Kyoto Univ
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Boundary states as exact solutions of (vacuum) closed string field theory

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based on

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with

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§1 Introduction

Analysis of nonperturbative phenomena

Soliton solutions are essential

Magnetic monopole
Instanton
} for Yang-Mills

Blackhole for Einstein gravity

They are solutions of

nonlinear equation of motion

↳ Possible to carry nontrivial topological charge

Soliton solutions in string theory

Magnetic 5 brane

D-brane ⟸ completely stringy description is possible by

$\boxed{\text{Boundary state}}$

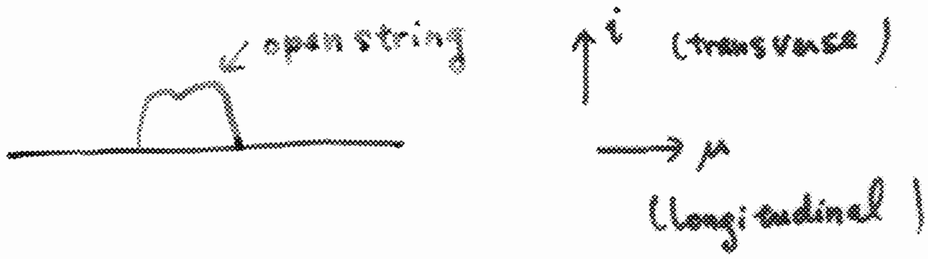
Boundary state

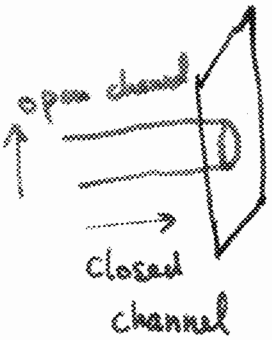
Implimentation of boundary conditions of *open* string in *closed* string Hilbert space

Example) D p-brane

$$\partial_\sigma X^\mu \Big|_{\sigma=0,\pi} = 0 \qquad \mu = 0, \cdots, p$$

$$\partial_\tau X^i \Big|_{\sigma=0,\pi} = 0 \qquad i = p+1, \cdots, d-1$$





$\Downarrow$ Modular transformation $(\sigma \leftrightarrow \tau)$

$$\begin{cases} \partial_\tau X^\mu \big|_{\tau=0,\pi} |B\rangle = 0 \\ \partial_\sigma X^i \big|_{\tau=0,\pi} |B\rangle = 0 \end{cases}$$

relations between left/right movers

$\Downarrow$

p=0 along Neumann direction

$$|B\rangle = \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} S_{\mu\nu}\, \alpha^\mu_{-n} \tilde{\alpha}^\nu_{-n} \right) |0; x^i\rangle$$

$X^i = x^i$ along Dirichlet direction

$$S_{\mu\nu} = \begin{pmatrix} -1 & & & & & \\ & -1 & & & & \\ & & \ddots & & & \\ & & & -1 & & \\ & & & & +1 & \\ & & & & & \ddots \\ & & & & & & +1 \end{pmatrix}$$

Note: *Background dependent* description

Background independent characterization

$$(L_n - \tilde{L}_{-n})|B\rangle = 0 \qquad \cdots\cdots \quad (*)$$

This linear condition is not enough!

Cardy condition

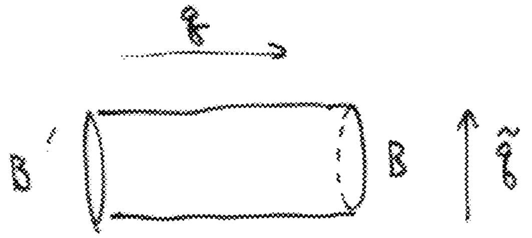
$$\langle B| q^{(L_0+\tilde{L}_0)/2} |B'\rangle = \chi(q)$$

$$\underset{\substack{\uparrow \\ \text{modular} \\ \text{transformation}}}{=} \sum_i n_{BB'}{}^i \chi_i(\tilde{q})$$

$\chi_i(\tilde{q})$ = Character of irreducible representation in open string channel

$$\boxed{n_{BB'}{}^i \ : \ \text{must be positive integer}}$$

$\Updownarrow$



nonlinear constraint which must be imposed with (*)

Can we translate it as the nonlinear equation in closed string field theory

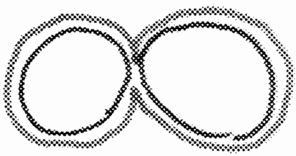
Main claim

Many (all ?) boundary states satisfy
universal nonlinear equation
(unipotency relation)

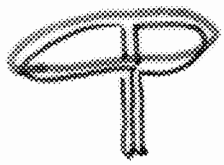
$$\boxed{|B\rangle * |B\rangle = (\text{constant})\, |B\rangle} \quad \cdots\cdots (*)$$

$*$: star product of closed string field theory

HIKKO type



Witten type

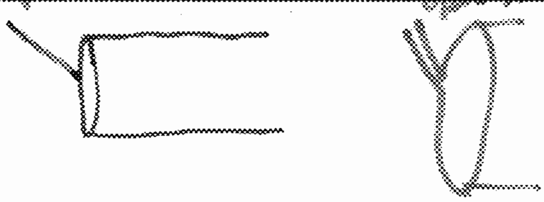


◦ Analogue of VSFT eq. of motion for open string

$$\mathcal{Q}\Psi + \Psi * \Psi = 0$$

$\mathcal{Q}$: pure ghost BRST operator

$$\left.\begin{array}{l}\text{Sliver} \\ \text{butterfly} \\ \text{identity}\end{array}\right\} \longleftrightarrow \text{Boundary state}$$



Variation of (*)

$$\delta |B\rangle = \oint \frac{d\sigma}{2\pi} V(\sigma) |B\rangle$$

Insertion of operators at the boundary

Note : Boundary state gives identification

$$X^L(\sigma) \Longleftrightarrow X^R(\sigma)$$

same as open string oscillator

Open string = Infinitesimal deformation of D-brane

example

$\phi'(x)$

Collective mode to describe transverse variation

We show that $\delta|B\rangle$ satisfies

$$|B\rangle * \delta|B\rangle + \delta|B\rangle * |B\rangle = (\text{const})\, \delta|B\rangle$$

iff

$\oint \frac{d\sigma}{2\pi} V(\sigma)$ is marginal deformation

$\Rightarrow$ unipotency relation (*) knows on-shell condition for open string

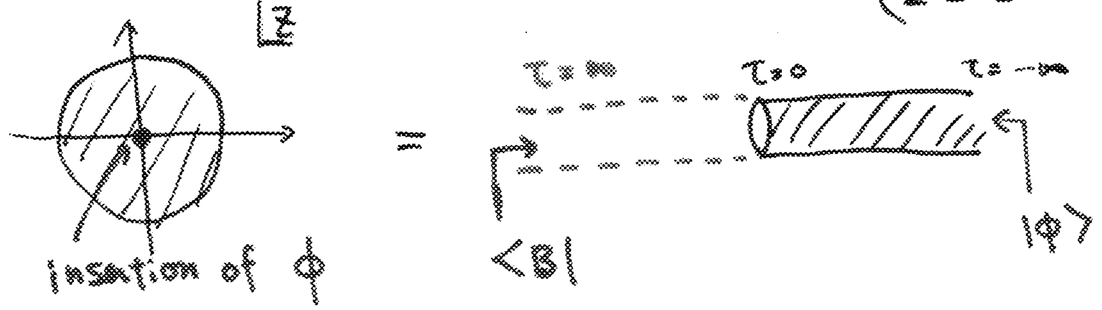
§2. Proof of unipotency relation

§2-1 Geometrical (Intuitive) proof

Boundary state

$\langle B|\phi\rangle$: 1 point function on Disk

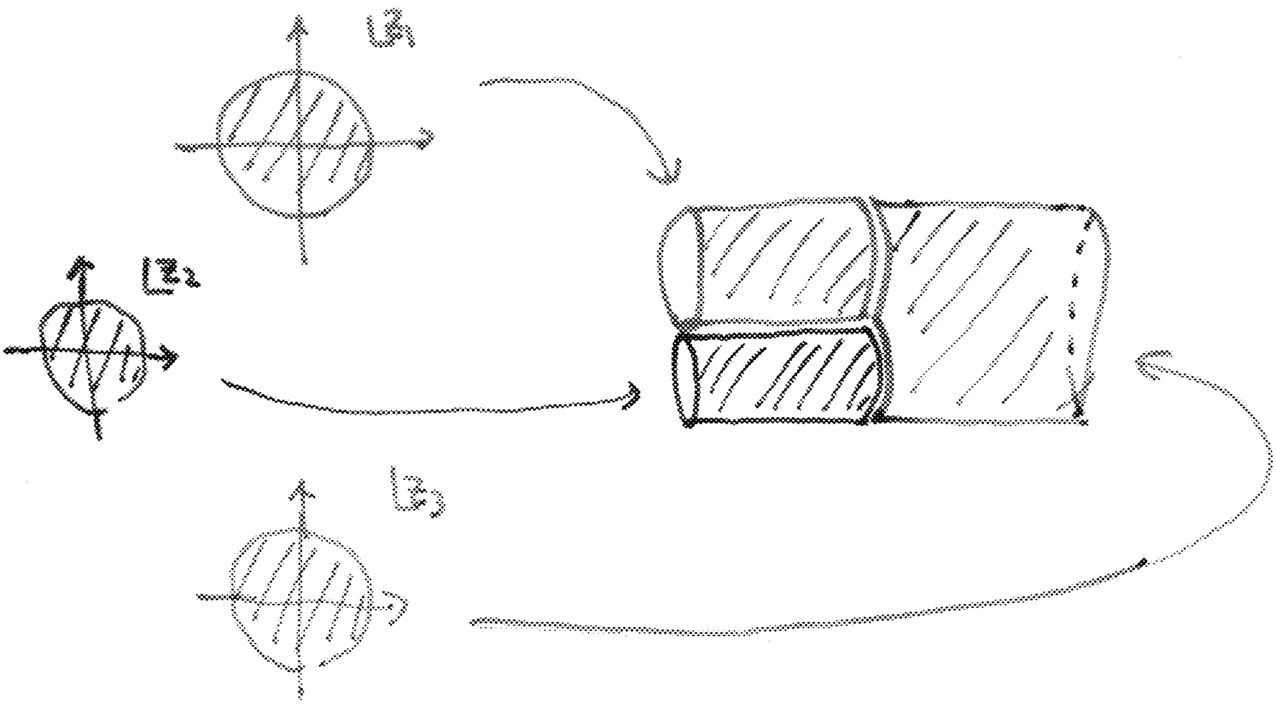
$(z = e^{\tau + i\theta})$



Role of boundary state

1. Cut the cylinder at $\tau = 0$
2. Set the boundary condition

3-string vertex (Lightcone type)

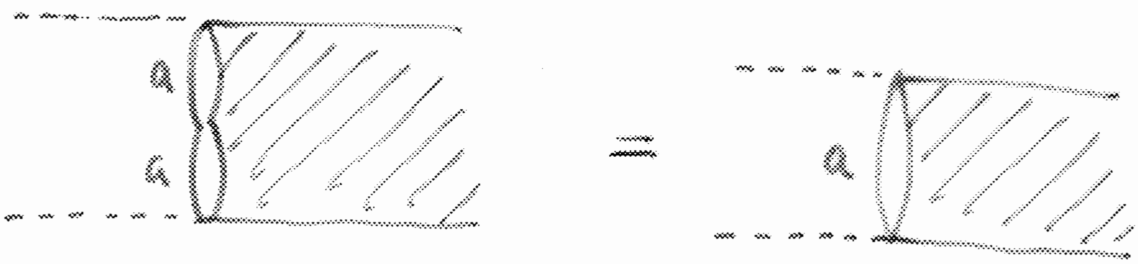


Patching together 3 world sheet

$|B_a\rangle * |B_b\rangle$

When $a = b$

$$|B_a\rangle * |B_a\rangle \propto |B_a\rangle \qquad (*)$$



- Boundary states are only states that have this property (stripping half cylinder)

$\Updownarrow$

$\boxed{\text{(*) unipotency relation is satisfied only by Boundary state ?}}$

§2.2 More explicit proof

① Boundary state (D p-brane with flux $F_{\mu\nu}$)

$$|B(F)\rangle = e^{-\sum_n a_n^{(+)\dagger} \cdot \mathcal{O} \cdot a_n^{(-)\dagger}} \, e^{\sum c_n^{(+)\dagger} \bar{c}_n^{(-)\dagger} + c_n^{(-)\dagger} \bar{c}_n^{(+)\dagger}} \, |p_\mu = 0, x^\perp\rangle$$

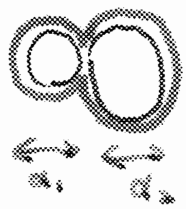
$$\cdot\ \mathcal{O}^\mu{}_\nu = \begin{cases} \left((1+F)^{-1}(1-F)\right)^\mu{}_\nu & \mu, \nu = 0, \cdots, p \\ -\delta^\mu{}_\nu & \mu, \nu = p+1, \cdots, d-1 \end{cases}$$

- $F_{\mu\nu}$: constant magnetic flux on D-brane
- $|x^\perp\rangle = \dfrac{1}{(2\pi)^{d-p-1}} \displaystyle\int d^{d-p-1} p^\perp \, e^{-ip x^\perp} \, |p^\perp\rangle$

To take HIKKO type $*$ product

- introduce α parameter

  (direct product with $|\alpha\rangle$)



- adjust ghost zero mode

$$\boxed{|B(F)\rangle \Rightarrow \Phi_F(x^\perp; \alpha) \equiv \bar{c}_0 \, |B(F)\rangle \otimes |\alpha\rangle}$$

② HIKKO's star product

$$|V(1,2,3)\rangle = \int \delta(1,2,3)\, [\mu(1,2,3)]^2\, \wp^{(1)}\wp^{(2)}\wp^{(3)}$$

$$\times \prod_{r=1}^{3} \left(1 + \frac{1}{\sqrt{2}} w_I^{(r)} \bar{c}_0^{(r)}\right) e^{F(1,2,3)}\, |p_1, \alpha_1\rangle |p_2, \alpha_2\rangle |p_3, \alpha_3\rangle$$

$$F(1,2,3) = \sum_{\pm} \sum_{r,s=1}^{3} \sum_{m,n\geq 1} \tilde{N}^{rs}_{mn} \left(\frac{1}{2} a_m^{(\pm)(r)\dagger} a_n^{(\pm)(s)\dagger} + \sqrt{m}\alpha_r C_m^{(\pm)(r)\dagger} (\sqrt{n}\alpha_s)^{-1} \bar{C}_n^{(\pm)(s)\dagger} \right)$$

$$+ \frac{1}{2} \sum_{\pm} \sum_{r} \sum_{n} \tilde{N}^r_n a_n^{(\pm)(r)\dagger} \mathbb{P} - \frac{\tau_0}{4\alpha_1\alpha_2\alpha_3} \mathbb{P}^2$$

$$\mathbb{P} = \alpha_1 p_2 - \alpha_2 p_1$$

$$w_I^{(r)} = \cdots$$

$$\mu(1,2,3) = \exp(-2 g(\beta)) \quad : \quad g(\beta) = (\beta^2+\beta+1)\left(\frac{\log|\beta|}{\beta+1} - \frac{\log|\beta+1|}{\beta}\right)$$

$$\tau_0 = \sum_{r=1}^{3} \alpha_r \log|\alpha_r| \qquad\qquad \beta = -\frac{\alpha_1}{\alpha_1+\alpha_2}$$

$$\int \delta(1,2,3) = \int d\alpha\, dp\, (2\pi)^d \delta^d(p_1+p_2+p_3)\, 2\pi\, \delta(\alpha_1+\alpha_2+\alpha_3)$$

$$\wp^{(i)} = \oint \frac{d\theta}{2\pi} e^{-i\theta(N_+^{(i)} - N_-^{(i)})} \quad : \quad N_\pm := \sum n \left(a_n^{(\pm)\dagger} a_n^{(\pm)} + c^\dagger \bar{c} + \bar{c}^\dagger c\right)$$

$$\left\{ \begin{array}{l} |\Phi_1 * \Phi_2\rangle_3 = \int d\bar{c}_0^{(1)} d\bar{c}_0^{(2)}\, {}_1\langle\tilde{\Phi}_1|\, {}_2\langle\tilde{\Phi}_2|\, V(1,2,3)\rangle \\ {}_2\langle\tilde{\Phi}| = \int d\bar{c}_0^{(1)} \langle \tilde{R}(1,2) | \Phi\rangle_1 \\ \langle \tilde{R}(1,2)| \ : \ \text{Reflector} \end{array} \right.$$

We prove (for $\alpha_1, \alpha_2 > 0$)

$$\boxed{\Phi_F(x_1, \alpha_1) * \Phi_F(x_2, \alpha_2) = \delta^{d-p-1}(x_1 - x_2)\, G \frac{\partial}{\partial c_0} \Phi_F(x_1, \alpha_1 + \alpha_2)}$$

$$G = \mu(1,2,3)^2 \left(\det(1 - r^2)\right)^{-1/2}$$

$$r_{mn} = \frac{(mn)^{3/2}}{m+n} \beta(\beta+1)\, \tilde{f}^{(3)}_m \tilde{f}^{(3)}_n$$

$$\tilde{f}^{(3)}_m = \frac{\Gamma(-m\beta)\, e^{m(\beta \log|\beta| - (\beta+1)\log|\beta+1|)}}{m!\, \Gamma(-m\beta + 1 - m)}$$

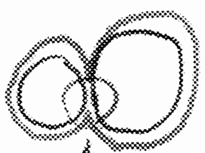
- So far, no closed form for determinant factor

After cutting-off r by size $L \times L$

G behaves numerically

$$G(\beta) \sim L^3 e^{7.07 + 0.866 g(\beta)}$$

Mildly divergent due to singularity at intersection?



singularity

Some detail of computation (matter part)

boundary state

$$|\Phi_1\rangle \otimes |\Phi_2\rangle = e^{\frac{1}{2} a^\dagger M a^\dagger} \underline{|p_1, \alpha_1\rangle \otimes |p_2, \alpha_2\rangle}$$

$\leftarrow$ before integrating along transverse direction

$$a^\dagger = \begin{pmatrix} a^{(+)\dagger} \\ a^{(-)\dagger} \end{pmatrix} \qquad a^{(\pm)\dagger} = \begin{pmatrix} a_n^{(1)(\pm)\mu\dagger} \\ a_n^{(2)(\pm)\mu\dagger} \end{pmatrix}$$

$$M = \begin{pmatrix} 0 & -O_{MN}\delta_{mn}\delta_{rs} \\ -O^T_{MN}\delta_{mn}\delta_{rs} & 0 \end{pmatrix}$$

Exponential factor in V_3

$$F(1,2,3) = \frac{1}{2} a^\dagger N a^\dagger + a^\dagger \mu + a^{(3)\dagger} \tilde{N}^{33} a^{(3)\dagger} - \frac{\tau_0}{4\alpha_1\alpha_2\alpha_3} \mathbb{P}^2$$

$$N = \eta_{MN} \begin{pmatrix} n & 0 \\ 0 & n \end{pmatrix} \qquad n = \begin{pmatrix} \tilde{N}^{11} & \tilde{N}^{12} \\ \tilde{N}^{21} & \tilde{N}^{22} \end{pmatrix}$$

$$\mu = \begin{pmatrix} \mu^{(+)} \\ \mu^{(-)} \end{pmatrix} \qquad \mu^{(\pm)} = \begin{pmatrix} \tilde{N}^{13} a^{(3)(\pm)\dagger} + \frac{1}{2} \tilde{N}^{1} \mathbb{P} \\ \tilde{N}^{23} a^{(3)(\pm)\dagger} + \frac{1}{2} \tilde{N}^{2} \mathbb{P} \end{pmatrix}$$

$$\left\{ \begin{array}{l} |\Phi_1 * \Phi_2\rangle = \det^{-1/2}(1 - MN)\, e^{H} |p_1 + p_2, \alpha_1 + \alpha_2\rangle \\[2ex] H = \frac{1}{2} a^\dagger N^{33} a^\dagger + \frac{1}{2} \tilde{N}^{3} (a^{(+)\dagger} + a^{(-)\dagger}) \mathbb{P} - \frac{\tau_0}{4\alpha_1\alpha_2\alpha_3} \mathbb{P}^2 \\[2ex] \qquad + \frac{1}{2} \mu^T \underline{M (1 - NM)^{-1}} \mu \end{array} \right.$$

$\uparrow$

How to manage this factor?

Neumann matrix

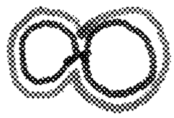
$$\tilde{N}^{rs}_{mn} = \delta_{rs}\delta_{mn} - 2\left(A^{(r)T}\Gamma^{-1}A^{(s)}\right)_{mn}$$

$$\tilde{N}^{r}_{m} = -\left(A^{(r)}\Gamma^{-1}B\right)_{m}$$

$$A^{(1)}_{mn} = -\frac{2}{\pi}\sqrt{mn}\,(-1)^{m+n}\frac{\beta\sin(m\pi\beta)}{n^2-m^2\beta^2}, \quad A^{(2)}_{mn} = \text{similar}, \quad A^{(3)}_{mn} = \delta_{m,n}$$

$$B_m = -\frac{2\alpha_3}{\pi\alpha_1\alpha_2}\, m^{-3/2}(-1)^m \sin(m\pi\beta)$$

$$\Gamma_{mn} = \delta_{mn} + \sum_{r=1,2}\left(A^{(r)}A^{(r)T}\right)_{mn}$$



$A^{(r)}$, B : overlap of Fourier basis

Essential property of $A^{(r)}$

HAVE INVERSE $D^{(r)}$

$$\sum_{r=1,2}\left(A^{(r)}D^{(r)}\right)_{mn} = \delta_{mn} \qquad \left(D^{(r)}A^{(s)}\right)_{mn} = \delta_{rs}\delta_{mn}$$

$$D^{(r)} \equiv -\frac{\alpha_3}{\alpha_r}\, C A^{(r)T} C^{-1} \qquad C_{mn} := m\,\delta_{mn}$$

In a sense, they are "rectangular" matrices

$$\begin{pmatrix} A^{(1)} & A^{(2)} \end{pmatrix}\begin{pmatrix} D^{(1)} \\ D^{(2)} \end{pmatrix} = 1$$

$$\begin{pmatrix} D^{(1)} \\ D^{(2)} \end{pmatrix}\begin{pmatrix} A^{(1)} & A^{(2)} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

One can compute the inverse now

$$(1-NM)^{-1} = \begin{pmatrix} (1-n^2)^{-1} & -\mathcal{O}\, n\,((1-n^2)^{-1} \\ -\mathcal{O}^T n\,(1-n^2)^{-1} & (1-n^2)^{-1} \end{pmatrix}$$

$$(1-n^2)^{rs} = 4\, A^{(r)T}\, \Gamma^{-2}\, A^{(s)}$$

$$\left((1-n^2)^{-1}\right)^{rs} = \frac{1}{4}\, D^{(r)}\, \Gamma^2\, D^{(s)T}$$

Some computations (after using this result) give

$$H_m = \frac{1}{2}\, a^+ M a^+ \qquad (= \text{quadratic part of } \Phi_F)$$

Determinant factor

$$\det(1-MN)^{1/2} \longrightarrow C(\beta)$$

We arrive at

$$\boxed{\Phi_F(p_1^\perp, \alpha_1) * \Phi_F(p_2^\perp, \alpha_2) = C(\beta)\, \frac{\partial}{\partial c_0}\, \Phi_F(p_1^\perp + p_2^\perp, \alpha_1 + \alpha_2)}$$

$\Big\Downarrow$ Fourier transformation for $p^\perp$

(Ishibashi state $\Rightarrow$ Cardy state)

$$\boxed{\Phi_F(x_1^\perp, \alpha_1) * \Phi_F(x_2^\perp, \alpha_2) = C(\beta)\, \delta(x_1^\perp - x_2^\perp)\, \frac{\partial}{\partial c_0}\, \Phi(x_1^\perp, \alpha_1 + \alpha_2)}$$

$C(\beta)$ [redacted] $\frac{\partial}{\partial c_0}$: universal factor for ANY Boundary state

§3. Derivation of open string spectrum

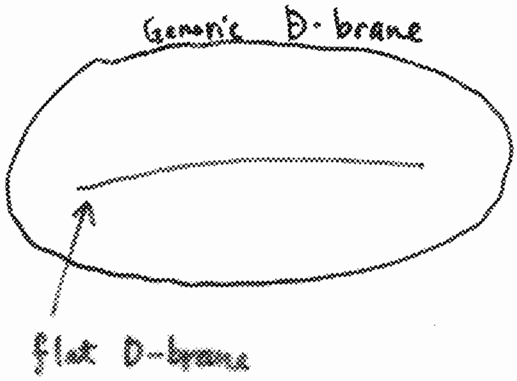
Question : Does unipotency relation

$$\Phi_\alpha * \Phi_\beta = C(B)\, \delta_{\alpha\beta} \frac{\partial}{\partial C_\alpha} \Phi_\alpha \qquad \cdots\cdots (*)$$

remains true for any (curved) D-brane?

First nontrivial test

· Check it for infinitesimal deformation around flat D-brane



$$\delta|B\rangle = \oint \frac{d\sigma}{2\pi} V(\sigma)\, |B\rangle$$

We examined two simplest examples

① Tachyon $\qquad V_T(\sigma) = e^{ikX(\sigma)}$

② Vector $\qquad V_V(\sigma) = (\zeta_\mu \partial X^\mu) \cdot e^{ikX}$

Variation of $(*)$

$$\delta\Phi * \Phi + \Phi * \delta\Phi = C(B) \frac{\partial}{\partial C_0} \delta\Phi$$

implies

$$\begin{cases} k_\mu G^{\mu\nu} k_\nu = 2 & \text{for ①} \\ k_\mu G^{\mu\nu} k_\nu = 0 & \text{for ②} \end{cases}$$

$G^{\mu\nu} \equiv \left((1+F)^{-1} 2 (1-F)^{-1}\right)^{\mu\nu}$ open string metric

$\boxed{\text{On-shell conditions of open string !}}$

① Derivation of tachyon mass-shell condition

$$|\delta_T \Phi(\alpha_1) * \Phi(\alpha_2)\rangle = \cdots = \underline{(-\beta)^{\frac{1}{2}kGk}} \; c_B | \delta_T \Phi(\alpha_1+\alpha_2)\rangle$$

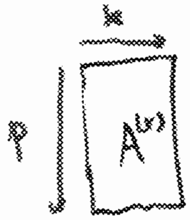
$$|\Phi(\alpha_1) * \delta_T \Phi(\alpha_2)\rangle = \cdots = \underline{(1+\beta)^{\frac{1}{2}kGk}} \; c_B | \delta_T \Phi(\alpha_1+\alpha_2)\rangle$$

$\therefore$ on-shell condition $\Leftrightarrow$ $(-\beta)^{\frac{1}{2}kGk} + (1+\beta)^{\frac{1}{2}kGk} = 1$

$$\Leftrightarrow \quad k_\mu G^{\mu\nu} k_\nu = 2$$

How we obtained $\begin{pmatrix} \log|\beta| \\ \log(1+\beta) \end{pmatrix}$ factor ?

Coefficient of $\frac{1}{2}kGk$ has following form

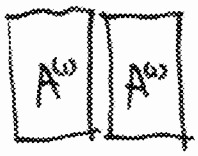


$$\sum_{k=1}^{\infty} \frac{1}{k} - \sum_{p=1}^{\infty} \frac{1}{p} \quad ; \quad \infty - \infty$$

indefinite

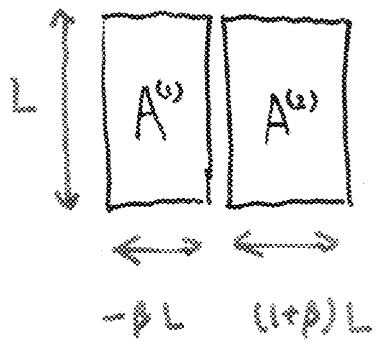
We need regularization (cut-off) of A

We note that



need to have inverse $\quad$ A is rectangular matrix

We need to consider following cut-off



* "string bit regularization"

$$\left(-\beta = \frac{\alpha_1}{\alpha_1+\alpha_2} \quad , \quad (1+\beta) = \frac{\alpha_2}{\alpha_1+\alpha_2} \right)$$

It implies coeff of $\frac{1}{2}kGk$ is given by

$$\lim_{L\to\infty} \left(\sum_{k=1}^{|\beta| L} \frac{1}{k} - \sum_{p=1}^{L} \frac{1}{p} \right) = + \log|\beta|$$

for 1st term

② Vector type fluctuation

$$| \delta_V \Phi(\alpha_1) * \Phi(\alpha_2) \rangle = (-\beta)^{+\frac{1}{2}kGk+1} \, | \Phi(\alpha_1+\alpha_2) \rangle + \underbrace{\cdots}_{(A)}$$

$$| \Phi(\alpha_1) * \delta_V \Phi(\alpha_2) \rangle = (1+\beta)^{\frac{1}{2}kGk+1} \, | \Phi(\alpha_1+\alpha_2) \rangle + \underbrace{\cdots}_{(B)}$$

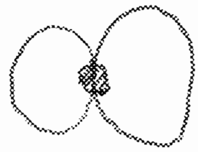
On-shell condition

$$(-\beta)^{\frac{1}{2}kGk+1} + (1+\beta)^{\frac{1}{2}kGk+1} = 1$$

$$\Longleftrightarrow \quad k_\mu G^{\mu\nu} k_\nu = 0$$

There are MANY TERMS in ... but they are cancelled exactly..

A subtlety.. singularity at interaction point



$\dddot{A}$ contains $\quad -i\zeta\cdot G\cdot k\left(2\,\delta(\pi-\sigma-\theta_1)\sum_{p=1}^{\infty}\frac{\sin^2\pi p\beta}{p}\right)$

$(-\pi\leq\sigma+\theta_1\leq\pi)$

$\dddot{B}$ contains $\quad i\zeta\cdot G\cdot k\left(2\,\delta(\sigma+\theta_2)\sum_{p=1}^{\infty}\frac{\sin^2\pi p\beta}{p}\right)$

$(0\leq\sigma+\theta_2\leq 2\pi)$

$\sum_{p=1}^{\infty}\frac{\sin^2\pi p\beta}{p}$ is divergent but A & B are cancelled exactly after integration over σ

$\Leftrightarrow$ No transversality condition

$$k_\mu G^{\mu\nu}\zeta_\nu=0$$

but it is subtle

① & ② are subtleties where we need more explicit regularization scheme

c.f. MSFT regularization

Note

- There exists $\boxed{\text{gauge symmetry}}$ for vector type deformation

off-shell

$$\zeta_\mu \to \zeta_\mu + \epsilon k_\mu$$

$$\delta_V |B\rangle \to \oint \frac{d\sigma}{2\pi} (\zeta_\mu + \epsilon k_\mu) \partial_\sigma X^\mu e^{ikX(\sigma)} |B\rangle$$

$$= \delta_V |B\rangle - i \underbrace{\oint \frac{d\sigma}{2\pi} \epsilon \, \partial_\sigma (e^{ikX}) |B\rangle}_{\parallel \atop 0}$$

Vector particle

Massless	○	same as (conventional) gauge particle
Gauge symmetry	○	
Transversality	×	

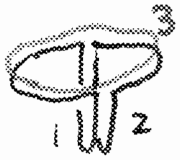
Conjecture

Variations of $\Phi * \Phi = c\,\Phi$ reproduces complete open string spectrum

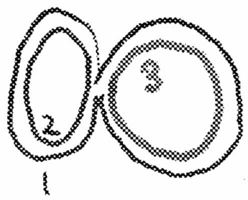
§ 4 Discussion

① Unipotency relation for other closed string vertex?

example:

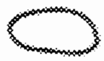


Witten type



HIKKO ($\alpha_1 \alpha_2 < 0$)

Yes! but VERY SINGULAR

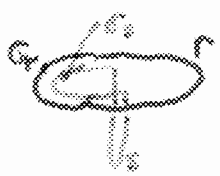
$\Phi * \Phi =$  $(\delta(0))^r \, \Phi$

r : number of overlap $= \infty$

Origin of singularity

Boundary state $\sim \prod_{\sigma} \delta(X(\sigma) - \hat{X}(\sigma))$

Vertex $\sim \prod_{r<s} \prod_{(\sigma_r, \sigma_s)} \delta(X_r(\sigma_r) - X_s(\sigma_s))$
$\times \, \delta(\tilde{X}_r(\sigma_r) - \tilde{X}_s(\sigma_s))$



$$\delta(X_1(\sigma_1) - \tilde{X}_1(\sigma_1))\, \delta(X_2 - \tilde{X}_2(\sigma_2))$$
$$\times \, \delta(X_1(\sigma_1) - X_2(\sigma_2))\, \delta(\tilde{X}_1(\sigma_1) - \tilde{X}_2(\sigma_2))$$
$$= \delta(0)\, \delta(X_1(\sigma_1) - X_2(\sigma_2))$$

Is it a disaster to construct VSFT like scenario?

May not be so serious!

Example:

Inner product of Boundary states

$$\langle B|B\rangle = (\delta(0))^{\infty}$$ same type of infinity

However there exist a regularization scheme

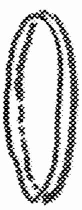
$$\langle\langle j|\, q^{\frac{1}{2}(L_0+\bar{L}_0)}\, |k\rangle\rangle = \delta_{jk}\, \chi_j(q)$$ ← regularization

$$= \delta_{jk}\, S_{jl}\, \chi_l(\tilde{q})$$

$$\langle\langle j|k\rangle\rangle = \lim_{q\to 1} \langle\langle j|\, q^{\frac{1}{2}(L_0+\bar{L}_0)}\, |k\rangle\rangle$$

$$= \lim_{\tilde{q}\to 0} \delta_{jk}\, S_{jl}\, \chi_l(\tilde{q})$$

$$= \delta_{jk}\, S_{j0}$$ (This is finite!)



infinitely thin strip

⟺ only vacuum channel in open string sector survives

② $\Phi * \Phi = \Phi$ as background independent characterization of D-brane

LPP formulation

$$\langle V_3 | \, |\varphi_1\rangle_1 |\varphi_2\rangle_2 |\varphi_3\rangle_3 \equiv \langle f_1 \circ \varphi_1 \; f_2 \circ \varphi_2 \; f_3 \circ \varphi_3 \rangle$$

f_i $(i = 1, 2, 3)$ glueing conformal mapping

Only conformal transformation is involved

$\Downarrow$

$*$ product is defined in background independent way

by LPP definition, we can prove

$$(L^{(1)}_n - \tilde{L}^{(1)}_{-n}) |B_1\rangle = 0$$

$$(L^{(2)}_n - \tilde{L}^{(2)}_{-n}) |B_2\rangle = 0$$

$$\Downarrow$$

$$(L^{(3)}_n - L^{(3)}_{-n}) (|B_1\rangle * |B_2\rangle) = 0$$

$*$ product of Boundary state is always boundary state

but without Cardy condition

We conjecture

$$|B\rangle * |B\rangle = |B\rangle$$

is satisfied only by Cardy state

In terms of Ishibashi states $|i\rangle\rangle$

this is equivalent to

$$|i\rangle\rangle * |j\rangle\rangle = \sum_k N_{ij}{}^k \, |k\rangle\rangle$$

$N_{ij}{}^k$: Verlinde's fusion coefficient

$$= \sum_\ell \frac{S_{\ell i} S_{\ell j} (S_{\ell k})^*}{S_{\ell 0}}$$

Cardy state

$$|a\rangle = \sum_\ell S_{a\ell} \, |\ell\rangle\rangle$$

$$|a\rangle * |b\rangle = \delta_{ab} \frac{1}{S_{a0}} |a\rangle$$

We check these relations for toroidal compactification

3 Similarity with VSFT

Open string ⟷ dual picture? ⟷ **Closed string**

Open string	Closed string
$\boxed{\text{Witten's CSFT}}$	$\boxed{\text{HIKKO or Witten type Closed SFT}}$
$\downarrow$ Tachyon vacuum	$\downarrow$ Tachyon vacuum?
VSFT $\quad \mathcal{Q}\Phi = \Phi * \Phi$	$\boxed{\Phi * \Phi = C_0 \Phi}$
$\Downarrow$	$\Uparrow$
D-brane $=$ projector?	$\boxed{\text{Boundary state} = \text{D-brane}}$

- Basic picture is very similar

Detail is quite different ...

cf. massless excitation (vector particle)

Other issues

- Supersymmetric extension
- Calculation of tension
- higher excited mode
- Regularization
- Application to rolling tachyon (time dependent process)
- Closed string propagation