Introduction to Open Superstring Field Theory

Nathan Berkovits[1]

Instituto de Física Teórica, Universidade Estadual Paulista
Rua Pamplona 145, 01405-900, São Paulo, SP, Brasil

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Abstract

I review the construction of an action for open superstring field theory which does not suffer from the contact term problems of other approaches. This action resembles a Wess-Zumino-Witten action and can be constructed in a manifestly d=4 super-Poincaré covariant manner.

[1] e-mail: nberkovi@ift.unesp.br

1 Problems with Conventional Approach

The construction of a field theory action for the superstring is an important problem since it may lead to information about non-perturbative superstring theory which is unobtainable from the on-shell perturbative S-matrix. This information might be useful for understanding the non-perturbative dualities of the superstring. Although there was much activity ten years ago concerning a field theory action for the bosonic string, there was not much progress on constructing a field theory action for the superstring.

The covariant string field theory action for the bosonic string is based on a BRST operator Q and a string field V of $+1$ ghost-number. In Witten's approach to open string field theory, the gauge-invariant action is [1]

$$S_{open} = Tr\langle \frac{1}{2} V Q V + \frac{1}{3} V^3 \rangle \tag{1}$$

where string fields are glued together at their midpoint.

In generalizing this approach to superstring field theory, the main difficulty comes from the requirement that the string field carries a definite "picture". Recall that each physical state of the superstring is represented by an infinite number of BRST-invariant vertex operators in the covariant RNS formalism [2]. To remove this infinite degeneracy, one needs to require that the vertex operator carries a definite picture, identifying which modes of the (β, γ) ghosts annihilate the vertex operator. For open superstring fields, the most common choice is that all Neveu-Schwarz (NS) string fields carry picture -1 and all Ramond (R) string fields carry picture $-\frac{1}{2}$.

Since the total picture must equal -2 for open superstrings, the obvious generalization of the action (1) is [3]

$$S = Tr\langle \frac{1}{2} V_{NS} Q V_{NS} + \frac{1}{2} V_R Q Y V_R + \frac{1}{3} Z\ V_{NS}^3 + \frac{1}{2} V_{NS} V_R V_R \rangle \tag{2}$$

where $Z = \{Q, \xi\}$ is the picture-raising operator of picture $+1$, $Y = c\partial\xi e^{-2\phi}$ is the picture-lowering operator of picture -1, and these picture-changing operators are inserted at the midpoint of the interacting strings. However, as shown by Wendt [4], the action of (2) is not gauge-invariant because of the contact-term divergences occuring when two Z's collide. One way to make the action gauge-invariant would be to introduce contact terms to cancel the divergences coming from colliding Z's. However, the coefficients of these contact terms would have to be infinite in the classical action since the divergences are present already in tree-level amplitudes. Note that infinite contact terms are also expected in light-cone superstring field theory (either in the RNS or Green-Schwarz formalisms) to cancel the divergences when interaction points collide [5].

Although one can choose other pictures for the string field V which change the relative factors of Z and Y [6][7], there is no choice for which the action is cubic and gauge-invariant [8]. For example, choosing V_{NS} in the zero picture leads to the action [9] [2]

$$S = Tr\langle \frac{1}{2} V_{NS} Y^2 Q V_{NS} + \frac{1}{2} V_R Q Y V_R + \frac{1}{3} Y^2 \ V_{NS}^3 + \frac{1}{2} Y V_{NS} V_R V_R \rangle. \qquad (3)$$

The kinetic term of (3) implies that $Y^2 Q V_{NS} = 0$ is the linearized equation of motion for the Neveu-Schwarz field. But since Y^2 has a non-trivial kernel, this equation of motion has additional solutions which are not in the cohomology of Q. Although one could restrict V_{NS} to only include states not in the kernel of Y^2, such a projection would break gauge-invariance since Witten's midpoint gluing prescription does not preserve this projection of the string field. Modifying the gluing prescription to preserve the projection would ruin its associativity properties. Note that a similar problem [13] exists for the Ramond kinetic term in the action of (2).

2 Open Neveu-Schwarz String Field Theory

In this section, it will be shown how to construct a ten-dimensional Lorentz-covariant action for the Neveu-Schwarz sector of open superstring field theory. It is not yet know how to extend this action to the Ramond sector in a ten-dimensional Lorentz-covariant manner. However, as will be shown in the next

[2] This action was recently used to compute the tachyon potential in NS string theory. However, besides the gauge invariance problems mentioned here, there appears to be an error in their computation of the D-brane tension by a factor of $\sqrt{2}$. When written in terms of the closed string coupling constant, the tension is background-dependent and picks up a factor of $\sqrt{2}$ if the D-brane is non-BPS. But when written in terms of the open string coupling constant, the tension is background independent and does not pick up a factor of $\sqrt{2}$ [10][11][12].

section, it can be generalized to a four-dimensional super-Poincaré covariant action which includes both the NS and R sectors.

To construct a field theory action, one first needs to define a NS string field. The first attempts to construct an action used the fermionic string field

$$V = ce^{-\phi}\psi^\mu A_\mu(x) + ...$$

where V was constrained to carry ghost number $+1$ and picture -1, and to satisfy $\eta_0 V = 0$, i.e. to be independent of the ξ_0 mode. Although quadratic actions were succesfully constructed using V, the cubic interaction term had problems due to the necessity of introducing the picture-raising operator Z.

The solution to this problem is to express V and ZV in terms of a more fundamental NS string field Φ which is bosonic and carries zero ghost-number and zero picture. If one defines $\Phi = \xi_0 V$, i.e.

$$\Phi = \xi ce^{-\phi}\psi^\mu A_\mu(x) + ...,$$

then $V = \eta_0\Phi$ and $ZV = Q\Phi$. However, as will be shown below, $\Phi = \xi_0 V$ is a specific gauge choice for Φ, and one needs a more gauge-invariant description for Φ in order to construct an NS string field theory action without contact term problems.

The quadratic action for Φ will be defined as

$$S = Tr\langle \frac{1}{2}\Phi\ Q\eta_0\ \Phi\rangle.$$

Because of the ξ zero mode, the non-vanishing inner product will be defined as in the "large" Hilbert space of [2], i.e.

$$\langle c\partial c\partial^2 c\xi e^{-2\phi}\rangle = 1.$$

Since $Q^2 = \eta_0^2 = \{Q, \eta_0\} = 0$, S is invariant under the linearized gauge transformation

$$\delta\Phi = \eta_0\tilde\Lambda + Q\Lambda$$

where $\tilde\Lambda$ and Λ are independent gauge parameters. Note that Φ carries picture 0, η_0 carries picture -1 and Q carries picture 0, so $\tilde\Lambda$ and Λ must carry picture $+1$ and 0 respectively.

Since $\{\eta_0, \xi_0\} = 1$, the $\tilde\Lambda$ parameter can be used to gauge $\Phi = \xi_0 V$ for some V. The equation of motion for S is

$$Q\eta_0\Phi = 0,$$

which in this gauge implies that

$$Q\eta_0(\xi_0 V) = QV = 0.$$

Furthermore, the remaining gauge parameter Λ generates the gauge transformation

$$\delta V = \eta_0(Q\Lambda) = Q\Omega$$

if one chooses $\Lambda = \xi_0\Omega$. So one recovers the desired linearized equations of motion and gauge invariances for V.

To include interactions, one needs to find an action which allows a non-linear generalization of the gauge invariances $\delta\Phi = \eta_0\tilde{\Lambda} + Q\Lambda$. This can be obtained by drawing an analogy with the two-dimensional Wess-Zumino-Witten (WZW) action

$$S_{WZW} = \frac{1}{2}Tr\int d^2z((g^{-1}\partial g)(g^{-1}\bar{\partial}g) - \int_0^1 dt(\hat{g}^{-1}\partial_t\hat{g})[\hat{g}^{-1}\partial\hat{g}\ ,\ \hat{g}^{-1}\bar{\partial}\hat{g}])$$

where $g(z,\bar{z})$ is a group-valued two-dimensional field and $\hat{g}(t,z,\bar{z})$ is any continuous group-valued three-dimensional field defined on the three-volume with boundary at $t=0$ and $t=1$ such that $\hat{g}(1,z,\bar{z}) = g(z,\bar{z})$ and $\hat{g}(0,z,\bar{z}) = 1$. Recall that S_{WZW} is invariant under the gauge transformation

$$g(z,\bar{z}) \to \bar{\Omega}(\bar{z})g(z,\bar{z}) + g(z,\bar{z})\Omega(z)$$

where $\partial\bar{\Omega}(\bar{z}) = \bar{\partial}\Omega(z) = 0$. If one writes $g = e^{\Phi}$ where Φ is Lie-algebra valued, the gauge transformation on Φ is $\delta\Phi = \bar{\Omega}(\bar{z}) + \Omega(z) + ...$ where ... depends on Φ. Furthermore, the WZW equation of motion $\bar{\partial}(g^{-1}\partial g) = 0$ implies that $\partial\bar{\partial}\Phi = ...$ where ... is non-linear in Φ.

This suggests writing the NS string field theory action as

$$S = \frac{1}{2}Tr\langle(e^{-\Phi}Qe^{\Phi})(e^{-\Phi}\eta_0 e^{\Phi}) - \int_0^1 dt(e^{-\hat{\Phi}}\partial_t e^{\hat{\Phi}})\{e^{-\hat{\Phi}}Qe^{\hat{\Phi}}\ ,\ e^{-\hat{\Phi}}\eta_0 e^{\hat{\Phi}}\}\rangle \quad (4)$$

where $e^{\Phi} = 1+\Phi+\frac{1}{2}\Phi*\Phi+...$ is defined using the midpoint gluing prescription, Φ is the NS string field discussed earlier, $\hat{\Phi}(t=0) = 0$ and $\hat{\Phi}(t=1) = \Phi$. One can show that S is invariant under the WZW-like gauge invariance

$$\delta(e^{\Phi}) = e^{\Phi}(\eta_0\tilde{\Lambda}) + (Q\Lambda)e^{\Phi},$$

and that on-shell,

$$\eta_0(e^{-\Phi}Qe^{\Phi}) = 0,$$

which generalize the linearized gauge invariance and equations of motion of the quadratic action.

To explicitly evaluate the action of (4), one first performs a Taylor expansion in Φ to obtain

$$S = Tr\langle \frac{1}{2}\Phi Q\eta_0\Phi - \frac{1}{6}\Phi\{Q\Phi, \eta_0\Phi\} + ...\rangle$$

where all string fields are multiplied together using the midpoint interaction. The cubic term can be evaluated in precisely the same manner as in the CS-like action by mapping three half circles for the external states into $2\pi/3$ wedges in the complex plane using the map

$$f_r^{(3)} = e^{\frac{2\pi i(r-1)}{3}}(\frac{1-iz}{1+iz})^{\frac{2}{3}}$$

for $r = 1$ to 3. To define the order N term, one uses the functions

$$f_r^{(N)} = e^{\frac{2\pi i(r-1)}{N}}(\frac{1-iz}{1+iz})^{\frac{2}{N}}$$

for $r = 1$ to N to map N half circles into $2\pi/N$ wedges in the complex plane. It has been shown by explicit computation [14] that the four-point tree amplitude is correctly reproduced by the action of (4), and there are indirect arguments based on gauge invariance that all N-point tree amplitudes are correctly reproduced by this action. The action of (4) has also been used to compute the NS tachyon potential [15][11][16][17] using the level truncation scheme of [18][19], and the results appear to agree with the predictions of Sen coming from D-brane analysis [10].

3 Open Superstring Field Theory

Although the action of the previous section is manifestly Lorentz invariant, it is not clear how to generalize it to include the Ramond sector. At the moment, the only action which includes all sectors of the open superstring is based on a hybrid formalism of the superstring with $\hat{c} = 2$ N=2 superconformal invariance. In order to understand the relation of this hybrid action with that of the previous section, it will be useful to first recall the relation of the bosonic open string field theory action and Chern-Simons theory.

As shown by Witten in [20], the action for open bosonic string field theory,

$$S = Tr\langle \frac{1}{2} VQV + \frac{1}{3} V^3 \rangle, \tag{5}$$

can also be used to describe the topological string theory version of Chern-Simons. This Chern-Simons string theory is defined by a $\hat{c} = 3$ $N = 2$ superconformal field theory constructed from the worldsheet variables $[x^j, \bar{x}_j, \psi^j, \bar{\psi}_j]$ for $j = 1$ to 3 with the twisted $N = 2$ generators:

$$T = \partial x^j \partial \bar{x}_j + \bar{\psi}_j \partial \psi^j,$$

$$G^+ = \psi^j \partial \bar{x}_j, \quad G^- = \bar{\psi}_j \partial x^j,$$

$$J = \psi^j \bar{\psi}_j.$$

If one identifies Q with $\int G^+$ and the ghost-number with U(1) charge, the action of (5) reproduces the Chern-Simon action

$$S_{CS} = Tr \int d^3x \epsilon^{jkl} [\frac{1}{2} A_j \partial_k A_l + \frac{1}{3} A_j A_k A_l].$$

To obtain the Chern-Simons action from (5), one uses the normalization that $\langle \psi^j \psi^k \psi^l \rangle = \epsilon^{jkl}$ and expands the ghost-number +1 string field V as

$$V = A_j(x)\psi^j + ... \quad .$$

The terms ... involve derivatives on ψ and x and describe massive fields which do not propagate. One can easily generalize the above construction to any $\hat{c} = 3$ N=2 superconformal field theory and the corresponding action computes topological quantities in the $N = 2$ superconformal field theory [21].

The action of (4) in the previous section involves operators Q and η_0, but η_0 has no analog in $\hat{c} = 3$ $N = 2$ superconformal field theory. However, as will now be shown, Q and η_0 do have analogs in $\hat{c} = 2$ $N = 2$ superconformal field theory, i.e. in critical $N = 2$ strings. When $\hat{c} = 2$, the operators J, $J^{++} = e^{\int J}$ and $J^{--} = e^{-\int J}$ generate an SU(2) affine Lie algebra. Commuting these SU(2) generators with the fermionic generators G^+ and G^- generate two new fermionic generators defined as

$$\tilde{G}^+ = [e^{\int J}, G^-], \quad \tilde{G}^- = [e^{-\int J}, G^+].$$

These four fermionic generators combine with the SU(2) generators and the stress tensor to form a set of "small" N=4 superconformal generators. After twisting by J, $(G^+, \tilde{G}^+)$ carry conformal weight +1 and $(G^-, \tilde{G}^-)$ carry conformal weight +2. So if the U(1) charge is identified with ghost-number, one can identify $\int G^+$ with Q and $\int \tilde{G}^+$ with η_0 [22].

There are three critical N=2 superconformal field theories which will be relevant here. The first is the self-dual string which describes self-dual Yang-Mills [23]. The worldsheet variables of the self-dual string consist of $[x^j, \bar{x}_j, \psi^j, \bar{\psi}_j]$ where $j = 1$ to 2. For the self-dual string, the N=4 generators after twisting are

$$\begin{gathered} T = \partial x^j \bar{x}_j + \bar{\psi}_j \partial \psi^j, \\ G^+ = \psi^j \partial \bar{x}_j, \quad \tilde{G}^+ = \epsilon_{jk} \psi^j \partial x^k, \\ G^- = \bar{\psi}_j \partial x^j, \quad \tilde{G}^- = \epsilon^{jk} \bar{\psi}_j \partial \bar{x}_k, \\ J^{++} = \frac{1}{2} \epsilon_{jk} \psi^j \psi^k, \quad J = \psi^j \bar{\psi}_j, \quad J^{--} = \frac{1}{2} \epsilon^{jk} \bar{\psi}_j \bar{\psi}_k. \end{gathered} \tag{6}$$

If one replaces Q with $\int G^+$ and η_0 with $\int \tilde{G}^+$ of (6), the action of (4) reproduces the Donaldson-Nair-Schiff action [24][25] for self-dual Yang-Mills,

$$S = \frac{1}{2} Tr \int d^4x ((e^{-\phi} \partial_j e^{\phi})(e^{-\phi} \partial^j e^{\phi}) + \int_0^1 dt (e^{-\hat{\phi}} \partial_t e^{\hat{\phi}})[e^{-\hat{\phi}} \partial_j e^{\hat{\phi}} \ , \ e^{-\hat{\phi}} \partial_j e^{\hat{\phi}}])$$

in terms of the Yang field $\phi(x)$. To obtains this action from (4), one uses the normalization that $\langle \psi^j \psi^k \rangle = \epsilon^{jk}$ and expands the ghost-number zero string field Φ as

$$\Phi = \phi(x) + ...$$

where the terms ... involve derivatives on ψ and x and correspond to non-propagating massive fields. Note that the equation of motion

$$\int \tilde{G}^+ (e^{-\Phi} \int G_+ e^{\Phi}) = 0$$

implies $\partial^j (e^{-\phi} \partial_j e^{\phi}) = 0$ which is Yang's equation for self-dual Yang-Mills.

A second critical N=2 superconformal field theory is given by the N=2 embedding of the RNS superstring. As shown in [27], any critical N=1 superconformal field theory (such as the ten-dimensional superstring) can be described by a critical N=2 superconformal field theory. The worldsheet fields of this critical N=2 superconformal field theory are the usual RNS worldsheet variables $[x^\mu, \psi^\mu, b, c, \xi, \eta, \phi]$ for $\mu = 0$ to 9 and the twisted N=4 generators are defined by [26][27]

$$\begin{gathered} T = T_{RNS}, \\ G^+ = j_{BRST}, \quad \tilde{G}^+ = \eta, \\ G^- = b, \quad \tilde{G}^- = \{Q, b\xi\} = bZ + \xi T_{RNS}, \\ J^{++} = c\eta, \quad J = bc + \xi\eta, \quad J^{--} = b\xi, \end{gathered} \tag{7}$$

where $Q = \int j_{BRST}$ is the standard BRST charge of the N=1 superstring and T_{RNS} is the sum of the stress tensors for the RNS matter and ghost variables. Since $Q = \int G^+$ and $\eta_0 = \int \tilde{G}^+$, this explains the relationship of the action in the previous section with the actions constructed in this section using the N=4 superconformal generators.

Finally, a third critical N=2 superconformal field theory is given by a hybrid version of the superstring which describes in $D = 4$ superspace the superstring compactified on a six-dimensional manifold [28]. The worldsheet variables of this superstring consist of $[x^m, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}, p_\alpha, \bar{p}_{\dot{\alpha}}, \rho]$ plus an N=2 c=9 superconformal field theory which describes the compactification manifold. p_α and $\bar{p}_{\dot{\alpha}}$ are the conjugate momenta to the superspace variables θ^α and $\bar{\theta}^{\dot{\alpha}}$, ρ is a chiral boson, $m = 0$ to 3 and $\alpha, \dot{\alpha} = 1$ to 2. As discussed in [28], these hybrid superstring variables are related to the RNS worldsheet variables by a field-redefinition.

In the hybrid version of the superstring, the twisted N=4 superconformal generators are

$$\begin{gathered} T = \frac{1}{2}\partial x^m \partial x_m + p_\alpha \partial\theta^\alpha + \bar{p}_{\dot{\alpha}} \partial\bar{\theta}^{\dot{\alpha}} + \frac{1}{2}\partial\rho\partial\rho + \frac{1}{2}\partial^2\rho + T_C, \\ G^+ = d^\alpha d_\alpha e^\rho + G_C^+, \quad \tilde{G}^+ = [e^{\int J}, G^-], \\ G^- = \bar{d}^{\dot{\alpha}} \bar{d}_{\dot{\alpha}} e^{-\rho} + G_C^-, \quad \tilde{G}^- = [e^{-\int J}, G^+], \\ J^{++} = e^{-\rho + \int J_C}, \quad J = -\partial\rho + J_C, \quad J^{--} = e^{\rho - \int J_C}, \end{gathered} \tag{8}$$

where

$$d_\alpha = p_\alpha + i\bar\theta^{\dot\alpha}\partial x_{\alpha\dot\alpha} - \frac{1}{2}(\bar\theta)^2\partial\theta_\alpha + \frac{1}{4}\theta_\alpha\partial(\bar\theta)^2,$$

$$\bar d_{\dot\alpha} = \bar p_{\dot\alpha} + i\theta^{\alpha}\partial x_{\alpha\dot\alpha} - \frac{1}{2}(\theta)^2\partial\bar\theta_{\dot\alpha} + \frac{1}{4}\bar\theta_{\dot\alpha}\partial(\theta)^2$$

are spacetime supersymmetric combinations of the fermionic momenta and $[T_C, G_C^+, G_C^-, J_C]$ are the c=9 N=2 superconformal generators representing the compactification.

In the RNS formalism, one needs to choose a picture for each off-shell state in the string field theory action. As discussed in [29], this choice is replaced in the hybrid formalism by restricting the ρ charge for any off-shell state to be $(-1, 0, +1)$. So the U(1)-neutral off-shell string field will be written in an $N = 1$ $d = 4$ super-Poincaré invariant manner as

$$\Phi = \Phi_{-1} + \Phi_0 + \Phi_1$$

where Φ_n are string fields carrying n units of ρ charge and $-n$ units of Calabi-Yau U(1) charge. In the massless sector, the four-dimensional super-Yang-Mills multiplet is contained in Φ_0 and the Calabi-Yau chiral and anti-chiral moduli are contained in Φ_{-1} and Φ_1.

Because of the different amounts of ρ charge, the linearized equations of motion $G^+\tilde G^+\Phi = 0$ generalize in the hybrid formalism to[29]

$$\tilde G_4^+ G_4^+ \Phi_{-1} + \tilde G_4^+ G_C^+ \Phi_0 + \tilde G_4^+ \tilde G_C^+ \Phi_1 = 0, \tag{9}$$

$$(\tilde G_C^+ G_C^+ + \tilde G_4^+ G_4^+)\Phi_0 + \tilde G_4^+ G_C^+ \Phi_1 + \tilde G_C^+ G_4^+ \Phi_{-1} = 0,$$

$$G_4^+ G_C^+ \Phi_{-1} + G_4^+ \tilde G_C^+ \Phi_0 + G_4^+ \tilde G_4^+ \Phi_1 = 0,$$

where $G_4^+ = d^\alpha d_\alpha e^\rho$, $\tilde G_4^+ = e^{-2\rho + \int J_C}\bar d^{\dot\alpha}\bar d_{\dot\alpha}$, $\tilde G_C^+ = [e^{-\rho+\int J_C}, G_C^-]$, and G^+ and $\tilde G^+$ will always mean $\int G^+$ and $\int \tilde G^+$ for the rest of this paper. Furthermore, the linearized gauge invariance $\delta\Phi = G^+\Lambda + \tilde G^+\tilde\Lambda$ generalizes in the hybrid formalism to

$$\delta\Phi_{-1} = G_4^+\Lambda_{-2} + G_C^+\Lambda_{-1} + \tilde G_C^+\Lambda_0 + \tilde G_4^+\Lambda_1, \tag{10}$$

$$\delta\Phi_0 = G_4^+\Lambda_{-1} + G_C^+\Lambda_0 + \tilde G_C^+\Lambda_1 + \tilde G_4^+\Lambda_2,$$

$$\delta\Phi_1 = G_4^+\Lambda_0 + G_C^+\Lambda_1 + \tilde G_C^+\Lambda_2 + \tilde G_4^+\Lambda_3$$

where Λ_n are gauge parameters carrying n units of ρ charge and $-1-n$ units of Calabi-Yau charge. Note that the cohomology of $\tilde{G}_4^+$ and G_4^+ is trivial, so the gauge transformations generated by Λ_3 and Λ_{-2} allow one to describe Φ_1 and Φ_{-1} using the gauge invariant string fields defined by

$$\Omega = \tilde{G}_4^+ \Phi_1, \quad \bar{\Omega} = G_4^+ \Phi_{-1}.$$

It was shown in [29] that the cohomology of the equations of motion and gauge invariances of (9) and (10) correctly reproduces the physical superstring spectrum.

The action which generates the non-linear version of these equations of motion and gauge invariances can be guessed using the action for the massless sector of the superstring. For the massless sector of the uncompactified superstring, the $d = 10$ super-Yang-Mills fields decompose into the four-dimensional superfields $V(x,\theta,\bar{\theta})$, $\omega^j(x,\theta)$ and $\bar{\omega}_j(x,\bar{\theta})$ for $j = 1$ to 3 where ω^j and $\bar{\omega}_j$ are covariantly chiral and anti-chiral superfields satisfying $\bar{D}_{\dot{\alpha}}\omega^j = e^{-V} D_\alpha (e^V \bar{\omega}_j) = 0$. The $\theta\bar{\theta}$ component of V is the four-dimensional gauge field while the lowest components of ω^j and $\bar{\omega}_j$ are the other six polarizations of the gauge field. These massless fields appear in the string fields as

$$\Phi_0 = V + ..., \quad \Omega = \frac{1}{2}\epsilon_{jkl} e^{-\rho} \omega^j \psi^k \psi^l + ..., \quad \bar{\Omega} = \bar{\omega}_j \psi^j + ...,$$

where $\bar{\psi}_j$ and ψ^j are the worldsheet fermions in the internal directions. One can easily check that the linearized equations of motion and gauge invariances of (9) and (10) reproduce the desired equations for these massless superfields.

As shown in [30], the complete action for ten-dimensional super-Yang-Mills can be written in terms of these four-dimensional superfields as

$$S = \frac{1}{2} \int d^{10}x [\, -2 \int d^2\theta \, W^\alpha W_\alpha \tag{11}$$

$$+ \int d^4\theta \left((e^{-V} \bar{\partial}_j e^V)(e^{-V} \partial^j e^V) - \int_0^1 dt (e^{-\hat{V}} \partial_t e^{\hat{V}}) \{ e^{-\hat{V}} \bar{\partial}_j e^{\hat{V}}, e^{-\hat{V}} \partial^j e^{\hat{V}} \}) \right)$$

$$+2 \int d^4\theta \left((\partial^j e^{-V}) \bar{\omega}_j e^V + e^V \omega^j (\bar{\partial}_j e^{-V}) + e^{-V} \bar{\omega}_j e^V \omega^j \right)$$

$$+ \int d^2\theta \epsilon_{jkl} (\omega^j \partial^k \omega^l + \frac{2}{3} \omega^j \omega^k \omega^l) + \int d^2\bar{\theta} \epsilon^{jkl} (\bar{\omega}_j \bar{\partial}_k \bar{\omega}_l - \frac{2}{3} \bar{\omega}_j \bar{\omega}_k \bar{\omega}_l) \,]$$

where $W_\alpha = \bar{D}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} (e^{-V} D_\alpha e^V)$ is the spinor field strength.

¿From the above action and using the normalization that

$$\langle (\theta)^2 (\bar\theta)^2 e^{-\rho} \psi^j \psi^k \psi^l \rangle = \epsilon^{jkl},$$

one can easily guess that the superstring field theory action is

$$S = \frac{1}{2} Tr \langle \ (e^{-\Phi_0} G_4^+ e^{\Phi_0})(e^{-\Phi_0} \tilde{G}_4^+ e^{\Phi_0}) \tag{12}$$

$$- \int_0^1 dt (e^{-\hat\Phi_0} \partial_t e^{\hat\Phi_0}) \{ e^{-\hat\Phi_0} G_4^+ e^{\hat\Phi_0}, e^{-\hat\Phi_0} \tilde{G}_4^+ e^{\hat\Phi_0} \}$$

$$+ (e^{-\Phi_0} G_C^+ e^{\Phi_0})(e^{-\Phi_0} \tilde{G}_C^+ e^{\Phi_0}) - \int_0^1 dt (e^{-\hat\Phi_0} \partial_t e^{\hat\Phi_0}) \{ e^{-\hat\Phi_0} G_C^+ e^{\hat\Phi_0}, e^{-\hat\Phi_0} \tilde{G}_C^+ e^{\hat\Phi_0} \}$$

$$+2 \left((\tilde{G}_C^+ e^{-\Phi_0}) \bar\Omega e^{\Phi_0} + e^{\Phi_0} \Omega (G_C^+ e^{-\Phi_0}) + e^{-\Phi_0} \bar\Omega e^{\Phi_0} \Omega \right)$$

$$- (\Omega \tilde{G}_C^+ \Phi_1 - \frac{2}{3} \Omega \Omega \Phi_1) + (\bar\Omega G_C^+ \Phi_{-1} + \frac{2}{3} \bar\Omega \bar\Omega \Phi_{-1}) \ \rangle.$$

The only subtle part of this generalization is that unlike the point-particle action, a WZW term is used for both the four-dimensional and six-dimensional parts of the string action for V. Note also that chiral F-terms in the point-particle action are annihilated by $\tilde{G}_4^+$ in the string action and, because $\tilde{G}_4^+$ has trivial cohomology, can be turned into D-terms by pulling $\tilde{G}_4^+$ off one of the Ω's. Since $\tilde{G}_4^+$ is the inverse of the ξ zero mode, turning F-terms into D-terms is like going from the small to the large RNS Hilbert space. To show that the superstring field theory action of (12) is correct, one can check that its linearized equations of motion and gauge invariances reproduce the on-shell conditions of physical vertex operators, and that the cubic term in the action produces the three-point tree-level scattering amplitude.

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