Activities of Matsuo's group

String Theory

Grand unification with gravity

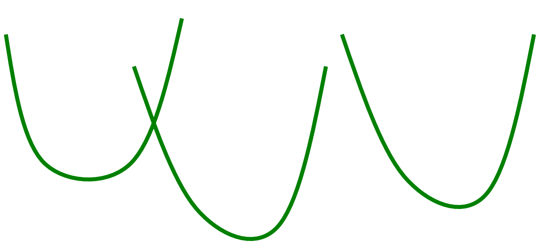
- *Compactification to four dimensions is needed*
- *Why specific compactification that reduces Standard model is selected?*
- *Nonperturbative off-shell formulation is needed*

Quantization of general relativity

✧*A framework that generalize Riemannian geometry is needed*

✧*T-duality, Noncommutative geometry*

Landscape of vacua



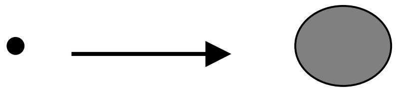
Modular invariance

→ ***Geometry should include full massive modes***

A First step toward stringy geometry

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}$$

- Inclusion of Kalb-Ramond Field $B_{\mu\nu}$
- Appearance of Moyal Plane



Noncommutative geometry

- Not so well-understood compared with Riemann geometry
- Topological charge = D-brane number
- Relation with K-theory in Mathematics
- D-brane on noncommutative torus

***Generalization of instanton number* (*= Chern class*)**

→ *Topological charges in operator theory* *(Matsuo)*

$$\int F^n \Leftrightarrow \mathrm{Tr}\pi(d\pi)^{2n}$$

$$\pi^2 = \pi$$: projection operator that represents D-brane

Why D-brane is represented as projector?

D-brane defines boundary condition for open string

✧Extended for Neumann
✧Pointlike for Dirichlet

$$\partial_\sigma X^\mu|_{\sigma=0,\pi} = 0 \quad : \quad \text{Neumann}$$
$$\partial_\tau X^\mu|_{\sigma=0,\pi} = 0 \quad : \quad \text{Dirichlet}$$

Distribution function

Commutative case:

$$f(x) = 1 \quad : \text{Neumann}$$
$$f(x) = \delta(x - x_0) \quad : \text{Dirichlet}$$

Noncommutative case:

$$f(x) \propto e^{-x^2/2\theta} \quad : \text{Neumann}$$

$$\longrightarrow \quad f(x) \star f(x) \propto f(x)$$

Noncommutative torus *(with T. Takayanagi, M. Kajiura, I. Bars)*

$\theta = p/q \longrightarrow$ reduces to matrix model

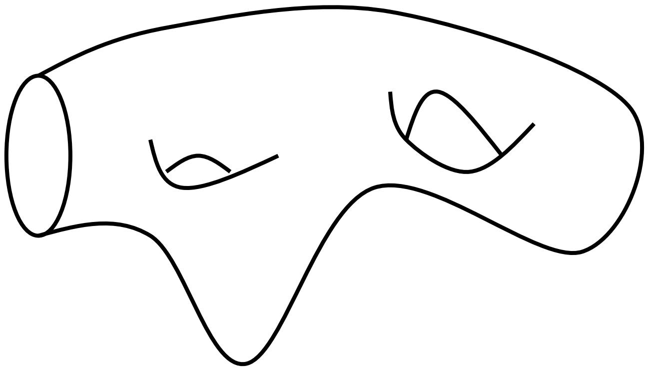
$\theta =$ irrational $\longrightarrow$ enjoys T-duality: nontrivial

✧*Nontrivial topological classes*
✧*Zero mode knows stringy T-duality symmetry*

Noncommutative p-brane *(with Y. Shibusa)*

$$S \propto \int \epsilon_{\mu_1 \cdots \mu_{p+2}} X^{\mu_1} dX^{\mu_2} \wedge \cdots \wedge dX^{\mu_{p+2}}$$

- *Generalization of topological open membrane*
- *Poisson bracket generates volume preserving diffeomorphism*
- *Natural realization of Nambu bracket*



Noncommutativity at the boundary appears whenever antisymmetric tensor field is nonvanishing

Limitation of noncommutativity

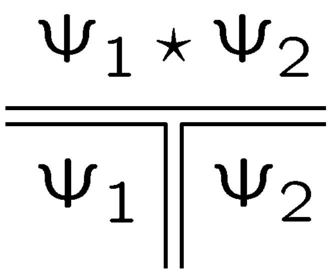
Although it captures some feature of string theory..
It has obvious drawbacks

- ✧ ***Only zero mode is present***
- ✧ ***Modular invariance does not exist***
- ✧ ***Difficult to cancel infinities and anomalies***

However, these are essential features of string theory!

In order to go beyond toy models, we have to include all the string massive modes into the geometry

→ Star product of open string field theory

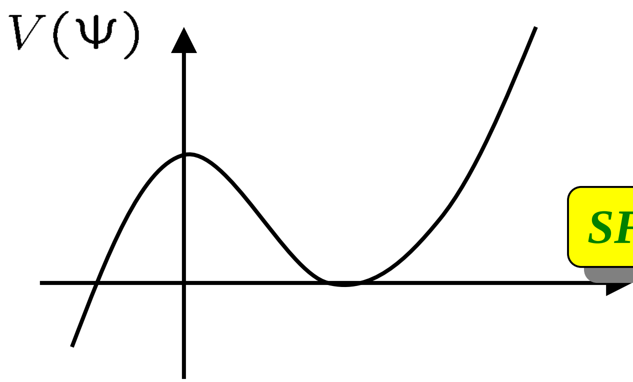


$\star$ *noncommutative and associative product*

String field theory

So far, unique nonperturbative formulation with all the string massive mode

$$S = \frac{1}{2}\int \Psi \star Q\Psi + \frac{g}{3}\int \Psi \star \Psi \star \Psi$$



$\Psi = 0$: Perturbative vacuum

$\Psi = \Psi_0$: No D-brane

SFT can describe creation/annihilation of D-branes

Moyal string field theory

(with I. Bars and I. Kishimoto)

- ✧ *Explicit mapping from Witten's star to Moyal's star*
- ✧ *Solved a subtlety at the midpoint → finite N approximation*
- ✧ *Simplification of the computation: No needs to use Neumann coefficients*
- ✧ *Many explicit calculation*
  - *Off-shell amplitudes*
  - *Tachyon vacuum*

Projector equation with all massive modes *(with I. Kishimoto and E. Watanabe)*

$$|B\rangle \star_c |B\rangle \propto |B\rangle$$

$\star_c$ *Star product for closed strings*

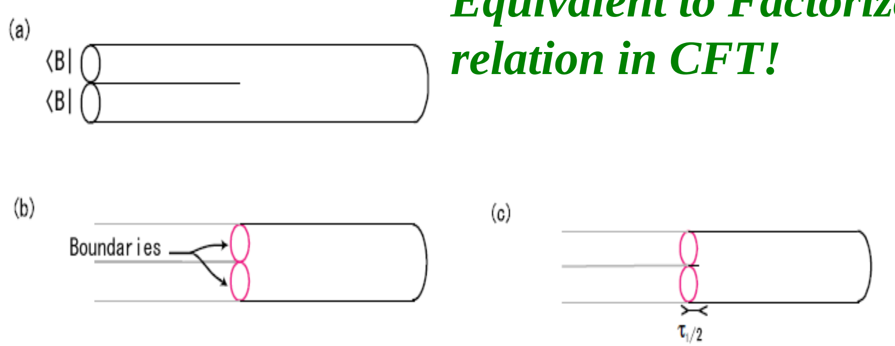
$|B\rangle$ *Boundary states*
(realization of boundary condition in closed string sector)

Modular dual description of the projector equation

It characterizes the D-brane remarkably although it is so simple

- *Every boundary states satisfies this equation*
- *Only boundary states satisfy this equation*

A simple geometrical picture



Equivalent to Factorization relation in CFT!

Discussion

Original goal:

Find an SFT with geometrical principle
Derive D-brane as their soliton solution

Where are we?

Noncommutative Geometry → T-duality can be derived
No massive mode included

Open string field theory → Explicit formula for off-shell formula
Analytic solution for vacuum
is not yet found

Closed string projector equation
→ Satisfactory to characterize D-brane
No geometrical/gauge principle