NSF-ITP-99-108

Holography and Non-Commutative Geometry

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We review the construction of the supergravity background dual to supersymmetric Yang-Mills theory on non-commutative spaces (NCSYM) in the spirit of AdS/CFT correspondence. Attempts to compute the physical observables of the NCSYM using the holographic principle are described.

This note is an expanded version of the lecture presented at Summer Institute 99. The basic motivation and the philosophy of the subject is already published [1]. In this note, I will concentrate on presenting more facts than ideas, so that they may be useful as a supplementary reference. Similar discussions on the same topic can be found in [2].

1 Non-commutative geometry and Yang-Mills theory

Non-commutative geometry [3] generalizes the notion of topological spaces so that, roughly speaking, manifolds have non-commuting coordinates

$$[x_\mu, x_\nu] = \theta_{\mu\nu}.$$

Non-vanishing components of $\theta_{\mu\nu}$, which has the dimension of length-squared, characterizes the scale of non-commutativity for pairs of coordinate dimensions. Non-commutative geometries are characterized by the space of functions defined over them. In a physicist's language, functions on spaces are fields. In other words, one can define quantum field theories, such as non-abelian gauge theories, on non-commutative spaces.

According to Connes, Douglas, and Schwarz [4], Supersymmetric Yang-Mills theory on non-commutative spaces (NCSYM) can be derived as a certain decoupling limit of open-strings living on the world-volume of D-branes in the presence of a constant background NSNS 2-form $B_{\mu\nu}$. Suppose we are interested in obtaining a 3+1 dimensional SYM with non-commutativity along the (23) plane. The scale of non-commutativity is defined by $\theta_{23} = \Delta^2$. The precise prescription for taking the decoupling limit was described in [4, 5] and are summarized as follows. Consider a D1-brane oriented along the (01) directions, compactify the (23) direction into a square torus of size R, and turn on a background of constant B_{23}. Now consider scaling $\alpha' \to 0$ keeping

$$\Sigma = \frac{\alpha'}{R}, \qquad \Delta^2 = \theta_{23} = \alpha' B_{23}, \qquad \text{and} \qquad \hat{g} = g\frac{\Sigma^2}{\alpha'} \tag{1}$$

fixed.

If we set $\Delta = 0$, the fact that R is shrinking compels us to go to the T-dual description [6], and in this limit, we simply obtain the $\mathcal{N} = 4$ SYM on a square torus of size Σ with coupling $\hat{g}$.

Now let us consider the case where Δ is non-vanishing. The radius R of the torus is still shrinking, but T-duality does not help here, since in the presence of background B_{23}, the configuration in the dual description is

$$\Sigma_B = \alpha' \frac{\Sigma}{\Delta^2}, \qquad B_{23} = \frac{\Delta^2}{\alpha'} \qquad \text{and} \qquad g = \hat{g}\frac{\alpha'}{\Delta^2}. \tag{2}$$

The reason for this drastically different behavior in the case of non-vanishing Δ can be understood better by applying T-duality in x_2 and x_3 directions separately. In the presence of non-vanishing B_{23}, T-duality along the x_3 direction gives rise to a skewed torus of width Σ and height α'/Σ (see figure 1). The isometry vector is now tilted dotted line in this figure, and the closed string winds along this cycle. The second T-duality must then be taken along that cycle, and this has the effect of shrinking the large cycle along the x_3 direction that we obtained after the first duality.

However, the open strings ending on the D-brane in the skewed torus which winds around x_2 will orient itself vertically instead of along the tilted isometry direction. This is because unlike the closed strings, open strings are not required to start and end at the same point, and the vertically wound configuration is energetically favored. Therefore, both the spectrum of open string winding modes along x_2 and the open string momentum modes along x_3 are quantized in units of $1/\Sigma$. One can Fourier transform the winding number to obtain a description of these degrees of freedom living on a space of size Σ. It is clear from the skewed torus description, however, that the endpoints of the open string winding modes are delocalized. The dynamics on Fourier dual torus is also non-local, and it has been shown that the nature of non-locality is precisely what one finds in non-commutative geometry [5, 7]. Roughly speaking, we can think of non-commutativity as having been induced when we deliberately T-dualized the skewed torus vertically which is not an isometry direction.

The essential point of figure 1 is the fact that descriptions related by T-duality and Fourier transforms are physically equivalent. The fact that D3-branes in background B-fields in the scaling limit (2) is equivalent to the NCSYM is also the main point of the recent paper by Seiberg and Witten [8].

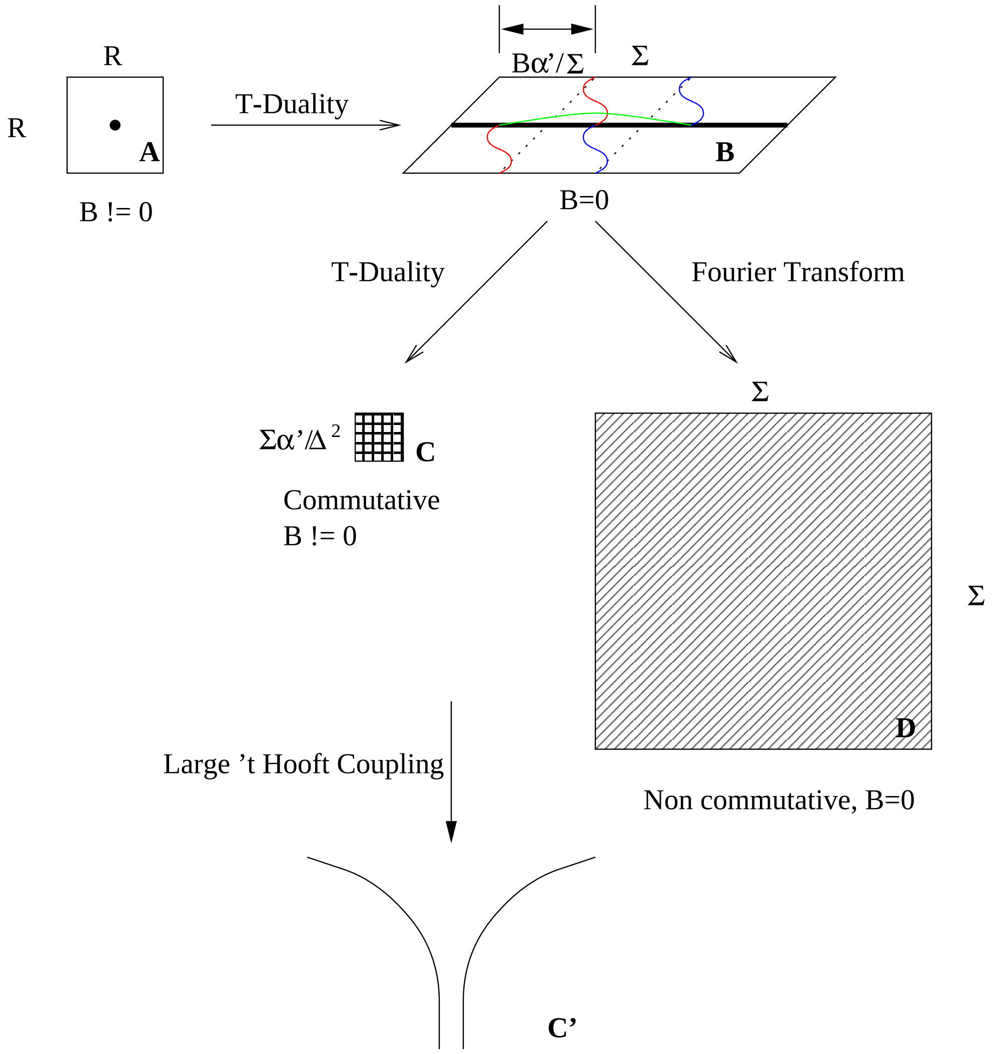

Figure 1: Various equivalent string theoretic descriptions for the system of open strings which becomes the non-commutative SYM in the decoupling limit $\alpha' \to 0$.

2 NCSYM and the AdS/CFT correspondence

Given the string theory/D-brane origin of the NCSYM, one can explore the strong coupling description by exploiting the supergravity correspondence of Maldacena [9]. The procedure of applying the same $\alpha' \to 0$ limit to the D-branes in supergravity description [10] is the essence of AdS/CFT correspondence and is by now quite standard [11]. We will perform precisely this procedure in the following.

Let us begin with the supergravity solution for a periodic array of D1-branes [10]

$$\begin{aligned} ds^2 &= f^{-1/2}(-dt^2 + dx_1^2) + f^{1/2}(dx_2^2 + dx_3^2 + dr^2 + r^2 d\Omega^2) \\ B_{23} &= \frac{\Delta^2}{\alpha'}, \qquad e^{\phi} = g f^{1/2} \end{aligned}$$

and apply the scaling (1). The harmonic function $f(r, x_2, x_3)$ is given by

$$\begin{aligned} f(r, x_2, x_3) &= 1 + \sum_{m,n} \frac{32\pi^2 g N \alpha'^3}{(r^2 + (x_2 + 2\pi n R)^2 + (x_3 + 2\pi m R)^2)^3} \\ &= (1 + \frac{4\pi g N \alpha'^3}{R^2 r^4}) \left(1 + \mathcal{O}\left(\frac{R^2}{r^2}\right)\right) \\ &= (1 + \frac{4\pi \hat{g} N}{\alpha^2 U^4}) \left(1 + \mathcal{O}\left(\frac{1}{\Sigma^2 U^2}\right)\right). \end{aligned}$$

We can take Σ to be much bigger than any other scale in the problem and forget about the $1/\Sigma^2 U^2$ corrections. In this limit, cycles in the (23) directions are shrinking, so it is natural to T-dualize in these directions, which gives

$$\begin{aligned} ds^2 &= f^{-1/2}(-dt^2 + dx_1^2) + \frac{\alpha'^2 f^{1/2}}{\alpha'^2 f + \Delta^4}(dx_2^2 + dx_3^2) + f^{1/2}(dr^2 + r^2 d\Omega^2) \\ B_{23} &= -\frac{\alpha' \Delta^2}{\Delta^4 + \alpha'^2 f}, \qquad e^{\phi} = \hat{g}\sqrt{\frac{\alpha'^2 f}{\alpha'^2 f + \Delta^4}}. \end{aligned}$$

This background is equivalent to the background of D1-D3 bound state which was written down in equation (2.12) of [12] with the identification $\tan(\theta) = \alpha'/\Delta^2$, $y_i = x_i/\sin(\theta)$. It is easy to read off the asymptotic geometry at large r and small α', but prior to taking the decoupling limit

$$\begin{aligned} ds^2 &= (-dt^2 + dx_1^2) + \frac{\alpha'^2}{\Delta^4}(dx_2^2 + dx_3^2) + (dr^2 + r^2 d\Omega^2) \\ B_{23} &= -\frac{\alpha'}{\Delta^2}, \qquad e^{\phi} = \hat{g}\frac{\alpha'}{\Delta^2}. \end{aligned} \tag{3}$$

To take the decoupling limit, we simply send $\alpha' \to 0$ keeping $U = r/\alpha'$ fixed. In the limit, we obtain the background

$$
\begin{aligned}
ds^2 &= \alpha' \left(\frac{U^2}{\sqrt{\lambda}}(-dt^2 + dx_1^2) + \frac{\sqrt{\lambda}U^2}{\lambda + \Delta^4 U^4}(dx_2^2 + dx_3^2) + \frac{\sqrt{\lambda}}{U^2} dU^2 + d\Omega^2 \right) \\
B_{23} &= -\frac{\alpha' \Delta^2 U^4}{\lambda + \Delta^4 U^4} \qquad e^{\phi} = \hat{g}\sqrt{\frac{\lambda}{\lambda + \Delta^4 U^4}}
\end{aligned} \tag{4}
$$

where $\lambda = 4\pi \hat{g} N$.

Starting from the supergravity solution describing D1-branes in background B_{23}, we have taken the same $\alpha' \to 0$ limit as the one used to derive the NCSYM. As a result, we ended up with a metric which has the form very much resembling the form of the $AdS_5 \times S_5$ metric which arises in the AdS/CFT correspondence for the $\mathcal{N} = 4$ SYM. Just as in the familiar cases, $\alpha' \to 0$ limit had the effect of scaling away the asymptotically flat region of space-time. Therefore, we conclude that we have constructed a supergravity description of decoupled NCSYM for large 't Hooft coupling in the spirit of AdS/CFT correspondence.

3 Physical Observables and the Holographic Principle

The AdS/CFT correspondence encodes the information about boundary field theories in the supergravity language. Finding the supergravity description for the NCSYM in the form of (4) is the first step in this program. With the explicit form of the background at hand, one can probe the physical observables of the NCSYM using the bulk/boundary correspondence. To date, boundary field theory observables that have been computed this way includes the entropy [13], the correlation functions [14, 15], and the Wilson loops [16, 17]. We will describe what one finds when one tries to compute these observables on (4).

3.1 Entropy

Thermodynamics for theories with boundary/bulk correspondences can be extracted from the bulk description of the finite temperature solution. In other words, we are interested in the generalization of (4) to a Euclidean solution with compact time direction. It is extremely straightforward to construct such a solution. All that one needs to do is to repeat the above decoupling procedure using the supergravity solution for a D1-brane at a finite temperature. When this is done, one finds

$$
\begin{aligned}
ds^2 &= \alpha' \left(\frac{U^2}{\sqrt{\lambda}}(h(U)dt^2 + dx_1^2) + \frac{\sqrt{\lambda}U^2}{\lambda + \Delta^4 U^4}(dx_2^2 + dx_3^2) + \frac{\sqrt{\lambda}}{U^2} h(U)^{-1} dU^2 + d\Omega^2 \right) \\
B_{23} &= -\frac{\alpha' \Delta^2 U^4}{\lambda + \Delta^4 U^4} \qquad e^{\phi} = \hat{g}\sqrt{\frac{\lambda}{\lambda + \Delta^4 U^4}}, \qquad h(U) = 1 - \frac{U_0^4}{U^4}.
\end{aligned} \tag{5}
$$

U_0 is the non-extremality parameter; the background reduces to (4) in the $U_0 \to 0$ limit. From this geometry, one can extract the entropy $S(T)$ as a function of temperature T using the standard method where one computes both the temperature $T(U_0)$ and the entropy $S(U_0)$ as a function of U_0.

The relation between T and U_0 follows from the requirement that there be no conical singularity in the (Ut)-plane near $U = U_0$. This requirement depends only on the form of G_{tt} and G_{UU}, but since both of these quantities are unaffected by Δ, the relation between U_0 and T is identical to the ordinary SYM.

S and U_0 are related by the standard relation $S = A_E/4$ where A_E is the area of the horizon $U = U_0$ in the Einstein frame

$$A_E = A_S e^{-2\phi(U_0)}.$$

When Δ is turned on, A_S picks up a factor of $\lambda/(\lambda + U^4\Delta^4)$. At the same time, $e^{2\phi}$ picks up a factor of $\lambda/(\lambda + U^4\Delta^4)$ because $\phi' = \phi - \frac{1}{4}\log(\det g/\det g')$ under T-duality [6]. These two effects cancel out, implying that $S(U_0)$ is also unaffected by Δ.

Since both $S(U_0)$ and $T(U_0)$ are unaffected by Δ, we find that $S(T)$ takes the standard form (ignoring the numerical coefficients)

$$S(T) = N^2 T^3 L^3$$

for large N and large λ. This conclusion could have been reached independently from the fact that to leading order in $1/N$, the only effect of the non-commutativity on planar diagrams for the n-point function of the NCSYM is to induce a phase factor depending only on the external momenta and the non-commutativity parameter $\theta_{\mu\nu}$ [18, 19]. Thermodynamic quantities are derived from partition functions which are zero-point functions, and are therefore unaffected by the non-commutativity.

3.2 Correlation Functions

Now let us consider the computation of correlation functions of gauge invariant operators using the methods of [14, 15], which amounts to considering small fluctuations of supergravity modes around the background (4). Strictly speaking one must systematically expand to quadratic order and diagonalize the Lagrangian along the lines of [20]. To simplify our discussion, we will consider a minimally coupled scalar in the background (4) and postpone the issue of identifying the diagonal fluctuations.

We are interested in computing the correlation functions for operators separated in the

x_2 direction. The minimally coupled scalar equation of interest is given by

$$\sqrt{G}G_{22}\frac{\partial^2}{\partial x_2^2}\psi(x_2,U)+\frac{\partial}{\partial U}\left(\sqrt{G}G_{UU}\frac{\partial}{\partial U}\psi(x_2,U)\right)=0$$

which for (4) simplifies to

$$\psi''_{p_2}(U)+\frac{5}{U}\psi'_{p_2}(U)-p_2^2\left(\Delta^2+\frac{\lambda}{U^4}\right)\psi_{p_2}(U),\qquad \psi(x_2,U)=e^{ip_2x_2}\psi_{p_2}(U).$$

The form of this equation is identical to that of minimal scalars propagating in the full D3-brane metric (before taking the decoupling limit) in type IIB supergravity, and have been solved in the context of computing the absorption cross-section [21]. The fact that we find the same wave equation as the full D3-geometry is a cause for alarm, since it suggests that the bulk/boundary correspondence for the NCSYM is as subtle as the bulk/boundary correspondences in asymptotically flat spaces [22]. Since the background (4) is not asymptotically AdS, new rules for extracting the holographic data need to be invented. The authors of [2] proposed to go back to the early days of gauge theory/supergravity correspondences [23, 24] and to interpret the absorption cross section directly as the correlation function in momentum space. This has the nice feature that the UV singularity of correlation function is smeared away (due to the fact that absorption cross sections are bounded between zero and one) while keeping the IR intact. This proposal however is only a well motivated "guess" and the interpretation deserves to be understood at a more fundamental level. The nature of the boundary/bulk correspondences as probed by correlation function is likely to be closely related to the similar issues in flat space-times and little string theories [25].

3.3 Wilson Loops

We will now describe the computation of Wilson loops. Let us begin by briefly reviewing the computation of quark potentials at strong coupling using the minimal surface technique. The basic idea stems from the fact that a quark can be represented in the AdS space as a string stretching from the boundary to the horizon. When a quark and an anti-quark are brought close to each other, these strings can lower their energies by joining together at the ends near the horizon and becoming a "U" shaped configuration of strings suspended from the boundary of AdS space with its ends separated by some distance which we interpret as the distance between the quark and the anti-quark (see figure 2).

In more concrete terms, suppose we wish to consider a minimal surface corresponding to quarks separated in the x_2 direction. The minimal surface will be parameterized by x_0 and x_2 and determined in terms of the curve $U(x_2)$. The energy of such a configuration is given

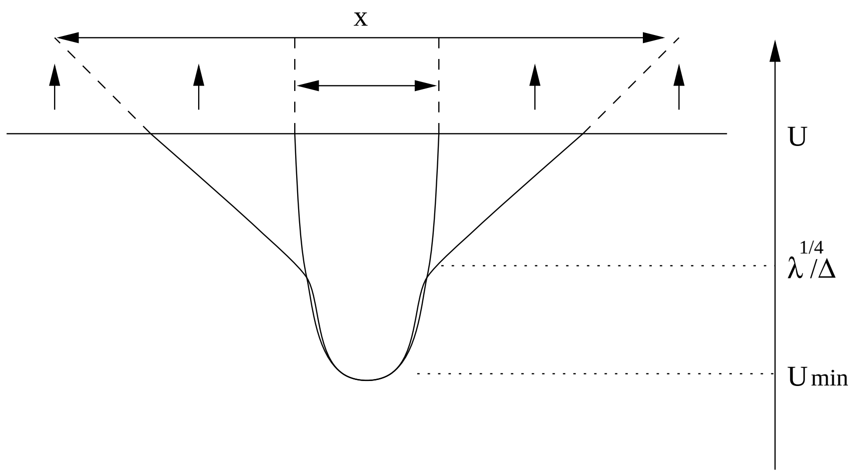

Figure 2: Typical configuration of minimal surfaces for fixed U_{min} which arises in the computation of quark anti-quark potential for ordinary and non-commutative SYM.

by the Nambu-Goto action

$$S = \frac{1}{\alpha'}\sqrt{-G_{00}(G_{22} + G_{UU}(\partial U(x_2))^2)} = \sqrt{(\partial U(x_2))^2 + \frac{U^4}{\lambda}} \tag{6}$$

which satisfies a simple conservation law

$$\text{fixed} = \frac{U^4/\lambda}{\sqrt{\partial U^2 + U^4/\lambda}} = \frac{U_{min}^2}{\sqrt{\lambda}} \tag{7}$$

owing to the absence of explicit dependence on x_2. U_{min} is the minimum U to which the string dips in the AdS bulk space. This relation can be solved for ∂U, which in turn can be used to relate U_{min} to x_2:

$$x_2 = \int_{U_{min}}^{\infty} \frac{dx}{dU} dU = \sqrt{\frac{\lambda U_{min}^4}{U^4(U^4 - U_{min}^4)}} \sim \frac{\sqrt{\lambda}}{U_{min}}. \tag{8}$$

Similarly, one can eliminate ∂U from the Nambu-Goto action and relate E to U_{min}:

$$E = \int_{U_{min}}^{\infty} \left(2\sqrt{\frac{U^4}{U^4 - U_{min}^4}} - 2\right) \sim U_{\mathrm{min}}. \tag{9}$$

The "-2" here corresponds to the subtraction of the quark self energies. Combining (8) and (9), we arrive at the standard coulombic potential

$$E \sim \frac{\sqrt{\lambda}}{x} \tag{10}$$

proportional to $\sqrt{\lambda}$.

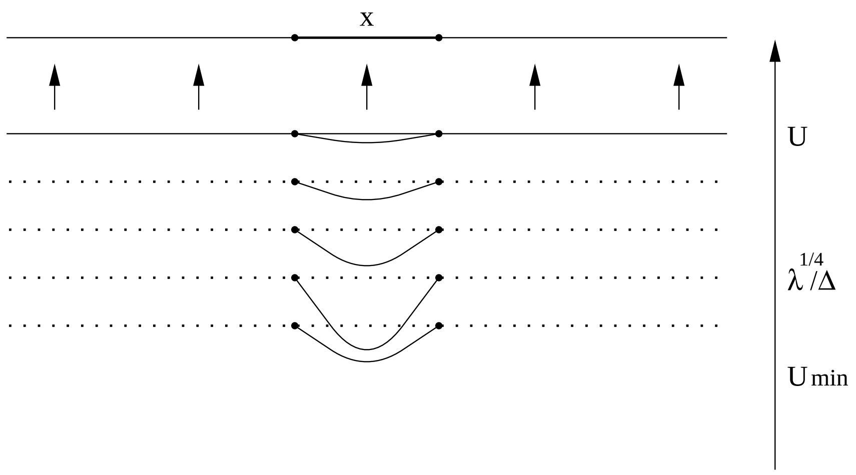

Figure 3: Configurations of minimal surfaces keeping x fixed (instead of U_{min}) at the cut-off scale U, and their behavior as U is sent to infinity.

At a purely mechanical level, it is trivial to repeat the analysis for the background (4) corresponding to the non-commutative case. Solving for $x_2(U_{min})$ this time gives rise to an integral expression

$$x_2 = \int_{U_{min}}^{\infty} \frac{U_{min}^2(\lambda + \Delta^4 U^4)}{U^2\sqrt{\lambda(U^4 - U_{min}^4)}}. \tag{11}$$

The integrand approaches $\Delta^4 U_{min}^2/\sqrt{\lambda}$ for large U, indicating that the integral diverges linearly. The configuration of the string world sheet exhibiting this behavior is illustrated in figure 2. The authors of [2] proposed computing the quark potential as a function of U_{min} and then to set $U_{min} \sim 1/x_2$ in order to recover (10) at large x_2. However, it is not a priori clear why or if the monomial relation between U_{min} and x_2 at large x_2 should persist to small x_2.

We would now like to propose an alternative interpretation for the divergence of $x_2(U_{min})$ when U_{min} is finite. The resolution is quite simple. in order to have x_2 finite, U_{min} must be infinite.

One can arrive at the same conclusion more systematically by considering the geodesic of strings ending on a surface of constant U and with endpoints separated by x_2. This is precisely the holographic realization of computing the quark potential with a UV cut-off at U. Now, imagine sending U to infinity keeping x_2 fixed. This process is illustrated in figure 3 and in the limit, one obtains a configuration of string world sheet stretching along a straight line at the boundary. Unlike in the case of AdS spaces, minimal surfaces do not like to penetrate the bulk of the large U region of (4). Similar "exclusion" of probe minimal surfaces have been encountered previously in the context of flat spacetimes [26, 27].

It is extremely straight forward to compute the energy for such a configuration

$$E = \int_0^x dx \, \frac{1}{\alpha'} \sqrt{G_{00} G_{22}} \Big|_{U \to \infty} = \frac{x}{\Delta^2} \tag{12}$$

using the background (4). This shows that the potential is linearly confining with the tension $1/\Delta^2$. This is an unexpected result; it does not asymptote to the usual SYM result in $\Delta \to 0$ limit or the large x limit.

4 Discussions

In the previous section we described the computation of various physical observables in the NCSYM using the AdS/CFT correspondence. The computation $U(N)$ of the entropy, which does not probe any local structure in non-commutative spaces, resulted in a sensible, yet uninteresting result identical to ordinary SYM. Correlation functions of local operators and quark anti-quark potentials are more interesting in that they actually probe the non-commutativity of space-time. Yet, these calculations only yielded strange, if not uninterpretable, results. The computation of correlation functions suggested that holography for the NCSYM and holography for flat space-times are related concepts. Since we do not really understand the latter, we can not understand the former either. The quark anti-quark potential also seem to give a strange answer which do not have a smooth $\Delta \to 0$ limit. We will conclude this note by proposing an interpretation for these pathologies.

To appreciate the correct interpretation of the observables computed in the previous section, it is useful to go back and examine the web of equivalences illustrated in figure 1. For concreteness let us enumerate the entries of figure 1:

A : D1 in the torus of size R
B : D2 in the slated torus
C : D3 in the torus of size $\alpha' \Sigma / \Delta^2$
D : Non-commutative SYM on torus of size Σ.

The point of figure 1 was to illustrate the fact that all descriptions are equivalent, in the sense that physical information contained in one description is encoded also in the other descriptions. The precise form of the encoding, however, can be very complicated. In particular, the equivalence between the **C** and **D** was the topic of [8], and the authors of that paper provided an explicit expression for first few terms of the map of the degrees of freedom as an expansion with respect to the non-commutativity parameter $\theta_{\mu\nu}$. The resummation of this series is not yet understood.

The supergravity background (4) was derived by T-dualizing twice the supergravity solution corresponding to D1-branes in a background B-field. By construction, this amounts to looking at the near horizon region of **C**. This could have been inferred also by looking at the geometry (3) in the asymptotically flat region prior to taking the decoupling limit. We will therefore refer to supergravity description (4) as **C'**. When we probe local objects in the non-commutative directions, we are probing local objects in a box of size $\alpha'\Sigma/\Delta^2$. In $\alpha' \to 0$ limit, the nature of such an observable, as well as the mapping between **C** and **D**, is bound to be pathological.

The bottom line appears to be that calculations which are natural on **C'** do not correspond to anything natural or nice on **D**. In order to compute the natural observables on **D**, one must first encode that observable in the language of **C'**, and then calculate it using techniques applicable for **C'**. This is a tall order since bulk/boundary correspondences are not currently powerful enough to compute arbitrary observables. We were lucky in quantities like entropy because these quantities do not probe any localized structures but rather sums over them indiscriminately. To probe localized structures in the NCSYM using supergravity, perhaps one should look for a more direct approach: invent a **D'**.

Acknowledgements

I would like to thank Sunny Itzhaki for the collaboration on which this lecture was based. I also thank the organizers and the participants of SI99 for providing a stimulating environment where some of the material presented in this note was developed. I am also grateful to D. Gross, K. Hashimoto, A. Kato, H. Ooguri, S. J. Rey, S. Shenker, E. Silverstein, and L. Susskind for illuminating discussions. This work was supported in part by the National science Foundation under Grant No. PHY94-07194.

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