Komaba Lectures on String Field Theory and Tachyon Condensation

Barton Zwiebach

Center for Theoretical Physics
Massachussetts Institute of Technology,
Cambridge, MA 02139, USA
E-mail: zwiebach@mitlns.mit.edu

Abstract

Cubic open string field theory is a powerful tool in the study of tachyon dynamics. After a brief introductory presentation of string field theory we examine the problem of tachyon condensation and the insights that have been gained in investigating the nature of the stable tachyon vacuum. *Based on lectures presented at the Komaba 2000 Workshop, November 14-16, 2000.*

Contents

1 A brief historical introduction

String field theories were first formulated in the light-cone gauge by Kaku and Kikkawa [1] in the early 70's using the interacting string picture of Mandelstam [2]. Both open and closed string field theories were written, in fact, the complete theory was an open/closed string field theory. As recognized by Siegel in the middle 80's [3], Lorentz covariant formulations of string field theory required the use of the BRST operator. Free gauge invariant (bosonic) string field theories were formulated by Siegel and Zwiebach [4] and Banks and Peskin [5]. The complete classical interacting open string field theory was given by Witten [6] who devised an associative multiplication rule for open string fields. One can imagine the product as a three string interaction, made by the two input strings and the output string, where the open strings overlap across half-strings.

The interacting closed bosonic string field theory took longer to formulate, basically due to the intricate structure of the moduli space of Riemann surfaces without boundaries.

The classical closed string theory required the nonpolynomial "polyhedral" setup of Saadi and Zwiebach [7], which incorporated and generalized a four-string tetrahedral interaction described by Kaku [8]. One can imagine the interactions by having each closed string lay along the edges making the boundary of a face on a polyhedron, with the various strings overlapping across the edges. The prescription for the required polyhedra was completed by Kugo, Kunitomo and Suehiro [9]. The quantum closed string field theory was obtained [10] by recognizing that all classical closed string diagrams were solutions to a modular invariant minimal area problem, and by demanding that the higher genus diagrams solve that same problem. It was then realized that the string diagrams of open string field theory solve the analogous minimal area problem for Riemann surfaces with boundaries. Thus by the early 90's nonpolynomial closed string field theory was seen to be the natural analog of cubic open string field theory.

The situation for Lorentz-covariant, gauge invariant superstring field theory is, even as of now, still incomplete. The early proposal of Witten for a cubic open superstring field theory [11] was found to have complications associated to the insertion of picture changing operators at interaction point [12][1] . In the middle 90's Berkovits [15] gave a Lorentz covariant formulation of the Neveu-Schwarz sector of open superstrings that avoided neatly these complications. For the Ramond sector the formulation does not have full $SO(1,9)$ Lorentz invariance. Finally, the formulation of closed superstring field theory remains a formidable problem, especially in the Ramond-Ramond sector. I do not believe there are unsurmountable difficulties with this construction; in fact, it is clear that even BRST cohomology has not yet been defined properly in the RR- sector [16, 17], and probably our understanding of supermoduli space is still rather insufficient. Thus, the lack of a superstring closed string field theory is probably just a reflection of our incomplete understanding of superstring perturbation theory where, indeed, complete proofs of the uniqueness of amplitudes or of their widely believed finiteness are still lacking.

String field theory had several achievements. Here is a list:

- Suggested the fundamental role of the BRST operator in string theory. This operator became a main tool of most studies of perturbative string theory. In particular, it gave a precise definition of the spectrum as BRST cohomology classes for open strings and BRST-semirelative cohomology classes for closed strings (all bosonic).

- Suggested a fundamental role for Batalin-Vilkovisky (BV) quantization in the eventual geometric formulation of the theory. The BV master equation was seen to encode the interplay between gauge invariance and the covering of the moduli spaces of Riemann surfaces. These appreciations led to much study of BV geometry.

[1] For some proposals that preserve the cubic nature of the theory see [13].

- Provided, via minimal area problems, a useful new parametrization of the moduli spaces of Riemann surfaces. This results in a manifestly Lorentz invariant and pertubatively unitary presentation of the string amplitudes. Allows the disentanglement of open string and closed string contributions to amplitudes in D-brane backgrounds.

- Background independence of string theory: two string theories formulated using two nearby conformal theories were shown to be the same theory expanded around different vacuum. Dilaton theorem; a shift of the dilaton was shown to shift the string coupling in the string field action. (These are properties of perturbative string theory that could not be established without use of the string field theory action).

As the reader will note, none of the above items speaks about the possibility of doing non-perturbative string physics using string field theory. In fact, doing such kind of physics was the main motivation for writing string field theory in the first place. By the middle 90's, with many exciting developments in understanding D-branes and dualities, it was widely assumed that string field theory was not a non-perturbative definition of string theory. It was thought that the string field was only defined as a fluctuating perturbative field, and nontrivial solutions, such as D-branes, were far away from the domain of validity of string field theory. The string field, it was said, could not be expected to reach such nontrivial configurations. All these expectations were proved wrong at the turn of the century.

2 Beginning with the basics

2.1 What is the string field ?

Briefly said, the string field is a state in the state space of the conformal field theory used to formulate the particular string theory. For the bosonic open string, we pick a matter CFT of $c = 26$, tensor it with the reparametrization ghosts (b,c) of $c = -26$ to get a total CFT of $c = 0$ with state space $\mathcal{H}$. If we are dealing with open strings these are conformal field theories on the disk, and the states are in one to one correspondance with the vertex operators that can be inserted on the boundary. The string field $|\Phi\rangle$ is written as

$$|\Phi\rangle = \sum_i |\Phi_i\rangle\, \phi^i\,, \quad |\Phi_i\rangle \in \mathcal{H}\,.$$

Here the expansion coefficients ϕ^i are the spacetime fields.

The state space $\mathcal{H}$, via its correspondance with vertex operators that can be inserted on the boundary of the disk and deform the original CFT, can be roughly though as the tangent space to the space of 2 dimensional field theories at the CFT in question. The string field is simply a vector in this tangent space. The question "is the string field big enough ?" is essentially the question whether working with the tangent space at a point in theory space we get a handle on faraway points in that theory space.

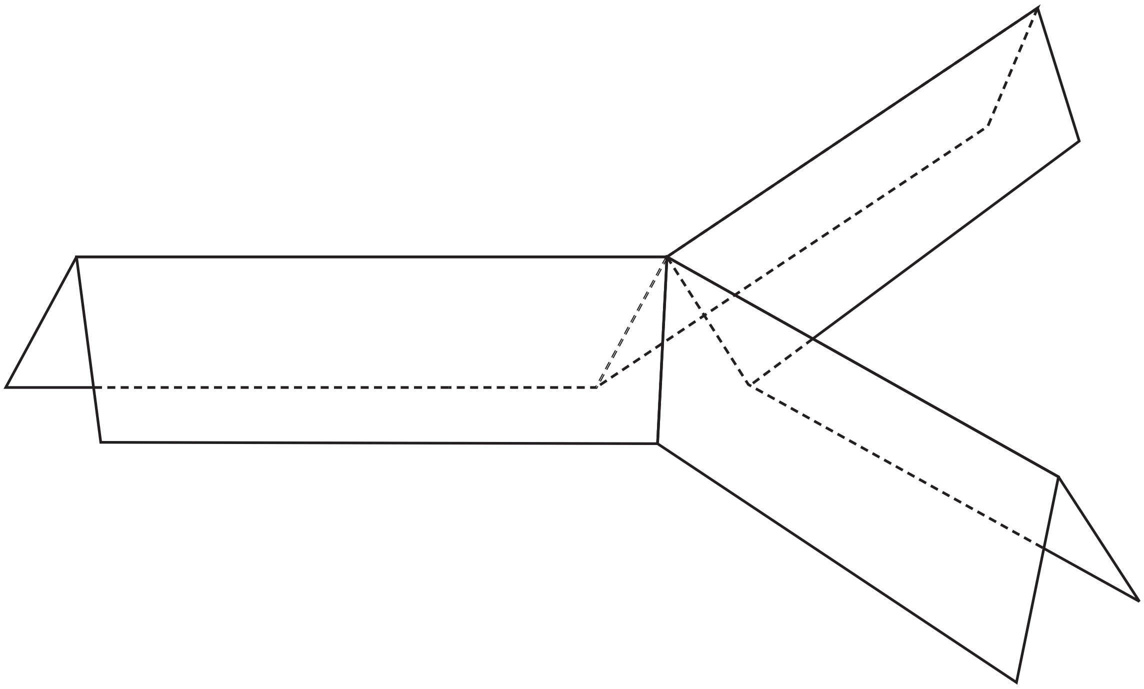

Figure 1: The interaction of three open strings.

2.2 How do OSFT and CSFT work ?

Open string field theory is defined by an action, $S(\Phi)$ which given an open string field Φ is gives us a number. The action is:

$$S(\Phi) = -\frac{1}{g_0^2}\left[\frac{1}{2}\langle\Phi, Q\Phi\rangle + \frac{1}{3}\langle\Phi, \Phi * \Phi\rangle\right]$$

Here g_o is the open string coupling. The various ingredients in this action are

- The BRST charge Q.
- A bilinear inner product $\langle\cdot\,,\,\cdot\rangle$.

- A multiplication rule $*$ for open string fields.

The star product $*$ is defined by a picture of three open strings interacting (see Fig. 1). Some explanation is required to turn this picture into a computation scheme, but suffice for our present purposes to say that they key information is encoded by showing the picture of the three world-sheets glued together. One can view two of the strings as those to be multiplied, and the last string as the product of these two. The above open string action is actually gauge invariant. These is the result of associativity of the multiplication plus a few other properties involving the BRST operator and the inner product. We will consider these properties in more detail below.

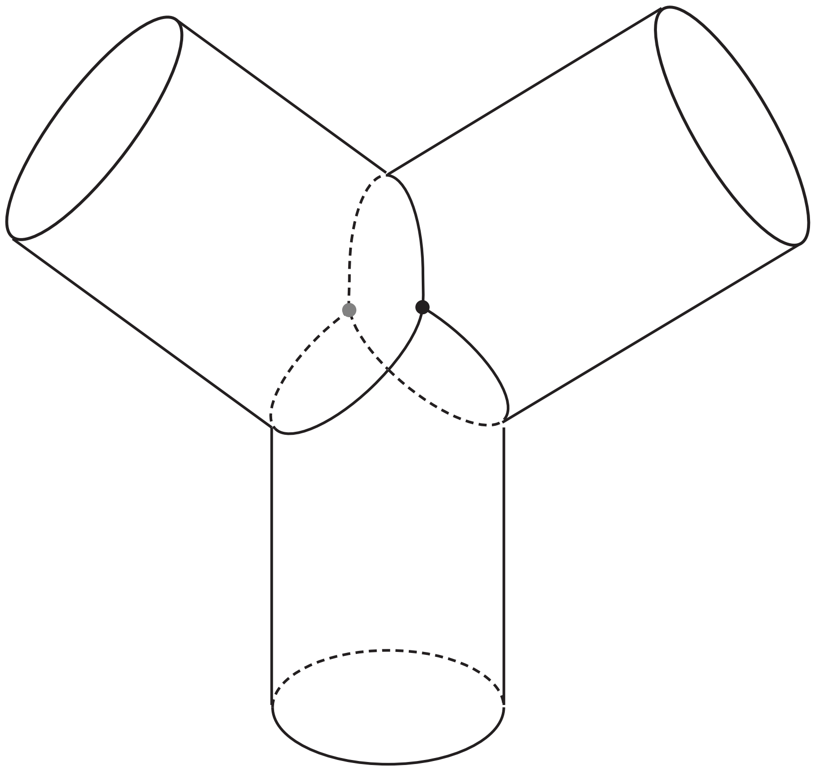

Figure 2: The interaction of three closed strings.

Closed string field theory, just like Einstein's gravity, is a nonpolynomial theory. Its action is written as

$$S(\Psi) = -\frac{1}{g_c^2}\left(\frac{1}{2}\langle\Psi, Q_c\Psi\rangle + \frac{1}{3!}\langle\Psi,[\Psi\Psi]\rangle + \frac{1}{4!}\langle\Psi,[\Psi\Psi\Psi]\rangle + \cdots\right)$$

Here g_c is the closed string coupling and Q_c is the closed string BRST operator. Moreover, while $[AB]$ denotes the product of two closed string fields A and B, intrinsically new higher products are required. The trilinear product $[ABC]$, for example, is not $[[AB]C]$, or any other object built with the bilinear product. There are pictures for each of these products. For the bilinear one we have Fig. 2. For higher ones, the tubes are glued to the faces of polyhedra. With each closed string of conventional circumference 2π the polyhedron is such that in addition to having each face of perimeter 2π, all nontrivial closed paths on the polyhedron have a length that exceeds 2π.

2.3 The issue of background independence

This issue arises because the OSFT and CSFT theories described above can only be written after choosing what amounts to a *classical solutions*, namely CFT's with $c = 0$. As a result the SFT actions S_1 and S_2 written using two different CFT's CFT_1 and CFT_2 are not manifestly equivalent. Fortunately, when CFT_1 and CFT_2 are nearby theories related by an infinitesimal marginal deformation, one can prove elegantly using CFT connections and generalized moduli spaces of Riemann Surfaces that S_1 and S_2 are the same action expanded around different solutions [18].

Another approach designed to implement more manifestly background independence is B-SFT (for boundary-SFT). It was proposed by Witten, and developed further by Witten, Li, and Shatashvili [19]. This approach uses the Batalin-Vilkovisky formalism (BV) to try to set up directly an action on the *"space of boundary field theories"*. It is extremely difficult to define precisely in general–but can be used to calculate the action on the space of special tachyon configurations. In fact, those suffice to compute the exact tachyon potential in this type of string field theory. This approach to string field theory is rather natural for open strings, but seems hard to implement for closed strings. Some work in this direction can be found in [20].

3 Beyond basics: The algebraic structure of OSFT

The open string field theory action was already introduced before. Since it is the key tool in the analysis of (open) string tachyon condensation, we examine it in more detail here:

$$S(\Phi) = -\frac{1}{g_o^2}\left[\frac{1}{2}\langle\,\Phi\,,\,Q\,\Phi\rangle + \frac{1}{3}\langle\,\Phi\,,\,\Phi * \Phi\rangle\right] \tag{3.1}$$

The consistency of this action hinges on properties we now discuss.

3.1 Main structure

The BRST operator Q must satisfy the following identities:

$$\begin{aligned} &Q^2 A = 0, \\ &Q(A * B) = (QA) * B + (-1)^A A * (QB) \\ &\langle\, QA, B\,\rangle = -(-)^A \langle A, QB\rangle\,. \end{aligned} \tag{3.2}$$

The first is the famous nilpotency condition, the second states that Q is a derivation of the star product, and the third describes the way Q moves between bras and kets.

There are also identities involving the inner product and the star operation

$$\begin{aligned} &\langle A, B\rangle = (-)^{AB}\langle B, A\rangle \\ &\langle\, A\,, B * C\,\rangle = \langle A * B\,,\, C\,\rangle \\ &A * (B * C) = (A * B) * C\,. \end{aligned} \tag{3.3}$$

In the sign factors, the exponents $A, B, \cdots$ denote the Grassmanality of the state, and should be read as $(-)^A \equiv (-)^{\epsilon(A)}$ where $\epsilon(A) = 0\,(\text{mod }2)$ for A Grassmann even, and $\epsilon(A) = 1\,(\text{mod }2)$ for A Grassmann odd. We also have that the star operation is an *even* product of degree zero. This, in plain english means that both the grassmanality and the ghost number of the star product of two string fields is obtained from those of the string fields without any additional offset:

$$\begin{aligned} &\epsilon(A * B) = \epsilon(A) + \epsilon(B) \\ &\text{gh}(A * B) = \text{gh}(A) + \text{gh}(B)\,, \end{aligned} \tag{3.4}$$

Finally, for use in the action, one defines a multilinear object that given three string fields it yields a number:

$$\langle\, A\,,\, B\,,\, C\rangle \equiv \langle\, A\,,\, B * C\,\rangle \tag{3.5}$$

The middle equation in (3.3) implies the *cyclicity* of the multilinear form. A small calculation immediately gives:

$$\langle\, A\,,\, B\,,\, C\rangle = (-)^{A(B+C)}\langle\, B\,,\, C\,,\, A\rangle \tag{3.6}$$

A basic consistency check of the signs above is that the cubic term $\langle\, \Phi\,,\, \Phi\,,\, \Phi\rangle$ in the action (3.2) is strictly cyclic for odd Φ, and therefore does not vanish. Equations (3.2), (3.3) and (3.4) guarantee that the string field action is invariant under the gauge transformations:

$$\delta\Phi = Q\Lambda + \Phi * \Lambda - \Lambda * \Phi\,, \tag{3.7}$$

for any Grassmann-even ghost-number zero state Λ. Variation of the action gives the field equation

$$Q\Phi + \Phi * \Phi = 0\,. \tag{3.8}$$

3.2 Twist properties

In open string theory twist properties are quite essential. Algebraically, this means the existance of an operator Ω with the following properties

$$\begin{aligned}
&\Omega(QA) = Q(\Omega A) \\
&\langle\, \Omega A\, ,\, \Omega B\, \rangle = \langle\, A\, ,\, B\, \rangle \\
&\Omega\,(A * B) = (-)^{AB+1}\,\Omega(B) * \Omega(A)
\end{aligned} \tag{3.9}$$

The first property means that the BRST operator has zero twist. The second property states that the bilinear form is twist invariant. The third property is crucial. Up to signs twisting the star product of string fields amounts to multiplying the twisted states in opposite order. The sign factor is important. For the string field Φ, which is grassmann odd, it gives

$$\Omega(\Phi * \Phi) = +\,(\Omega\Phi) * (\Omega\Phi) \tag{3.10}$$

with the plus sign. This result, together with the first two equations in (3.9) immediately implies that the string field action in (3.2) is twist invariant

$$S(\Omega\Phi) = S(\Phi)\,. \tag{3.11}$$

Since the string field is grassman odd, we have that the Sl(2,R) vacuum $|0\rangle$ is Grassmann even. We also recall that the star product of the vacuum with itself is the vacuum plus Virasoro descendents

$$|0\rangle * |0\rangle = |0\rangle + \cdots \tag{3.12}$$

This equation indicates that the twist eigenvalue of $(|0\rangle * |0\rangle)$ must equal the twist eigenvalue of $|0\rangle$. Letting $\Omega|0\rangle = \omega_0|0\rangle$, and using the last equation in (3.9) we find

$$\Omega(|0\rangle * |0\rangle) = (-)^{1\cdot 1+1}\,\omega_0^2(|0\rangle * |0\rangle) = -\omega_0^2(|0\rangle * |0\rangle) \tag{3.13}$$

The above-mentioned equality of twist eigenvalues now gives $-\omega_0^2 = \omega_0$, and we deduce that $\omega_0 = -1$, and therefore the vacuum is twist odd:

$$\Omega|0\rangle = -|0\rangle\,. \tag{3.14}$$

The zero momentum tachyon is the state $c_1|0\rangle$ and is twist even. This is the case because oscillators with L_0 eigenvalue n have twist eigenvalue $(-1)^n$. We will see later on that we can require the full string field to be twist even when searching for a critical point of the tachyon string field potential.

4 Tachyon Condensation to the Vacuum

The question here is whether the string field can reach the tachyon vacuum. We only discuss the bosonic D-brane. This brane is unstable and it carries no conserved charge. Its instability is reflected by the tachyon of the open string field theory describing the dynamics of this brane. For the open string field theory described before, the associated D-brane mass is: $M = 1/(2\pi^2 g_o^2)$

Tachyon Conjecture (Ashoke Sen [21, 22]) The stationary stable point of the tachyon potential represents a vacuum with no D-brane.

This conjecture requires that the energy difference between the unstable and the stable vacua in the tachyon potential must equal the D-brane energy. Thus the tachyon potential, written as $V(T) = Mf(T)$, with $f(0) = 0$ must have the property that the calculable function $f(T)$ must have a stationary point $T = T_c$, such that

$$f(T_c) = -1\,.$$

4.1 Testing the tachyon conjecture

This requires calculating the tachyon potential in the open string field theory. The method used is level truncation, a scheme devised by Kostelecky and Samuel [23], who themselves calculated much about this potential before the conjectures were formulated. The tachyon conjecture was first tested in SFT by Sen and Zwiebach in [24]. The test was extended to very high level by Moeller and Taylor [25].

If one truncates the string field to the tachyon field t one finds :

$$\frac{f^{(0)}(t)}{2\pi^2} = -\frac{1}{2}\,t^2 + \frac{3^3\sqrt{3}}{2^6}\,t^3\,.$$

This has a local minimum at

$$t = t_c = \Big(\frac{4}{3\sqrt{3}}\Big)^3 \simeq 0.456\,.$$

At this minimum:

$$f^{(0)}(t_c) = -\frac{\pi^2}{3}\cdot\Big(\frac{4}{3\sqrt{3}}\Big)^6 = -\frac{4096}{59049}\,\pi^2 \simeq -0.684\,.$$

We get about 70% of the desired energy. If we include the two scalar fields (u, v) that appear at level 2, the multiscalar tachyon potential is

$$\frac{f^{(4)}(T)}{2\pi^2} \;=\; -\frac{1}{2}\,t^2 + \frac{3^3\sqrt{3}}{2^6}\,t^3 - \frac{1}{2}\,u^2 \;+\; \frac{1}{2}\,v^2$$

$$
\begin{aligned}
&+\frac{11\cdot 3\sqrt{3}}{2^6}\,t^2\,u-\frac{5\cdot 3\sqrt{39}}{2^6}\,t^2\,v\\
&+\frac{19}{2^6\sqrt{3}}\,t\,u^2+\frac{7\cdot 83}{2^6\cdot 3\sqrt{3}}\,t\,v^2-\frac{11\cdot 5\sqrt{13}}{2^5\cdot 3\sqrt{3}}t\,u\,v\,.
\end{aligned}
$$

$f^{(4)}(T)$ has a stationary point at

$$t_c \simeq 0.542,\ u_c \simeq 0.173, v_c \simeq 0.187$$

at which

$$f^{(4)}(T_c) \simeq -0.949$$

This is about 95% of the expected answer: -1!

If we include the six level 4 fields (A, B, C, D, E, F), we need two pages to write the potential (not shown!). The critical point near the previous one is:

$$
\begin{aligned}
&t_c = 0.5482, \quad u_c = 0.2043, \quad v_c = 0.2045,\\
&A_c = -0.00495, \quad B_c = -0.00056, \quad C_c = -0.0549,\\
&D_c = 0.0183, E_c = 0.0317, \quad F_c = -0.0066\,.
\end{aligned}
$$

The value of the potential at this stationary point is given by

$$f^{(8)}(T_c) = -0.9864\,.$$

This is only 1.36% away from the expected answer $f(T_c) = -1$.
¿From Moeller–Taylor, the results up to level 10 (with interactions to level 20) give $f(T_c) = -0.99912$. At this level one includes 252 scalars and 138,202 distinct cubic vertices, and the accuracy is about one in one thousand.

4.2 Trimming the string field with the twist condition

The above calculations of the string field condensate were made under the assumption that only twist even fields do condense. As was discussed before, the zero-momentum tachyon is twist even and a simple argument, to be given below shows that only twist even scalars need to acquire expectation values.

The basic general argument for trimming is that a scalar that need not acquire an expectation value, as determined by its field equations, does not acquire it. Consider the set of fields split into a set Y that yes — the fields acquire expectation values, and a set N of fields that no— they do not acquire expectation values. It must then happen that there is no term in the action (expanded polynomially in the fields) where there is just one N field. If this is the case, the N field equations are satisfied setting all the N fields equal to zero, and one can consistently search for a solution of the field equations using only the Y set of fields.

For the case of twist symmetry we now claim that only the twist even part of the string field, where the zero momentum tachyon lies, need to acquire an expectation value. First split the string field as

$$\Phi = \Phi_+ + \Phi_- \tag{4.15}$$

where Φ_+ is twist even and Φ_- is twist odd: $\Omega\Phi_\pm = \pm\Phi_\pm$. Let us now explain why there are no terms in the string field action where Φ_- appears linearly. To begin consider the term

$$\langle \Phi_+, Q\Phi_- \rangle \, . \tag{4.16}$$

Using the first two identities in (3.9) we have

$$\langle \Phi_+, Q\Phi_- \rangle = \langle \Omega\Phi_+, \Omega Q\Phi_- \rangle = -\langle \Phi_+, Q\Phi_- \rangle \, , \tag{4.17}$$

and therefore the term must vanish. Consider now a term in the interaction where Φ_- appears linearly

$$\langle \Phi_+, \Phi_+, \Phi_- \rangle \, . \tag{4.18}$$

By the cyclicity property (3.6) this is the general term where Φ_- appears linearly. Using now the identities (3.5), (3.9) and (3.10) we have

$$\langle \Phi_+, \Phi_+, \Phi_- \rangle = \langle \Phi_+, \Phi_+ * \Phi_- \rangle = \langle \Omega\Phi_+, \Omega(\Phi_+ * \Phi_-) \rangle = -\langle \Phi_+, \Phi_+, \Phi_- \rangle \, , \tag{4.19}$$

which implies the vanishing of such term. This shows that the twist odd fields do not appear linearly in the string field action, and can be consistently set to zero in the field equations.

4.3 Trimming the string field with a gauge condition

The above calculations of the string field condensate were all made in the Siegel gauge:

$$b_0 |\Phi\rangle = 0$$

Given that string field theory is a gauge theory, the tachyon potential has flat directions representing gauge equivalent configurations. A gauge condition is therefore necessary to get tractable equations in any numerical approximation.

The use of the Siegel gauge allows one to drop many potential scalars from consideration, thus giving a very large trimming of the tachyon string field. In addition, the computation of kinetic terms is much simplified since Q can be effectively replaced by $c_0 L_0$ in this gauge.

While the Siegel gauge is well-known to be an allowed gauge for the linearized string field theory, there had been no previous evidence of its non-perturbative validity. In fact, the star operation $*$ takes us out of the gauge:

$$\Phi * \Phi \equiv c_0(\Phi^2) + [\Phi^2]$$

Given that $Q_o = c_0 L_0 + b_0 M + \widetilde{Q}$, the string field equation $Q\Phi + \Phi * \Phi = 0$ reduces to

$$L_0\Phi + (\Phi^2) = 0 \quad \text{the usual equation}$$

and

$$\widetilde{Q}\Phi + [\Phi^2] = 0 \quad \text{an additional equation}$$

Hata and Shinohara [28] tested this last equation successfully in the level expansion. This confirmed that the solution found using the Siegel gauge, solves, as it should, the full gauge invariant field equations. This is, in turn, evidence for the nonperturbative validity of the gauge. These authors also noted that other spurious critical points that appear in the level-truncated tachyon potential do not seem to satisfy both equations. This is a nice way to rule out such spurious solutions.

5 Universality and su(1,1) symmetry

In this section we discuss the universality of the tachyon potential and of the tachyon string field. The universal basis contains less scalars than the standard background dependent basis, so working in the universal basis simplifies computations. We also discuss an su(1,1) symmetry of the ghost sector of the theory in the Siegel gauge which allows a further drastic trimming of the string field.

5.1 Virasoro universality of tachyon condensation

The statement of universality of the tachyon potential is very simple: A single tachyon potential describes all bosonic D-branes regardless of their dimensionality, how they are curled up, the compactification, etc. Similarly, a single tachyon potential describes all superstring D-brane anti–D-brane pairs as shown by Sen [26].

At a practical level, this also implies that not all scalar states in $\mathcal{H}$ need to acquire expectation values when the tachyon does. In fact one can restrict oneself to the space of states

$$\mathcal{H}_1 = \{L_{-n}^{matt}, c_{-n}, b_{-n}\}|0\rangle \quad gh\# = 1.$$

namely the states obtained by acting with matter Virasoro operators and ghost oscillators on the vacuum. This subspace of $\mathcal{H}$ is certainly canonical and universal.

A computational scheme for the string action using this universal basis was developed by Rastelli and Zwiebach[27]. In particular they showed that $\mathcal{H}_1$ can be presented as

$$\mathcal{H}_1 = \{L_{-n}^{matt}, L_{-n}^{gh}\} c_1|0\rangle$$

and thus all computations reduced to Virasoro algebra manipulations.

The main object we want to compute here is the cubic term of the string field theory, which schematically involves the calculation of

$$\langle V_{123}|(L_{-I}c_1|0\rangle)^{(1)}(L_{-J}c_1|0\rangle)^{(2)}(L_{-K}c_1|0\rangle)^{(3)} \tag{5.20}$$

Conservation laws were used to obtain off-shell Virasoro Ward Identities that allow a recursive calculation of the above. The conservation laws take the form:

$$\langle V_3|L_{-k}^{(2)} = \langle V_3| \left(A^k \cdot c + \sum_{n\geq 0} a_n^k L_n^{(1)} + \sum_{n\geq 0} b_n^k L_n^{(2)} + \sum_{n\geq 0} c_n^k L_n^{(3)} \right) ,$$

where the calculable constants A^k, a_n^k, b_n^k, c_n^k capture the geometry of the interaction vertex. An example of such an identity is

$$\begin{aligned} 0 = \langle V_3| &\Big(-\frac{8}{27}\, L_0 + \frac{80}{81\sqrt{3}} L_1 - \frac{112}{243} L_2 + \frac{304}{729\sqrt{3}} L_3 - \frac{400}{19683} L_4 \cdots \Big)^{(1)} \\ +\langle V_3| &\Big(L_{-2} + \frac{5}{54}\, c + \frac{16}{27}\, L_0 - \frac{19}{243} L_2 + \frac{800}{19683} L_4 + \cdots \Big)^{(2)} \\ +\langle V_3| &\Big(-\frac{8}{27}\, L_0 - \frac{80}{81\sqrt{3}} L_1 - \frac{112}{243} L_2 - \frac{304}{729\sqrt{3}} L_3 - \frac{400}{19683} L_4 \cdots \Big)^{(3)} . \end{aligned}$$

As the reader can see, this equation allows one to replace the negatively moded L_{-2} operator, by positively moded ones. This type of identity can be used to eliminate, one by one, the negatively moded operators in (5.20). The background independent basis is likely to be useful to understand properly tachyon condensation.

5.2 Trimming with su(1,1)

It was noted in Ref. [28] that the three-string vertex coupling three open string fields in the Siegel gauge is invariant under a discrete $\mathbf{Z}_4$ symmetry transformation of the ghost and antighost oscillators:

$$b_{-n} \to -n\, c_{-n}\,, \quad c_{-n} \to \frac{1}{n}\, b_{-n}\,. \tag{5.21}$$

This, for example, was seen to imply that the modes associated to the level-four states $(3b_{-1}c_{-3})|\Omega\rangle$ and $(b_{-3}c_{-1})|\Omega\rangle$ must acquire identical expectation values in the tachyon condensate. Modes odd under this discrete transformation need not acquire expectation values in the tachyon condensate. This implies a trimming of the string field down to $\mathbf{Z}_4$ singlets.

In view of the above observations two natural questions arise:

- Is this discrete $\mathbf{Z}_4$ symmetry a symmetry of any three-string vertex, or is it a special property of the associative vertex of cubic string field theory ?
- Is this discrete $\mathbf{Z}_4$ symmetry part of a larger symmetry acting on the ghost coordinates that impose a larger set of constraints on the string field ?

The answers are simple– only special vertices are $\mathbf{Z}_4$ symmetric, and the $\mathbf{Z}_4$ symmetry is a subgroup of an su(1,1) symmetry acting on the ghost coordinates.

Indeed, the vertex coupling the three string fields is of the form

$$|v_3\rangle \sim \exp\left(E_{matt}\right)\ \exp\Big(-\sum_{r,s=1}^{3}\sum_{n,m=1}^{\infty} c^r_{-n}\, X^{rs}_{nm}\, b^s_{-m}\Big)\,|\Omega\rangle_{123}\,, \tag{5.22}$$

where we have focused on the ghost sector. The $\mathbf{Z}_4$ symmetry (5.21) requires that

$$X^{rs}_{nm} = \frac{n}{m} X^{sr}_{mn}\,, \quad n,m \geq 1\,. \tag{5.23}$$

One can show [29] that this property is not generic and imposes constraints on the geometry of the three string vertex. Happily, these constraints are satisfied by the cubic vertex of OSFT.

Note, however, that the symmetry transformation in (5.21) can be embedded in a continuous $U(1)$ symmetry as follows

$$\begin{aligned} b_{-n}(\theta) &= b_{-n}\cos\theta - n c_{-n}\sin\theta\,, \\ c_{-n}(\theta) &= c_{-n}\cos\theta + \frac{1}{n} b_{-n}\sin\theta\,. \end{aligned} \tag{5.24}$$

These transformations, valid for all $n \neq 0$, coincide with the action of the $\mathbf{Z}_4$ generator for $\theta = \pi/2$. One readily finds that

$$b_{-n}(\theta) = e^{\theta\mathcal{S}_1}\, b_{-n} e^{-\theta\mathcal{S}_1}\,, \quad c_{-n}(\theta) = e^{\theta\mathcal{S}_1}\, c_{-n} e^{-\theta\mathcal{S}_1}\,, \tag{5.25}$$

where the operator $\mathcal{S}_1$ is given by

$$\mathcal{S}_1 = \sum_{n=1}^{\infty}\Big(\frac{1}{n}\, b_{-n}b_n - n c_{-n}c_n\Big)\,. \tag{5.26}$$

We now explain why the vertex $|v_3\rangle$ is invariant under this $U(1)$ symmetry. For this we must have

$$\Big(\mathcal{S}_1^{(1)}+\mathcal{S}_1^{(2)}+\mathcal{S}_1^{(3)}\Big)|v_3\rangle_{123}=0\,. \tag{5.27}$$

Given eqn. (5.23), the argument of the exponential in $|v_3\rangle_{123}$ (see (5.22)) can be written as a sum of terms of the form (r,s,n,m, not summed)

$$X^{rs}_{nm}\,c^r_{-n}b^s_{-m}+X^{sr}_{mn}\,c^s_{-m}b^r_{-n}=\frac{1}{m}X^{sr}_{mn}\Big(nc^r_{-n}b^s_{-m}+mc^s_{-m}b^r_{-n}\Big)\,. \tag{5.28}$$

It is now simple to verify that such terms commute with the $U(1)$ generators, namely

$$\Big[\sum_{k=1}^{3}\mathcal{S}_1^{(k)}\,,\,nc^r_{-n}b^s_{-m}+mc^s_{-m}b^r_{-n}\Big]=0\,. \tag{5.29}$$

This, together with $\mathcal{S}_1^{(k)}|\Omega\rangle_{123}=0$ establishes (5.27), and therefore the u(1) invariance of the vertex. The generator $\mathcal{S}_1$, together with the ghost number generator $\mathcal{G}$

$$\mathcal{G}=\sum_{n=1}^{\infty}\Big(c_{-n}b_n-b_{-n}c_n\Big)\,, \tag{5.30}$$

which is also conserved: $\Big(\mathcal{G}^{(1)}+\mathcal{G}^{(2)}+\mathcal{G}^{(3)}\Big)|v_3\rangle_{123}=0$, generate an algebra:

$$[\mathcal{S}_1\,,\,\mathcal{G}\,]=2\mathcal{S}_2\,,\quad [\mathcal{S}_2\,,\,\mathcal{G}\,]=2\mathcal{S}_1\,,\quad [\mathcal{S}_1\,,\,\mathcal{S}_2\,]=-2\mathcal{G}\,. \tag{5.31}$$

where $\mathcal{S}_2=\sum_{n=1}^{\infty}\Big(\frac{1}{n}b_{-n}b_n+nc_{-n}c_n\Big)$. These relations show that $\{\mathcal{S}_1,\mathcal{S}_2,\mathcal{G}\}$ generate the algebra of su(1,1). In summary, the three string vertex is fully su(1,1) invariant. Defining $X=(\mathcal{S}_2-\mathcal{S}_1)/2$, $Y=(\mathcal{S}_2+\mathcal{S}_1)/2$, and $H=\mathcal{G}$ we recover the conventional definition of the isomorphic (real) Lie algebra $sl(2,R)$, with brackets $[X,Y]=H,\ [H,X]=2X,\ [H,Y]=-2Y$. One can verify that $T_+=-2X$ is the operator multiplying b_0 in the BRST operator. Actually T_+ is conserved on the full (gauge invariant) string field vertex. This follows because $T_+\sim\{Q,c_0\}$ and both Q and c_0 are conserved on the three string vertex [27].

It is now simple to argue that the tachyon condensate can be restricted to su(1,1) singlets. This implies a very large trimming of the tachyon string field.

6 Final Remarks

In this set of lectures I have only covered a subset of the important developments that have occurred in the study of tachyon condensation. The reader may be interested in the study of lump solutions and marginal deformations[30, 31, 32, 33, 34, 35], or in developments in

BSFT and models of tachyon condensation [36, 37, 38, 39, 40, 41]. More recently, a direct approach to the study of the tachyon vacuum has been proposed in [42, 43]. Additionally, the reader should look into the insights non-commutative geometry gives into tachyon condensation, as reviewed in Harvey's lectures. Approaches to superstring field theory were discussed in Berkovits's lectures, and further analysis of the tachyon potential was given in Taylor's lecture.

Acknowledgements: I am grateful to Yoichi Kazama for his invitation to lecture at the Komaba Workshop and for this kind hospitality during my visit to Tokyo.

References

[1] M. Kaku and K. Kikkawa, *The Field Theory Of Relativistic Strings, Pt. 1: Trees,* Phys. Rev. D **10**, 1110 (1974); *The Field Theory Of Relativistic Strings. Pt. 2: Loops And Pomerons,* Phys. Rev. D **10**, 1823 (1974).

[2] S. Mandelstam, *Interacting String Picture Of Dual Resonance Models,* Nucl. Phys. **B64**, 205 (1973).

[3] W. Siegel, *Covariantly Second Quantized String,* Phys. Lett. **B142**, 276 (1984); *Covariantly Second Quantized String. 2,* Phys. Lett. **B149**, 157 (1984); *Covariantly Second Quantized String. 3,* Phys. Lett. **B149**, 162 (1984).

[4] W. Siegel and B. Zwiebach, *"Gauge String Fields,* Nucl. Phys. **B263**, 105 (1986).

[5] T. Banks and M. E. Peskin, *"Gauge Invariance Of String Fields,* Nucl. Phys. **B264**, 513 (1986).

[6] E. Witten, *Noncommutative Geometry And String Field Theory,* Nucl. Phys. **B268**, 253 (1986).

[7] M. Saadi and B. Zwiebach, *Closed String Field Theory From Polyhedra,* Annals Phys. **192**, 213 (1989).

[8] M. Kaku, *Geometric Derivation Of String Field Theory From First Principles: Closed Strings And Modular Invariance,* Phys. Rev. D **38**, 3052 (1988); M. Kaku and J. Lykken, *"Modular Invariant Closed String Field Theory,* Phys. Rev. D **38**, 3067 (1988).

[9] T. Kugo, H. Kunitomo and K. Suehiro, *Nonpolynomial Closed String Field Theory,* Phys. Lett. **B226**, 48 (1989); T. Kugo and K. Suehiro, *Nonpolynomial Closed String Field Theory: Action And Its Gauge Invariance,* Nucl. Phys. **B337**, 434 (1990).

[10] B. Zwiebach, *Closed string field theory: Quantum action and the BV master equation.* Nucl.Phys. **B390**(1993) 33-152, hep-th/9206084.

[11] E. Witten, *Interacting Field Theory Of Open Superstrings,* Nucl. Phys. **B276**, 291 (1986).

[12] C. Wendt, *Scattering amplitudes and contact interactions in Witten's superstring field theory*, Nucl. Phys. **B 314** 209 (1989).
J. Greensite and F.R. Klinkhamer, *Superstring Amplitudes and Contact Interactions,* Nucl. Phys. **B304** (1988) 108.

[13] I. Y. Arefeva, P. B. Medvedev and A. P. Zubarev, *Background Formalism For Superstring Field Theory,* Phys. Lett. **B240**, 356 (1990);
I. Y. Arefeva, P. B. Medvedev and A. P. Zubarev, *New Representation For String Field Solves The Consistence Problem For Open Superstring Field,* Nucl. Phys. **B341**, 464 (1990);
C. R. Preitschopf, C. B. Thorn and S. A. Yost, *Superstring Field Theory,* Nucl. Phys. **B337**, 363 (1990).

[14] I. Y. Aref'eva, D. M. Belov, A. S. Koshelev and P. B. Medvedev, *Tachyon condensation in cubic superstring field theory,* hep-th/0011117.

[15] N. Berkovits, *Super-Poincare Invariant Superstring Field Theory,* Nucl. Phys. **B450** (1995) 90, [hep-th/9503099];
A New Approach to Superstring Field Theory, proceedings to the 32^{nd} International Symposium Ahrenshoop on the Theory of Elementary Particles, Fortschritte der Physik (Progress of Physics) **48** (2000) 31, [hep-th/9912121].

[16] N. Berkovits and B. Zwiebach, *On the picture dependence of Ramond-Ramond cohomology,* Nucl. Phys. **B523**, 311 (1998) [hep-th/9711087]

[17] K. Junemann and O. Lechtenfeld, *"Chiral BRST cohomology of $N = 2$ strings at arbitrary ghost and picture number,* Commun. Math. Phys. **203**, 53 (1999) [hep-th/9712182].

[18] A. Sen and B. Zwiebach, *Background independent algebraic structures in closed string field theory,* Commun. Math. Phys. **177**, 305 (1996) [hep-th/9408053]; A. Sen and

B. Zwiebach, *Quantum background independence of closed string field theory,* Nucl. Phys. **B423**, 580 (1994) [hep-th/9311009]; A. Sen and B. Zwiebach, *A Proof of local background independence of classical closed string field theory,* Nucl. Phys. **B414**, 649 (1994) [hep-th/9307088].

[19] E. Witten, *On background independent open string field theory*, Phys. Rev. **D46**, 5467 (1992) [hep-th/9208027]; E. Witten, *Some computations in background independent off-shell string theory*, Phys. Rev. **D47**, 3405 (1993) [hep-th/9210065]; K. Li and E. Witten, *Role of short distance behavior in off-shell open string field theory*, Phys. Rev. **D48**, 853 (1993) [hep-th/9303067]; S.L. Shatashvili, *Comment on the background independent open string theory*, Phys. Lett. **B311**, 83 (1993) [hep-th/9303143]; S.L. Shatashvili, *On the problems with background independence in string theory*, hep-th/9311177.

[20] B. Zwiebach, *Building string field theory around non-conformal backgrounds,* Nucl. Phys. **B480**, 541 (1996) [hep-th/9606153].

[21] A. Sen, *Descent relations among bosonic D-branes*, Int. J. Mod. Phys. **A14**, 4061 (1999) [hep-th/9902105].

[22] A. Sen, *Non-BPS states and branes in string theory,* hep-th/9904207.

[23] V.A. Kostelecky and S. Samuel, *The Static Tachyon Potential in the Open Bosonic String Theory,* Phys. Lett. **B207** (1988) 169.

[24] A. Sen and B. Zwiebach, *Tachyon Condensation in String Field Theory*, JHEP **0003**, 002 (2000) [hep-th/9912249].

[25] N. Moeller and W. Taylor, *Level truncation and the tachyon in open bosonic string field theory*, Nucl. Phys. **B583**, 105 (2000) [hep-th/0002237].

[26] A. Sen, *Universality of the tachyon potential,* JHEP **9912**, 027 (1999) [hep-th/9911116].

[27] L. Rastelli and B. Zwiebach, *Tachyon potentials, star products and universality,* hep-th/0006240.

[28] H. Hata and S. Shinohara, *BRST invariance of the non-perturbative vacuum in bosonic open string field theory,* JHEP **0009**, 035 (2000) [hep-th/0009105].

[29] B. Zwiebach, "Trimming the tachyon string field with SU(1,1)," hep-th/0010190.

[30] J.A. Harvey and P. Kraus, *D-Branes as unstable lumps in bosonic open string field theory*, JHEP **0004**, 012 (2000) [hep-th/0002117].

[31] B. Zwiebach, *A solvable toy model for tachyon condensation in string field theory*, JHEP **0009**, 028 (2000) [hep-th/0008227].

[32] N. Moeller, A. Sen and B. Zwiebach, *D-branes as tachyon lumps in string field theory,* hep-th/0005036.

[33] R. de Mello Koch and J.P. Rodrigues, *Lumps in level truncated open string field theory*, hep-th/0008053.

[34] N. Moeller, *Codimension two lump solutions in string field theory and tachyonic theories*, hep-th/0008101.

[35] A. Sen and B. Zwiebach, *Large marginal deformations in string field theory,* JHEP **0010**, 009 (2000) [hep-th/0007153].

[36] A.A. Gerasimov and S.L. Shatashvili, *On exact tachyon potential in open string field theory*, hep-th/0009103.

[37] D. Kutasov, M. Marino and G. Moore, *Some exact results on tachyon condensation in string field theory*, hep-th/0009148.

[38] D. Kutasov, M. Marino and G. Moore, "Remarks on tachyon condensation in superstring field theory," hep-th/0010108.

[39] J.A. Minahan and B. Zwiebach, *Field theory models for tachyon and gauge field string dynamics*, JHEP **0009**, 029 (2000) [hep-th/0008231].

[40] J. A. Minahan and B. Zwiebach, *Effective tachyon dynamics in superstring theory,* hep-th/0009246.

[41] J. A. Minahan and B. Zwiebach, *Gauge fields and fermions in tachyon effective field theories,* hep-th/0011226.

[42] L. Rastelli, A. Sen and B. Zwiebach, *Classical solutions in string field theory around the tachyon vacuum,* hep-th/0102112.

[43] L. Rastelli, A. Sen and B. Zwiebach, *String field theory around the tachyon vacuum,* hep-th/0012251.