Physics of an extra dimension at the TeV

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1 Introduction

It is widely believed that the standard model (SM) is the low-energy limit of a more fundamental theory including gravity. It is also believed that one of the requirements for this fundamental theory is the existence of more than three spatial dimensions, which would be compact and with a radius R of Planckean size. Recently, however, it has been suggested that the extra dimensions can appear at much lower energies. A first possibility was given in Refs. [1, 2] in the context of string theory. It was shown that large extra dimensions do not necessarily spoil the gauge coupling unification of the 4D theory. A more radical possibility, proposed in Ref. [3], is to decrease the scale of unification of gravity with the gauge interactions down to the TeV. This can be realized by means of two sub-millimeter extra dimensions in which only gravity propagates. Although the gauge interactions would not feel these sub-millimeter dimensions, a fundamental scale (string scale) in the TeV region suggests the possibility of compact dimensions of this size where the SM fields do propagate.

Large ($\approx$ TeV^{-1}) extra dimensions find also an interesting motivation as a framework to break supersymmetry [4]. This has been studied in detail in Refs. [5, 6, 7], where it was predicted a compactification scale around $3 - 20$ TeV. In the next section we will explain how supersymmetry can be broken by compactifycation and what predictions one obtains.

In section 3 we study the effects of extra dimensions on electroweak observables. If the SM gauge bosons can propagate in a compact dimension, their (quantized) momentum along this dimension can be associated to the mass n/R ($n = 1, .., \infty$) of a tower of Kaluza-Klein (KK) excitations. As a consequence the relations between electroweak observables will be modified with respect to those of the 4D SM [8, 9]. We calculate these effects and show how to put bounds on the size of an extra dimension from high-precision electroweak data. We find limits on $1/R$ from 1 to 3 TeV from different observables, with a model independent lower bound of ≈ 2.5 TeV [9].

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2 Supersymmetry breaking by Scherk-Schwarz compactification

Theories with extra dimensions, and in particular five dimensional theories, allow the use of the Scherk-Schwarz (SSSB) mechanism to break supersymmetry [4, 5]. This consists in imposing to the 5D fields a different periodicity condition for bosons and fermions under a $2\pi R$ translation of the extra dimension:

$$\Phi(x^\mu, y + 2\pi R) = e^{2\pi i q_\Phi}\, \Phi(x^\mu, y)\,, \tag{1}$$

where q_Φ is the R-symmetry charge of the field Φ. Due to the periodicity condition (1), the fields are Fourier expanded as

$$\Phi(x^\mu, y) = \sum_{n=-\infty}^{\infty} e^{iy(n+q_\Phi)/R}\, \Phi^{(n)}(x^\mu)\,, \tag{2}$$

where y is assumed to be compactified on the circle S^1. Reducing the theory from 5D to 4D, eq. (2) leads to a tower of KK states with a fermion-boson mass splitting inside each KK supermultiplet:

$$\begin{aligned} m_B^2 &= (n + q_B)^2\, M_c^2\ , \\ m_F^2 &= (n + q_F)^2\, M_c^2\ , \quad n = 0, \pm 1, \pm 2, \ldots\,, \end{aligned} \tag{3}$$

where $M_c \equiv 1/R$, and q_B and q_F are the charges of the bosons and fermions, respectively.

In orbifold compactifications, the expansion (2) is truncated, since only the invariant states under the orbifold group remain in the theory. For example in S^1/Z_2 compactifications, the Z_2 parity, $y \to -y$, acts on the KK-states as $\Phi^{(n)} \to \Phi^{(-n)}$. Therefore the Z_2 projects the KK-tower into Z_2-even ($\Phi_+^{(n)}$) or Z_2-odd ($\Phi_-^{(n)}$) states:

$$\begin{aligned} \Phi_+^{(n)} &= \Phi^{(n)} + \Phi^{(-n)}, \qquad n = 0, 1, 2, \ldots\,, \\ \Phi_-^{(n)} &= \Phi^{(n)} - \Phi^{(-n)}, \qquad n = 1, 2, \ldots\,. \end{aligned} \tag{4}$$

Massless fields arise in the theory only if either q_B or q_F are zero. We are interested in the $q_B = 0$ case, in which only the vector and scalar boson $n = 0$ states remain in the massless spectrum of the theory, and can be associated with the bosonic sector of the SM. Although vector bosons remain massless because of the gauge symmetry, the $n = 0$ complex scalar field, ϕ, will get a mass at the one-loop level due to the breaking of supersymmetry by the SSSB mechanism. Each level of KK excitations contribute to the scalar mass. This is a crucial difference with respect to 4D theories with softly broken supersymmetry.

The one-loop m_ϕ^2 induced by a tower of KK with a mass splitting (3) is given by

$$m_\phi^2 = \frac{1}{2} \mathrm{Tr}\, \frac{dM^2(\phi)}{d|\phi|^2}\bigg|_{\phi=0} \sum_{n=-\infty}^{\infty} \Pi_n(0)\,, \tag{5}$$

$$\Pi_n(0) = \int \frac{d^4p}{(2\pi)^4} \left[\frac{1}{p^2 + (n + q_B)^2 M_c^2} - \frac{1}{p^2 + (n + q_F)^2 M_c^2} \right]\,. \tag{6}$$

where the trace is over the degrees of freedom of the KK tower and $M^2(\phi)$ is the ϕ-dependent mass of the KK states. As in finite temperature calculations, we must first sum over the infinite tower of KK states and then perform the momentum integral. Eq. (5) thus leads to [2]

$$m_\phi^2 = \frac{1}{32\pi^4}\left[\Delta m^2(q_B) - \Delta m^2(q_F)\right] \mathrm{Tr}\, \frac{dM^2(\phi)}{d|\phi|^2}\bigg|_{\phi=0} , \tag{7}$$

where

$$\Delta m^2(q) = \frac{1}{2}(Li_3(z) + Li_3(1/z))\, M_c^2 , \tag{8}$$

with $z \equiv e^{i2\pi q}$ and $Li_n(z)$ are the polylogarithm functions $Li_n(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^n}$. For q ranging from 0 to 1/2, $(Li_3(z) + Li_3(1/z))/2$ goes from 1.2 to -0.9.

We have just considered a theory compactified on S^1. For a compactification on the S^1/Z_2 orbifold, the contribution to m_ϕ^2 from a KK-tower must sum from $n = 0$ to ∞ (see eq. (4)). Nevertheless, the contribution to m_ϕ^2 of an even field (Π_n^+) can be always added to the one of an odd field (Π_n^-) such that they can be considered as the contribution of a single KK-tower where n goes from $-\infty$ to ∞ [5]:

$$\sum_{n=0}^{\infty} \Pi_n^+ + \sum_{n=1}^{\infty} \Pi_n^- = \sum_{n=-\infty}^{\infty} \Pi_n , \tag{9}$$

where $\Pi_{\pm n} \equiv \Pi_n^\pm$. Thus, in S^1/Z_2 orbifold compactifications, the effective number of states of the KK-tower is reduced by a half with respect to that in S^1.

The contribution (7) is finite and ultraviolet independent. There are different ways to understand this result. The simplest way is to notice that a theory with SSSB supersymmetry breaking keeps a clear analogy with a theory at finite temperature T (a quantum field theory with the time compactified) where M_c plays the role of T. At finite temperature, the T-dependent effective potential is not affected by the ultraviolet cutoff due to the Boltzmann suppression of the heavy states. Of course, the T-independent part of the effective potential is ultraviolet sensitive. In our case, however, $M_c = 0$ corresponds to the supersymmetric limit and therefore the corrections to $V(\phi)$ are zero.

Let us now consider the fields living in the 4D boundary. These fields do not have associated KK excitations and are massless at tree level. Nevertheless, if they couple to the fields living in the 5D bulk, the supersymmetry breaking will be transmitted from the bulk to the boundary and, as a consequence, the scalars living in the boundary will get masses at the one-loop level. Let us consider again the case of an S^1/Z_2 orbifold. The possible interactions between the bulk and the boundary fields can be found in ref. [7]. The gauge interactions couple the fields in the boundary to the KK-towers of the gauge boson, gaugino and the auxiliary D-field. At the one-loop, we find that the boundary scalars get a mass given by (for $q_B = 0$)

$$m_i^2 = \frac{g^2 C(R_i)}{4\pi^4}\left[\Delta m^2(0) - \Delta m^2(q_F)\right] , \tag{10}$$

[2] For further details see ref. [7].

where R_i is the representation of the gauge group under which the boundary field transforms, and $\Delta m^2(q)$ is given in eq. (8). The boundary field can also couple to an $N = 1$ chiral supermultiplet that consists in the KK-towers of a complex scalar, a bispinor and the auxiliary F-field. In this case we find that the scalar field of the boundary gets a mass given by

$$m_i^2 = \frac{Y^2}{16\pi^4}\left[\Delta m^2(q_B) + \Delta m^2(2q_F - q_B) - 2\Delta m^2(q_F)\right], \tag{11}$$

where Y is the Yukawa coupling between the bulk and boundary fields. The first term in eq. (11) arises from the scalar KK-tower, the second term from the auxiliary F-field KK-tower and the third from the bispinor KK-tower. Again these contributions are also finite and ultraviolet independent.

2.1 Superparticle spectrum

Let us apply the above calculation to our model, where gauginos and Higgsinos in the bulk are given the same boundary conditions, q_F, corresponding to a common R-symmetry charge. Since gauge and Higgs bosons live in the 5D bulk their corresponding $n = 0$ fermions, gauginos and Higgsinos, will get masses of order M_c:

$$\begin{aligned} m_\lambda &= q_F M_c\,, &(12)\\ m_{\widetilde{H}} &= q_F M_c\,. &(13)\end{aligned}$$

Therefore the massless states of the 5D bulk correspond to the gauge and Higgs bosons of the SM. Quarks and leptons superfields reside in the 4D boundary, and then they also remain massless at the tree-level. Nevertheless, since supersymmetry is broken in the bulk, squarks and sleptons will get masses at the one-loop level through the gauge and Yukawa interactions, leaving only the fermion sector of the SM in the massless spectrum. Using eqs. (10) and (11), we obtain the squarks and sleptons masses:

$$\begin{aligned} m^2_{\widetilde{Q}} &= \left(\frac{8}{6}\alpha_3 + \frac{3}{4}\alpha_2 + \frac{1}{60}\alpha_1\right)\Delta m_g^2 + \frac{1}{2}\alpha_t \Delta m_H^2\,, &(14)\\ m^2_{\widetilde{U}} &= \left(\frac{8}{6}\alpha_3 + \frac{4}{15}\alpha_1\right)\Delta m_g^2 + \alpha_t \Delta m_H^2\,, &(15)\\ m^2_{\widetilde{D}} &= \left(\frac{8}{6}\alpha_3 + \frac{1}{15}\alpha_1\right)\Delta m_g^2\,, &(16)\\ m^2_{\widetilde{L}} &= \left(\frac{3}{4}\alpha_2 + \frac{3}{20}\alpha_1\right)\Delta m_g^2\,, &(17)\\ m^2_{\widetilde{E}} &= \frac{3}{5}\alpha_1 \Delta m_g^2\,, &(18)\end{aligned}$$

where

$$\Delta m_g^2 = \left[\Delta m^2(0) - \Delta m^2(q_F)\right]/\pi^3\,, \tag{19}$$

and

$$\Delta m_H^2 = \left[\Delta m^2(0) + \Delta m^2(2q_F) - 2\Delta m^2(q_F)\right]/(2\pi^3)\,, \tag{20}$$

with $\Delta m^2(q)$ given in eq. (8).

The above equation gives us a very predictive spectrum for the squarks and sleptons. It only depends on two free parameters, q_F and M_c. Notice also that the above contributions are positive as they are necessary to avoid color or charge breaking. The ratio of masses is given by (for $q_F = 1/2$)

$$5\, m_{\widetilde{Q}} \sim 5\, m_{\widetilde{D}} \sim 5\, m_{\widetilde{U}} \sim 10\, m_{\widetilde{L}} \sim 20\, m_{\widetilde{E}} \sim m_{\lambda,\widetilde{H}}\,. \tag{21}$$

Therefore the gauginos and Higgsinos are the heaviest supersymmetric particles and the right-handed slepton is the lightest (LSP) one. In an R-parity conserving theory, the right-handed slepton will be stable and will cross the detector leaving an ionizing track. It can be discovered by looking at anomalous ionization energy loss, dE/dx, in the tracking detector gas [12]. Cosmological arguments exclude charged, stable particles with masses in the ~ 100 GeV to ~ 10 TeV range. To evade such constraints, one can consider that the charged LSP is unstable. A simple way to accomplish this is to postulate an R-parity breaking interaction. Indeed, if R-parity is violated, the right-handed slepton will decay into SM leptons. Another possibility to avoid a right-handed slepton LSP occurs in theories with right-handed neutrinos. These must have miniscule Yukawa coupling to account for the observed smallness of the neutrino masses. Since the right-handed sneutrinos are electroweak singlets and also have miniscule Yukawa couplings, they will get tiny masses of order of the (Dirac) neutrino masses in the sub-eV range and will be the LSP. We are not aware whether such an LSP passes all the necessary cosmological safety tests, but it does not seem to us to be obviously excluded.

2.2 Electroweak Breaking

To study the breaking of the electroweak symmetry, we must analyze the soft masses of the Higgs(es). In the MSSM, we need two Higgs $SU(2)_L$-doublets, H_1 and H_2, in order to give masses to all the fermions living in the 4D boundary. After imposing the supersymmetry breaking with the SSSB compactification, these two Higgses can either arise as two (tree-level) massless states, or as a massless and a massive state. This depends on the R-charges of the Higgses, and different possibilities have been proposed in refs. [5]. The simplest case corresponds to having a unique massless $SU(2)_L$-doublet, ϕ, that will be responsible for the electroweak symmetry breaking.

Here we will be only interested in knowing whether ϕ gets a vacuum expectation value and therefore breaks the electroweak symmetry. For this purpose we must calculate the quantum corrections to its mass. Using eq. (7), we have

$$m_\phi^2(M_c) = \left(\frac{3}{4}\alpha_2 + \frac{3}{20}\alpha_1\right) \Delta m_g^2\,. \tag{22}$$

This mass is positive. Nevertheless, we must also consider the correction to the Higgs mass due to the stop. This correction arises at the two-loop level but it is important since the stops are heavier than the Higgs. This is given by

$$m_\phi^2(m_Z) \simeq m_\phi^2(M_c) - \frac{3\alpha_2 m_t^2}{8\pi m_W^2}(m_{\widetilde{Q}}^2 + m_{\widetilde{U}}^2)\ln\frac{M_c^2}{m_{\widetilde{Q}}^2}\,. \tag{23}$$

As in theories of gravity or gauge-mediation supersymmetry breaking, this contribution turns the Higgs mass to negative values and triggers the breaking of the electroweak symmetry. Imposing the minimization condition of the Higgs potential

$$-m_\phi^2(m_Z) = \frac{m_Z^2}{2}\cos^2 2\beta\,, \tag{24}$$

one can derive the value of the compactification scale M_c. Taking $q_F = 1/2$, $m_t \simeq 175$ GeV and $\cos 2\beta \sim 1$, we obtain

$$M_c \sim 3.7 \text{ TeV}\ , \qquad m_{\widetilde{Q}} \sim 400 \text{ GeV}\ , \qquad m_{\widetilde{E}} \sim 100 \text{ GeV}\,. \tag{25}$$

2.3 Contrast with the MSSM and Gauge Mediation

Sparticle spectroscopy in SSSB is strikingly different from that of more familiar gauge mediated and MSSM. All fermionic sparticles are a factor between 5 and 20 heavier than the bosonic ones, a smoking gun for this framework. In particular, the gauginos and higgsinos are ~ 20 times heavier than the right-handed sleptons. A consequence of this, following from the present lower limit of 82.5 GeV on the mass of the right-handed sleptons, is that the compactification scale must be no less than ~ 3 TeV. Consequently, the KK excitations of ordinary particles would not be accessible at LHC.

Another point of contrast is the ease with which SSSB generates a mass μ for the higgsinos. This occurs as an integral part of the SSSB and naturally accounts for the equality of the μ term and the supersymmetry breaking scales. In contrast, in gauge mediation one needs to work hard to accomplish this task.

Finally there are some similarities. As in gauge-mediated theories, the breaking of supersymmetry is communicated to the squarks by the gauge interactions. Therefore the theory does not have dangerous flavor violating interactions.

3 Electroweak observables and constraints on M_c

The fact that the gauge boson have a tower of KK-excitations with masses nM_c, $n = 1, 2, ...$ implies that the electroweak relations will be modified at tree-level. There are two kinds of effects. The first one is due to the presence of mixing between the zero and the n-modes of the W and Z bosons. This leads to a modification of the W and Z masses and their couplings to the fermions. The second effect arises from the exchange of KK-excitations of the W, Z and γ

vector bosons, which induces extra contributions to four-fermion interactions [8]. These effects have been calculated in ref. [9]. For the Fermi constant G_F, the leptonic Z-width $\Gamma(l^+l^-)$, the electroweak charge Q_W and the ρ parameter, we have

$$
\begin{aligned}
G_F &= G_F^{SM}\left[1-2(s_\beta^4+2s_\beta^2-1)\sum_{n=1}^{\infty}\frac{m_W^2}{n^2M_c^2}\right], && (26)\\
\Gamma(l^+l^-) &= \Gamma(l^+l^-)^{SM}\left[1+2\left[(s_\beta^2-1)^2\sin^2\theta_W-1\right]\sum_{n=1}^{\infty}\frac{m_Z^2}{n^2M_c^2}\right], && (27)\\
Q_W &= Q_W^{SM}\left[1+2(s_\beta^2-1)^2\sin^2\theta_W\sum_{n=1}^{\infty}\frac{m_Z^2}{n^2M_c^2}\right], && (28)\\
\rho &= \rho^{SM}\left[1-2s_\beta^4\sin^2\theta_W\sum_{n=1}^{\infty}\frac{m_Z^2}{n^2M_c^2}\right]. && (29)
\end{aligned}
$$

where the superindex SM refers to the SM value and s_β ranges from 0 to 1 depending if the Higgs scalars, whose VEV lead to electroweak symmetry breaking, live in the 5D bulk or on the 4D boundary [9].

The excellent agreement between the SM predictions and the experimental values implies that the effects (26)-(29) must be small. This allows us to place bounds on M_c. These are shown in Fig. 1 of ref. [9]. We find that the strongest lower bound on M_c comes from the leptonic Z width, an observable that seems to be very appropriate to constrain models with extra gauge bosons. This is because (a) it is measured at the level of 0.1%, (b) the SM loop corrections are calculated with an even better precision and (c) its dependence on s_β is very mild. We find an absolute bound of $M_c \gtrsim 2.5$ TeV. For new bounds see refs. [10, 11]

KK-excitations also affect the differential cross-sections for $e^+e^- \to f^+f^-$ measured at high energies, $q^2 > m_Z^2$. These experiments can be used to test four-fermion contact interactions, and consequently to put lower bounds on the masses of the KK-excitations [2]. We find that the largest bound comes from limits on the vector four-fermion interaction, $\epsilon_V[e^+\gamma^\mu e^-][f^+\gamma_\mu f^-]$. In our model these are mediated (predominantly) by the KK-tower of the photon and gives $\epsilon_V = -2q_f^2e^2\sum_{n=1}^{\infty}1/(n^2M_c^2)$. The minus sign indicates that the contribution interferes destructively with the SM one. The strongest constraint on ϵ_V is found in the LEP2 experiment, that gives $\epsilon_V < 4\pi/(9.3)^2$ TeV^{-2} for leptons at the 95% CL [12]. This implies $M_c \gtrsim 1.5$ TeV. Constraints on M_c can also be obtained from direct searches for Z' at Tevatron [13]. The present limit for a SM-like Z' is $M_{Z'} > 690$ GeV. In our model, however, we must consider that the coupling of the KK-excitations to fermions is a factor $\sqrt{2}$ larger than that of the Z, and the cross-section production is enhanced by a factor of 4. ¿From Ref. [13], we get the limit $M_c \gtrsim 820$ GeV. Similarly, from searches for W' [14] we obtain the bound $M_c \gtrsim 780$ GeV. Limits in future experiments can be found in ref. [15, 10].

4 Concluding remarks

There are well motivated theoretical arguments that imply the existence of more than three spatial dimensions. In order to be consistent with all observations, of course, the extra dimensions must be compactified at some high-energy scale. If this scale is around the TeV, their presence can be useful for supersymmetry breaking.

The first interesting consequence of our analysis is the pattern of supersymmetry breaking. While gaugino and Higgsino masses are of the order of the compactification scale, scalar masses are generated at one loop level via gauge interactions and are naturally one order of magnitude lighter. Thus, flavor universality is guaranteed as in gauge mediated models. On the other hand, again as in gauge mediation, the stop correction to the Higgs mass-squared drives it to negative values, breaking the electroweak symmetry. The resulting spectrum consists of heavy charginos and neutralinos ($2-3$ TeV), squarks at $400-500$ GeV, and the right-handed slepton as the lightest supersymmetric particle with mass close to the electroweak scale. Moreover, the higher dimensional nature of the theory and the softness of the Scherk-Schwarz breaking replace effectively the ultraviolet cutoff with the compactification scale, keeping the loop corrections to the Higgs mass due to the heavy gauginos small. Thus, one obtains a pattern of supersymmetry breaking with hierarchical structure, which is very different from all other scenaria. In addition, the models we study are extremely predictive, since they have no free parameters, other than a discrete option of boundary conditions, and the superparticle spectrum is fully determined.

The presence of extra TeV dimensions affect the SM electroweak predictions currently being tested at high precision experiments. We have shown how the associated tower of KK-excitations of the SM fields modify the relations between different electroweak observables. We have considered the most general case by taking the SM Higgs doublet as a combination of a field living in the 5D bulk and another living on the 4D boundary of the manifold. We have compared with the present electroweak data and have put constraints on the compactification scale. We have shown that, if an extra dimension exists, it must be compactified at a scale larger than ≈ 2.5 TeV. This bound will be improved with a better experimental determination of, for example, the W mass. Also new LEP2 data on differential cross-sections for $e^+e^- \to f^+f^-$ can, as discussed above, be very useful to establish the maximum length of an extra dimension.

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