncation, we have the pure tachyon potential $V = -\frac{1}{2}\phi^2 + g\kappa\phi^3$, whose vacuum energy in the nontrivial vacuum $\phi = 1/3g\kappa$ is given by

$$V = -0.6846\, T_{25}$$

In the level (4, 8) truncation we have [10, 8]

$$V = -0.9864\, T_{25}$$

We have solved the quadratic equations at each level truncation up to (10, 20), and we find the following values for the tachyon VEV and the vacuum energy

| level | $g\langle\phi\rangle$ | $V/T_{25}$ |
|---|---|---|
| (0, 0) | 0.91236 | -0.68462 |
| (2, 4) | 1.08318 | -0.94855 |
| (2, 6) | 1.08841 | -0.95938 |
| (4, 8) | 1.09633 | -0.98640 |
| (4, 12) | 1.09680 | -0.98782 |
| (6, 12) | 1.09602 | -0.99514 |
| (6, 18) | 1.09586 | -0.99518 |
| (8, 16) | 1.09424 | -0.99777 |
| (8, 20) | 1.09412 | -0.99793 |
| (10, 20) | 1.09259 | -0.99912 |

Note that the energy converges very rapidly; the error in the vacuum energy goes down by a factor of $\sim 1/3$ at each level. The values of the tachyon and the other fields also converge very rapidly as the level of truncation is increased. It is interesting that both the vacuum energy and the field values converge much more rapidly than the perturbative coefficients $c_n$.

As we mentioned previously, a good check on our calculations is to verify that the fields in the vacuum lie in $\mathcal{H}_1$. This condition leads to various linear relations which must be satisfied by the field expectation values. For example, the level 4 fields obey

$$\langle\psi^4\rangle - 2\langle\psi^5\rangle - 4\langle\psi^6\rangle = 0.$$

We have checked these relations and find that they are satisfied to the accuracy (10 decimal places) with which we have calculated the vacuum field values in each level truncated theory.

Part of the original motivation for carrying out high-level calculations of the vacuum configuration in string field theory was to get enough data so that patterns might emerge which might give a hint as to how an analytic description of the vacuum might be attained. An example of such a pattern was suggested by Sen and Zwiebach [8], who found that fields

$$\begin{aligned} u &= -\psi^3 \\ v &= 2\sqrt{13}\psi^2 \end{aligned}$$

differ by $\approx 0.1\%$ in the vacuum computed at level (4, 8). Hata and Shinohara have suggested that this equality may be related to a level-by-level vanishing of the norm of the vacuum state [16]. We have looked for patterns involving approximate equalities between fields in our data for the stable vacuum configuration. Unfortunately, even the agreement between $u$ and $v$ at level (4, 8) appears to be a red herring. At level (10, 20), the expectation values of these fields differ by more than 5%, indicating that the close agreement at level (4, 8) is a coincidence. So far the only apparent pattern in the vacuum expectation values of the fields arises from a symmetry which involves exchanging the ghost and antighost modes $c_n, b_n$; this symmetry is discussed in the lectures of Zwiebach at this meeting.

In the analysis described above of the asymptotic form of the coefficients in the tachyon effective potential we found evidence for a singular point near

$$\phi \approx -0.25/g\,. \tag{14}$$

In the level (2, 4) truncation, Kostelecky and Samuel exactly solved the quadratic equations for the level 2 fields and found an interesting branch structure for the tachyon effective potential, with a singular point at $\phi \approx -0.35/g$, corresponding to a square root branch point. We have studied the branch structure of the tachyon effective potential and found that this singular point persists in higher level truncations. As the level of truncation increases, we find that the tachyon effective potential has a branch point singularity which approaches a point near $\phi \approx -0.25/g$. There is also a singularity when the tachyon exceeds the value where the stable vacuum lies; this second singularity is located near $\phi \approx 1.5/g$. An explanation for these singularities was recently found: these branch points are gauge artifacts which arise when the tachyon and the massive string fields on the curve associated with the tachyon effective potential leave the region in field space where Feynman-Siegel gauge is a valid gauge choice [17]. (Note: when this lecture was given, I did not understand the reason for these branch points, and suggested several alternative explanations for their appearance. At this point, however, the evidence that these branch point are gauge artifacts is fairly conclusive.)

It is interesting to compare the tachyon effective potential found in Feynman-Siegel gauge-fixed cubic string theory with the analogous expression $V = (1 + T)e^{-T}$ calculated in background independent string field theory (BSFT) [18, 19, 20]. While in the BSFT picture the tachyon effective potential continues to arbitrarily negative values of $T$ without encountering the gauge artifact just mentioned, in the BSFT picture the effective potential does not continue past the minimum at $T \to \infty$ associated with the stable vacuum, unlike the cubic SFT picture where the potential continues past the stable vacuum.

# 5 U(1) on the brane

In this section we describe some interesting features of the U(1) gauge field $A_\mu$ living on the D25-brane. In principle, integrating out all massive fields as well as the tachyon should give rise to an effective U(1) action for this gauge field which should be related to the usual Born-Infeld action through a field redefinition [21]. To begin with, we note that the SFT action contains the term

$$\frac{16}{9}\kappa g\ \phi A_\mu A^\mu$$

which gives rise to a term in the effective U(1) action

$$\kappa^2 g^2 \left(\frac{128}{81}\right)(A^\mu A_\mu)^2 \tag{15}$$

after integrating out $\phi$. Naively, this seems to break gauge invariance. Worse, than that, this term seems to lift the zero modes of the gauge field which should arise after compactification; in the T-dual picture this would remove the translation 0-modes of D$p$-branes with $p < 25$ in transverse directions. Fortunately, we find that the term (15) seems to cancel when we integrate out higher mass fields [1]. Integrating out the two level 2 fields cancels 76% of the offending term, and including out all fields of level level $\leq 16$ fields cancels 97% of this term.

It is interesting to generalize this consideration to the nonabelian gauge theory. Including Chan-Paton indices, we find a term in the effective nonabelian gauge theory action

$$-\kappa^2 g^2 \left(\frac{\gamma_-}{2}\right) \mathrm{Tr}\ \left([A^\mu, A^\nu]^2\right) \tag{16}$$

with $\gamma_- \to\approx 0.70$ after levels $\leq 16$ have been integrated out. While we expect this form of term from the nonabelian Yang-Mills or Born-Infeld theory, naively this term does not agree with the quadratic and cubic terms in the SFT action for $A_\mu$. It was pointed out by David [14] that this apparent disagreement may be resolved by a field redefinition which is necessary to relate the SFT field $A_\mu$ to the usual gauge field in the Born-Infeld action. We will return to this field redefinition momentarily. It would be interesting to generalize the calculation of the term (16) to higher order terms in the effective action for $A_\mu$ using the cubic SFT. This could lead to new information and insight about the bosonic nonabelian Born-Infeld theory.

In the discussion so far we have integrated out the tachyon $\phi$ as well as the massive fields to arrive at an effective action for $A_\mu$. It is also of interest to integrate out the massive fields without integrating out $\phi$, giving an effective action for $\phi$ and $A_\mu$ together. This gives an action with leading terms

$$\begin{aligned} V &= -\frac{1}{2}\phi^2 + \kappa g\phi^3 + \frac{16}{9}\kappa g\phi A_\mu A^\mu \\ &\quad -\frac{128}{81}\kappa^2 g^2 (A^\mu A_\mu)^2 + \cdots \end{aligned}$$

This action is invariant under transformations

$$\begin{aligned} \delta\phi &= \frac{32}{9}\kappa g A^\mu \partial_\mu \Lambda + \cdots \\ \delta A_\mu &= \partial_\mu \Lambda + \cdots \end{aligned}$$

With a field redefinition whose leading terms are [21]

$$\phi \to \tilde{\phi} + \frac{16}{9} g\kappa A_\mu A^\mu + \cdots$$

this symmetry becomes the more familiar U(1) gauge symmetry

$$\begin{aligned} \delta\tilde{\phi} &= 0 \\ \delta A_\mu &= \partial_\mu \Lambda + \cdots \end{aligned}$$

and the action becomes

$$V = -\frac{1}{2}\tilde{\phi}^2 + \kappa g\tilde{\phi}^3 + \cdots$$

In principle such a field redefinition can be written to arbitrary order in the fields $\phi, A_\mu$. Since we are only considering constant fields in the abelian theory, all dependence on the $U(1)$ field should be removed by this field redefinition, and the action should become simply the usual cubic potential for the tachyon. Unfortunately, however, it is only known how to construct this field redefinition in a perturbative fashion around the origin of field space. As we will discuss in the next section, there is no reason that we should expect this field redefinition to be globally valid.

# 6 Physics in the stable vacuum

Part (c) of Sen's conjecture predicts that the open strings will decouple in the true vacuum. We still expect closed strings in full theory. These closed strings are difficult to construct in the open string language, however, where they are only visible as intermediate states in loop diagrams. A complete demonstration that the open string states decouple from the theory in the stable vacuum would require a complete study of the spectrum of states in the stable vacuum, including all momentum-dependent terms in the action. Some progress in this direction was made in [10], where it was found that at level 2 the lowest mass squared of a physical scalar state was slightly above 1 (in string units), which is the cutoff associated with level 2 states. Work in progress with Ellwood [17] has extended this analysis to higher levels. We find that at level 4 the lowest mass squared of a physical scalar state is slightly above 3, which gives further evidence that all physical states below the level truncation are removed from the spectrum. Recently further encouraging results were found by Hata and Teraguchi [22], who analyzed the BRST-invariant scalar excitations at level 2 and found approximately vanishing momentum-dependent terms for these fields.

While a complete understanding of the decoupling of open string states in the stable vacuum is still a difficult unsolved problem, there is a much simpler question which we can address using the momentum-independent part of the string field theory action. This question is that of the fate of the zero modes of the gauge field in the stable vacuum. After compactification on a torus $T^k$, near the unstable vacuum shifting the gauge field in a compact direction by a constant $\delta A_\mu = a_\mu$ is a zero mode of the theory. These zero modes are T-dual to translation zero modes of D$(p-k)$-branes $\delta X^\mu = a^\mu$. While the infinitesimal form of these zero modes is quite simple, a finite shift in $A_\mu$ requires all vector fields $C_\mu, \ldots$ to be turned on (again due to a field redefinition from the usual U(1) field.) These zero modes of the U(1) field were studied in [23]. Since the true vacuum of the theory should be unique, we expect that these zero modes should be lifted in stable vacuum. That is, there should be no translation zero modes after a D$(p-k)$-brane decays into the stable vacuum. The mechanism for lifting these zero modes is not obvious since no fields are charged under $A_\mu$, so a Higgs mechanism is not possible.

The key to this question lies in the terms of the form

$$\phi_i A_\mu A^\mu \to (\phi_i^* + \delta\phi_i) A_\mu A^\mu$$

in the SFT action. In the stable vacuum these terms give mass terms

$$\mu^2 A_\mu A^\mu$$

to the gauge field $A_\mu$. The massless vector field $A_\mu$ couples to other massive vector fields $\eta_\mu^{(m)}$, so to determine whether there is truly a zero mode for $A_\mu$ we must calculate the mass matrix for the vector fields up to some fixed level $L$, and then integrate out the massive fields $\eta_\mu^{(m)}$, giving an effective action

$$S_{\phi,A} = V(\phi) + G(\phi) A_\mu A^\mu + \mathcal{O}(A^4)$$

We have carried out this procedure up to level (9, 10, 20), where 9 is the maximal level of the massive vector fields, 10 is the maximal level of scalars, and 20 is the maximum interaction level [3]. The results of this calculation are summarized in the following table

| level | $N_v$ | $N_s$ | $G(\phi_*)$ |
|---|---|---|---|
| (1, 2, 4) | 1 | 3 | 0.67289 |
| (3, 4, 8) | 4 | 10 | 0.63343 |
| (5, 6, 12) | 17 | 31 | 0.62738 |
| (7, 8, 16) | 61 | 91 | 0.62491 |
| (9, 10, 20) | 197 | 252 | 0.62362 |

We find that the quantity $G(\phi_*)$ converges in a very similar fashion to the energy $V(\phi_*)$ in the stable vacuum, and that $G(\phi_*)$ seems to converge to a nonzero number. This result seems to indicate that the zero modes of the U(1) gauge field are indeed lifted in the stable vacuum. A related calculation was performed by Sen and Zwiebach at level (3, 4, 8) [23]. We have compared our calculation with theirs in detail and find complete agreement.

This calculation seems to imply that the gauge field $A_\mu$ acquires a mass in the true vacuum. Naively, this result seems to contradict gauge invariance, since it seems difficult to remove a mass term by a field definition such as those discussed above. However, there is no conflict with gauge invariance if the true vacuum is outside the range of the field redefinition. Sen has given a nice interpretation of this picture [24], in terms of an analogy with Cartesian and polar coordinates. If we have a function on the plane which is independent of $\theta$, with a minimum at $\rho = 0$ and a circle of local maxima at $\rho = \rho_0$, then around a point on the circle of maxima, $\rho, \theta$ are good coordinates, and $\theta$ acts as a zero mode. In polar coordinates, $\theta$ also appears to be a zero mode at the origin, but this is an artifact of the coordinate system. There is a singularity in the coordinate transformation to Cartesian coordinates. In the Cartesian coordinate system, the potential at the origin can be of the form $\rho^2 = x^2 + y^2$, and thus have no zero mode. It seems that the cubic SFT variables play the role in this story of the Cartesian coordinates, which are good coordinates around the true vacuum, while the Born-Infeld field plays the role of the angular polar coordinate, which is a bad coordinate near the true vacuum. This picture seems to neatly resolve the issue of how the massless gauge field decouples from the theory in the true vacuum.

# 7 Conclusions

We conclude with a brief summary of the results presented here and a list of some further directions in which this work might be extended.

We have discussed the calculation of the action for the complete (10, 20) truncation of scalar open bosonic string field theory. We have calculated the coefficients in the perturbative effective tachyon potential, and found that this effective potential has branch point singularities where Feynman-Siegel gauge breaks down. We have determined the stable vacuum to 99.91% accuracy. No obvious structure was found in the field expectation values in the vacuum which might lead to an analytic solution. We have found that zero modes of the theory are lifted by mass terms in the true vacuum. All evidence so far agrees with Sen's conjecture regarding the decoupling of open strings in the true vacuum, but further work in this direction needs to be done.

One of the most important open questions in this area is to understand in detail the structure of the theory in the true vacuum. We should first find conclusive evidence that all open strings decouple in the stable vacuum, and then move on to look for signs of the closed strings as asymptotic states in the theory around this vacuum. We have not touched here on the issues of studying $p-k$-brane solitons, or tachyon condensation in superstring field theory. These subjects have been discussed by the other lecturers here. It would clearly be of interest to do high-level analyses of these problems analogous to the work discussed here. As mentioned above, it would be interesting to carry the study of the nonabelian Born-Infeld action further using string field theory techniques. The connection with BSFT is important, and should be better understood. Finally, it would be nice to develop a better understanding of the algebraic structure of the star product and the 3-string vertex in the cubic SFT. This might lead to a deeper understanding of the structure of this interesting and subtle theory.

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